YEAR 9 – REASONING WITH NUMBER...

Numbers

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative
Rational: a number that can be made by dividing two integers
Irrational: a number that cannot be made by dividing two integers
Inverse operation: the operation that reverses the action
Quotient: the result of a division
Product: the result of a multiplication
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2 : 1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

HCF/LCM (R)

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18
30: 1, 2, 3, 5, 6, 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54
12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Standard form (R)

Any number between 1 and less than 10 × 10^{n} ← Any integer

$6 \times 10^5 + 8 \times 10^5$

$= 600000 + 800000$

$= 1400000$

$= 1.4 \times 10^5$

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$15 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

Directed number (R)

Addition

$2 + -4 = -2$

Zero pair ($-1 + 1 = 0$)

Two "–1" left $= -2$

Generalisation: + − = −

Subtraction

$= 2$

Representation for calculation

"Subtract" – means take away or remove

Generalisation: − − = +

$2 - -1 = 3$

Take away one

$= 3$

Start with the representation of 2

Multiplication

= −1
= 1

The act of making counters into their negative is turning them over

$-2 \times -3 = 6$

Divisions are the inverse operations

a = 5 b = −4

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

Addition/ Subtraction of fractions (R)

$\frac{4}{5} - \frac{2}{3}$

$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplication/ Division of fractions (R)

$\frac{3}{4} \times \frac{2}{3}$

Shade in 3 parts

Repeat it on this many rows

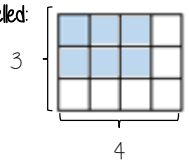
This many columns

This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ ← Parts shaded

Total number of parts in the diagram

Modelled:



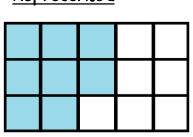
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

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Using Percentages

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What do I need to be able to do?

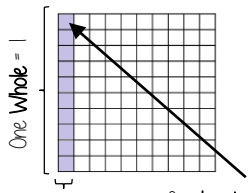
By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 – written using the % symbol
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).
Multiplier: the number you are multiplying by.
Profit: the income take away any expenses/ costs.

FDP Equivalence (R)



$\frac{10}{100} = \frac{1}{10} = 0.10$

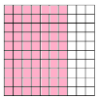
One **hundredth** (one whole split into 100 equal parts)

Percentage
100% = a whole = 100 hundredths

10 hundredths
10 out of 100
10%

ones	tenths	hundredths
	●	●

Converting FDP (R)

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares → **70 "hundredths"** = **7 "tenths"** = **0.7** →  → 70 hundredths = 70%

Using a **calculator**

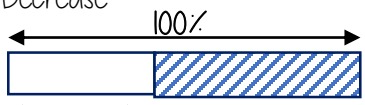
$\frac{\blacksquare}{\square}$ → S⇔D Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
e.g. $\frac{1}{3} = 0.3333333$
$= 0.\dot{3}$
The dot above the 3

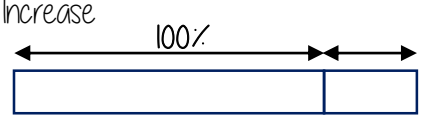
Percentage Increase/ Decrease (R)

Decrease



100 – 0.58 = 0.42 ← **Multiplier** Less than 1

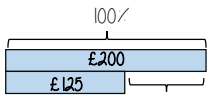
Increase



100% + 12% = 112%
100 + 0.12 = 1.12 ← **Multiplier** More than 1

Percentage change (R)

I bought a phone for £200. A year later sold it for £125



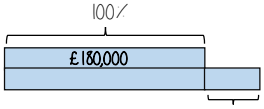
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

$$\frac{\textit{Difference in values}}{\textit{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000



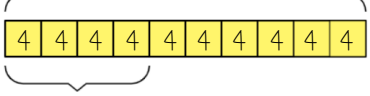
Percentage profit

Money made (profit value) → $\frac{36000}{180000} \times 100 = 20\%$

Reverse Percentages

40% of my number is 16. What am I thinking of?

Original Number (100%)

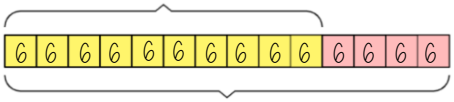


16

40% = 16
10% = 4
100% = 40

140% of my number is 84. What is the original number?

Original Number (100%)



84

140% = 84
10% = 6
100% = 60

Try to scale down to 10% or 1% and then scale back up to 100%

YEAR 9 – REASONING WITH NUMBER...

Maths & Money

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What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/ outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses.
Unitary: one – the cost of one.

Bills and Bank Statements

Bills – tell you the amount items cost and can show how much money you need to pay.

Some can include a total

Look for different units (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Sept	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same

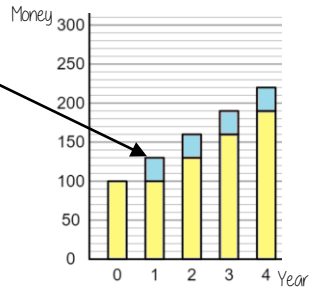
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account

e.g Invest £100 at 30% simple interest for 4 years

$\frac{100 \times 30 \times 4}{100} = £120$

This account earned **£120** interest
At the end of year 4 they have **£220**



Compound Interest

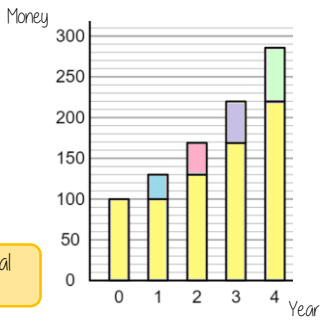
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$100 \times 1.3^4 = £285.61$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

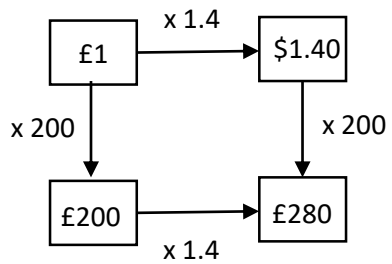
Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half – means 1.5 times their hourly rate
Double – 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies

United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges £1	5 cupcakes £1.20

$4 = £1.00$ ÷ 2 → $2 = £0.50$ ÷ 2 → $1 = £0.25$

$5 = £1.20$ ÷ 5 → $1 = £0.20$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

More to folllow