YEAR 9 – REASONING WITH NUMBER...

Numbers

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative

Rational: a number that can be made by dividing two integers

Irrational: a number that cannot be made by dividing two integers

Inverse operation: the operation that reverses the action

Quotient: the result of a division

Product: the result of a multiplication

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat.

The square root of a negative is not a real number and cannot be found

HCF/LCM R

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, (6), 9, 18
30	1, 2, 3, 5, (6), 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, (36), 45, 54
12	12, 24, (36), 48, 60

LCM = 36

The first time their multiples match

Standard form R

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

$6 \times 10^5 + 8 \times 10^5$

= 600000 + 800000

= 1400000

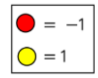
= 1.4×10^5

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \quad \times 10^5 \div 10^3$

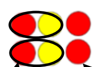
= 5×10^2

Directed number R



Addition

2 + - 4 = -2

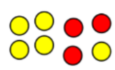


Zero pair (-1 + 1 = 0)

Two " – 1 " left = -2

Generalisation: + – = –

Subtraction



= 2

Representation for calculation

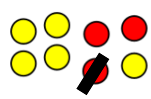
Generalisation: – – = +

"Subtract" – means take away or remove

2 - - 1 = 3

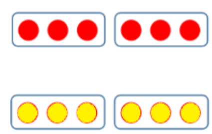
Take away one

Start with the representation of 2



= 3

Multiplication



The act of making counters into their negative is turning them over

-2 x -3 = 6

Divisions are the inverse operations

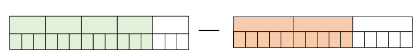
a = 5 b = -4

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = \underline{14}$

Addition/ Subtraction of fractions R

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplication/ Division of fractions R

Shade in 3 parts → $\frac{3}{4} \times \frac{2}{3}$ ← Repeat it on this many rows

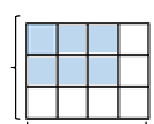
This many columns | This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded (6)

Total number of parts in the diagram (12)

Modelled:



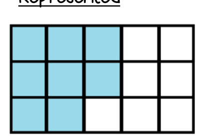
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 9 – REASONING WITH NUMBER...
Using Percentages

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What do I need to be able to do?

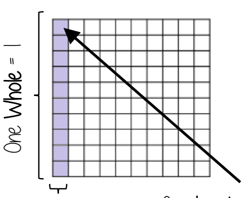
By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 – written using the % symbol
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).
Multiplier: the number you are multiplying by.
Profit: the income take away any expenses/ costs.

FDP Equivalence (R)



Percentage
100% = a whole = 100 hundredths

10 hundredths
10 out of 100
10%

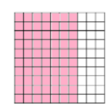
$\frac{10}{100} = \frac{1}{10} = 0.10$

One hundredth
(one whole split into 100 equal parts)

ones	tenths	hundredths
.	●	

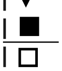
Converting FDP (R)

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares, 70 "hundredths" = 7 "tenths", 0.7 →



→ 70 hundredths = 70%

Using a calculator

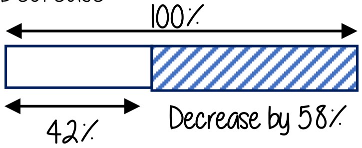


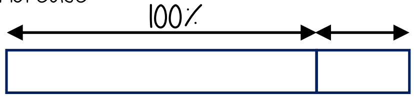
→ S⇌D Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333
= $0.\dot{3}$
The dot above the 3

Percentage Increase/ Decrease (R)



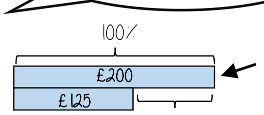


Multiplier
Less than 1
1.00 – 0.58 = 0.42

100% + 12% = 112%
1.00 + 0.12 = 1.12
Multiplier
More than 1

Percentage change (R)

I bought a phone for £200.
A year later sold it for £125.



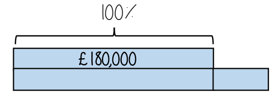
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

$$\frac{\textit{Difference in values}}{\textit{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000.

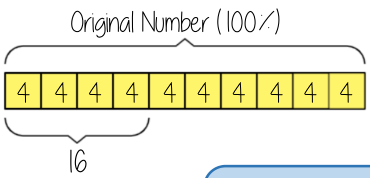


Percentage profit

Money made (profit value) → $\frac{36000}{180000} \times 100 = 20\%$

Reverse Percentages

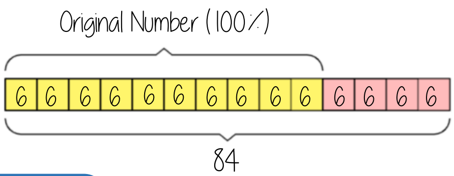
40% of my number is 16. What am I thinking of?



40% = 16
10% – 4
100% = 40

Try to scale down to 10% or 1% and then scale back up to 100%

140% of my number is 84. What is the original number?



140% = 84
10% – 6
100% = 60

YEAR 9 – REASONING WITH NUMBER… Maths & Money

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What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/ outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses.
Unitary: one – the cost of one.

Bills and Bank Statements

Bills – tell you the amount items cost and can show how much money you need to pay

Some can include a total

Look for different units (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Setp	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same

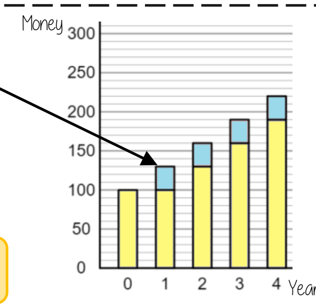
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account.

e.g. Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest. At the end of year 4 they have **£220**



Compound Interest

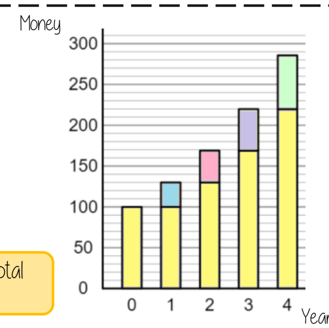
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g. Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

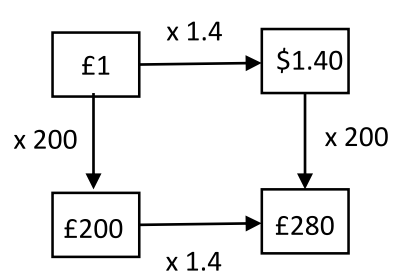
Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half – means 1.5 times their hourly rate
Double – 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies		
United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges £1	5 cupcakes £1.20

$4 = £1.00$ $\div 2$
$2 = £0.50$ $\div 2$
$1 = £0.25$

$5 = £1.20$ $\div 5$
$1 = £0.20$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

YEAR 9 – REASONING WITH GEOMETRY...

Deduction

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What do I need to be able to do?

By the end of this unit you should be able to:

- Identify angles in parallel lines
- Solve angle problems
- Make conjectures with angles
- Make conjectures with shapes

Keywords

Parallel: two straight lines that never meet with the same gradient.
Perpendicular: two straight lines that meet at 90°
Transversal: a line that crosses at least two other lines.
Sum: the result of adding two or more numbers.
Conjecture: a statement that might be true but is not proven.
Equation: a statement that says two things are equal
Polygon: a 2D shape made from straight edges.
Counterexample: an example that disproves a statement

Alternate angles

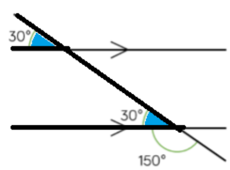
R

Because alternate angles are equal the highlighted angles are the same size

Corresponding angles

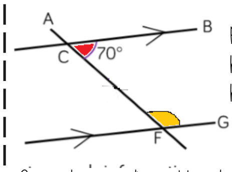
R

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

R



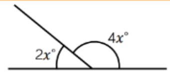
Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Solving angle problems

Angles on a straight Line
180°

Link angle facts to algebra



Form an equation

$$2x + 4x = 180°$$

State the reason

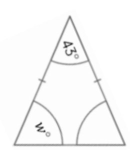
The sum of angles on a straight line is 180°

Solve

$$2x + 4x = 180°$$
$$6x = 180°$$
$$x = 30°$$

Vertically opposite angles
Equal

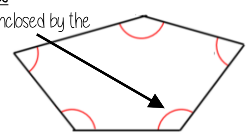
Angles around a point
360°



Triangles
Sum of angles is 180°

Isosceles have the same base angles

Interior Angles
The angles enclosed by the polygon



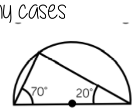
(number of sides – 2) x 180

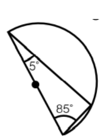
Making conjectures with angles

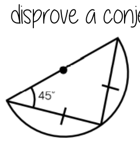
True
Always
Never
False
Sometimes

Proving a conjecture
A pattern is noticed for many cases

Disproving a conjecture
Only one counterexample is needed to disprove a conjecture







Apply the angle rules

The sum of angles in a triangle is 180°

Test the theory

$180 - 70 - 20 = 90$
$180 - 85 - 5 = 90$
$180 - 45 - 45 = 90$

Make conjecture

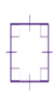
The angle that meets the circumference in a semi circle is 90°

Making conjectures with shapes

Keywords and facts to recall with shape

Area: the amount of space inside a shape
Perimeter: the length around a shape
Regular Polygons: All sides and angles are equal

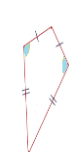
Quadrilateral Facts



Square
All sides equal size
All angles 90°
Opposite sides are parallel

Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel



Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Rhombus
All sides equal size
Opposite angles are equal

YEAR 9 – REASONING WITH GEOMETRY...

Reflection, rotation and translation (part 1)

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

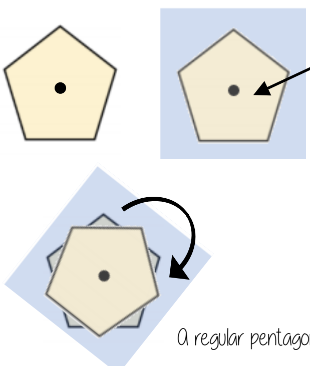
- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

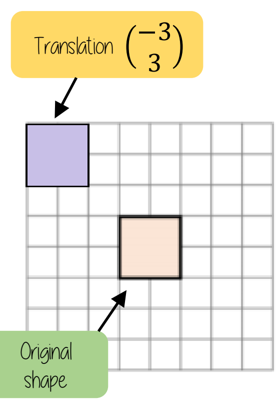
A regular pentagon has rotational symmetry of order 5

Translation and vector notation

Vector Notation → $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

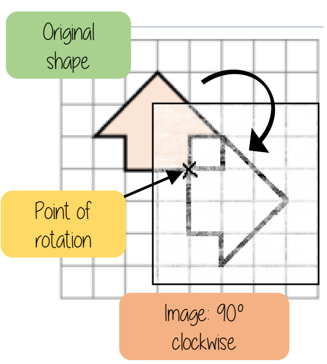
How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

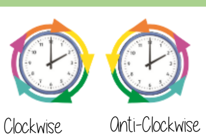


Every vertex has been translated by the same amount

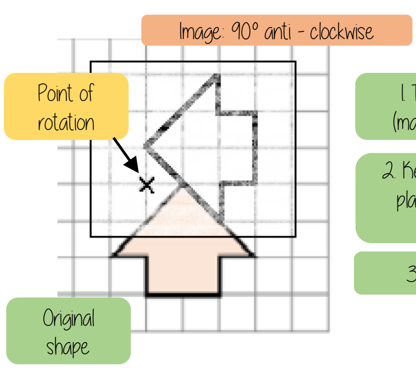
Rotate from a point (in a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

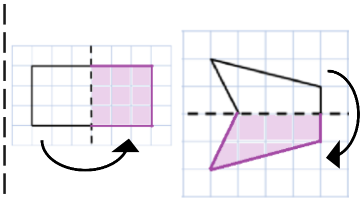


Rotate from a point (outside a shape)



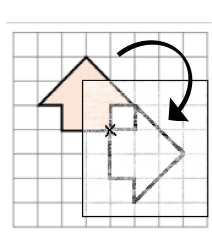
1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

Compare rotations and reflections



R Reflections are a mirror image of the original shape.

Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

Year 9 - Developing geometry

Reflection, rotation and translation (part 2)

What do I need to be able to do?

By the end of this unit you should be able to:

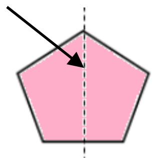
- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

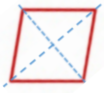
Keywords

Mirror line: a line that passes through the center of a shape with a mirror image on either side of the line
Line of symmetry: same definition as the mirror line
Reflect: mapping of one object from one position to another of equal distance from a given line.
Vertex: a point where two or more-line segments meet.
Perpendicular: lines that cross at 90°
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)

Lines of symmetry

Mirror line (line of reflection)



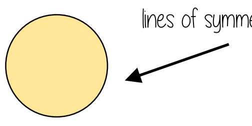


Rhombus
two lines of symmetry

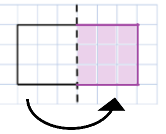
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry….
This regular polygon (a regular pentagon has 5 lines of symmetry)

A circle has an infinite amount of lines of symmetry

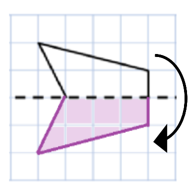


Reflect horizontally/ vertically (1)



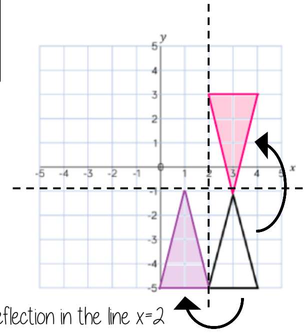
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

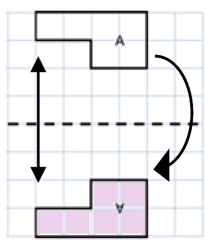


Reflection in the line y=-2

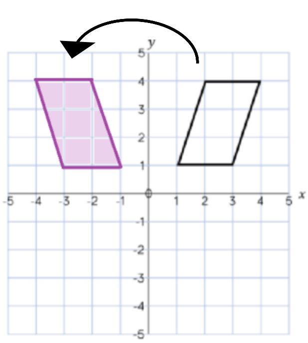
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



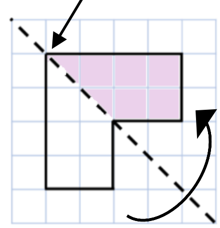
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

Reflect Diagonally (1)

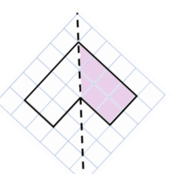
Points on the mirror line don't change position

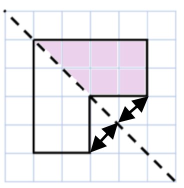


Fold along the line of symmetry to check the direction of the reflection

Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)



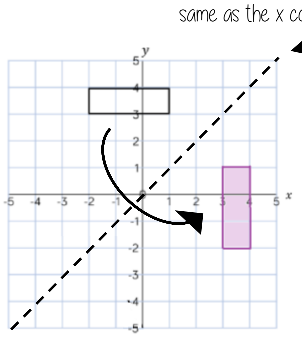


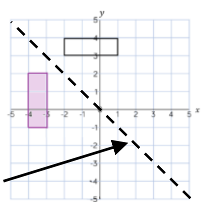
Drawing perpendicular lines

Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

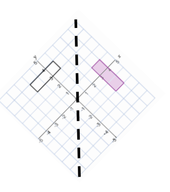
Reflect Diagonally (2)

This is the line **y = x** (every y coordinate is the same as the x coordinate along this line)





This is the line **y = - x**
The x and y coordinate have the same value but opposite sign



Turn your image

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YEAR 9 – REASONING WITH GEOMETRY...

Pythagoras' theorem

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

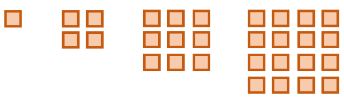
- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself
Square root: a value that can be multiplied by itself to give a square number
Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.
Opposite: the side opposite the angle of interest
Adjacent: the side next to the angle of interest

Squares and square roots

R



This can also be written as 6^2

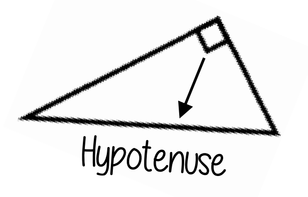
$\sqrt{\ }$ is the square root symbol

eg $\sqrt{64} = 8$
Because $8 \times 8 = 64$

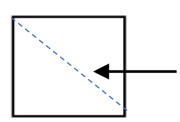
1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

Square numbers

Identify the hypotenuse

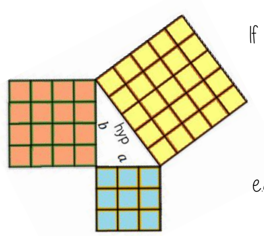


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

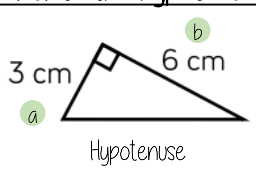
eg $a^2 + b^2 = hypotenuse^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$3^2 + 6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

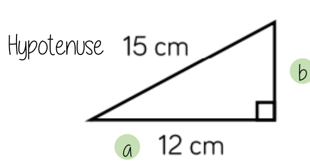
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$

$\mathbf{6.71\,cm} = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2 + b^2 = 15^2$

1. Substitute in the values you are given

$144 + b^2 = 225$
$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

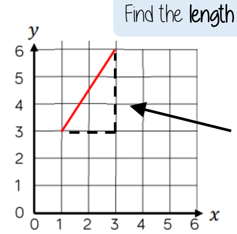
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\,cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes