YEAR 8 - ALGEBRAIC TECHNIQUES...

Brackets, Equations & Inequalities

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

Simplify: grouping and combining similar terms
Substitute: replace a variable with a numerical value
Equivalent: something of equal value
Coefficient: a number used to multiply a variable
Product: multiply terms
Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)
Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

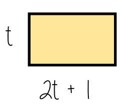
Form expressions

For unknown variables, a letter is normally used in its place

More than – ADD
Less than/ difference – SUBTRACT

eg 4 more than t ⟶ $t + 4$
8 less than k ⟶ $k - 8$

Only similar terms can be grouped together



eg Find the perimeter of this shape
(Perimeter = length around outside of shape)

$t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$

Directed numbers

$+\ + \longrightarrow +$

$-\ - \longrightarrow +$

$+\ - \longrightarrow -$

$-\ + \longrightarrow -$

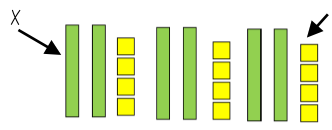
eg $a = -5$ and $b = 2$
$a^2 = a \times a = -5 \times -5 = 25$
$b + a = 2 + -5 = -3$

Multiply single brackets

$3(2x + 4)$

	2x	4
3	3 x 2x 6x	3 x 4 12

$6x + 12$



2x + 4			2x + 4			2x + 4		
x	x	4	x	x	4	x	x	4
6x + 12								

Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket

$8x + 4$

	2x + 1
4	8x + 4

⟵ Try and make this the **highest common factor**

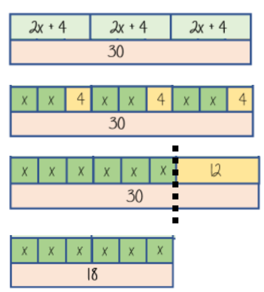
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Solve equations with brackets

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \qquad -12$

Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$
$\div 6 \qquad \div 6$

$x = 3$

Simple Inequalities

$<$ less than
$\leq$ Less than or equal to
$>$ More than
$\geq$ More than or equal to

$x < 10$
Say this out loud
"x is a value less than 10"

$10 > x$
Say this out loud
"10 is more than the value"

Note:
$x < 10$ and $10 > x$ represent the same values

$x + 2 \leq 20$
"my value + 2 is less than or equal to 20"
$x \leq 18$
The biggest the value can be is 18

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

Form

$x \longrightarrow \times 3 \longrightarrow +2 \longrightarrow 11$

$\boxed{3x + 2 > 11}$

Solve

$x \longleftarrow \div 3 \longleftarrow -2 \longleftarrow 11$

$\boxed{x > 3}$

Check

This would suggest any value bigger than 3 satisfies the statement

$3 \times 3 + 2 = 11$ ✓ $\qquad 10 \times 3 + 2 = 32$ ✓

Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term
A single number or variable

Identity
An equation where both sides have variables that cause the same answer includes $\equiv$

Formula
A rule written with all mathematical symbols
eg area of a rectangle $A = b \times h$

YEAR 8 - ALGEBRAIC TECHNIQUES...

Sequences

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What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Linear: the difference between terms increases or decreases (+ or -) by a constant value each time
Non-linear: the difference between terms increases or decreases in different amounts, or by x or ÷
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

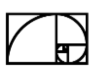
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

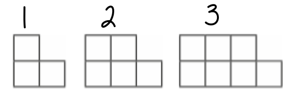
Each term is the sum of the previous two terms



Sequence in a table and graphically

Position: the place in the sequence

1 2 3



3 5 7

"The term in position 3 has 7 squares"

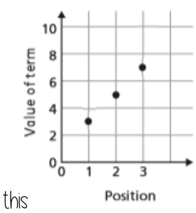
Term: the number or variable (the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ → Substitute the number of the term you are looking for in place of 'n'

eg

1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$3n - 4 = 201$

Algebraic rule

Term to check

Solving this will find the position of the term in the sequence.
ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

eg

1st term = $2 \times 1^2 = 2$
2st term = $2 \times 2^2 = 8$
100th term = $2 \times 100^2 = 2000$

$(2n)^2$

2 times n then square the answer

eg

1st term = $(2 \times 1)^2 = 4$
2st term = $(2 \times 2)^2 = 16$
100th term = $(2 \times 100)^2 = 40000$

$n(n + 5)$ ←

eg

1st term = 1 (1 + 5) = 6
2st term = 2 (2 + 5) = 14
100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20...

$4n$

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 8 - ALGEBRAIC TECHNIQUES...

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Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

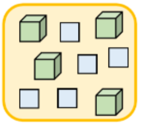
Keywords

Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Coefficient: The number used to multiply a variable
Simplify: To reduce a power to its lowest term
Product: Multiply

Addition/ Subtraction with indices

Coefficient Power

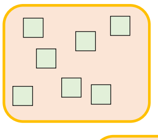
$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}}$ →



Each square represents x^2 and each cube represents x^4

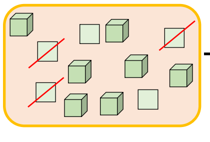
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →



→ $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →



→ $2x^2 + 7x^4$

Divide expressions with indices

$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$

$\frac{5a^3b^2}{15ab^6} \rightarrow \underbrace{\frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \rightarrow \frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$3^5 \times 3^2 \rightarrow 3^7$

$= \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)}$

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

YEAR 8 - DEVELOPING NUMBER...
Fractions & Percentages

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What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between FDP less than and more than 100.
- Increase or decrease using multipliers.
- Express an amount as a percentage.
- Find percentage change.

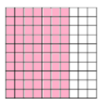
Keywords

Percent: parts per 100 – written using the % symbol
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).

Convert FDP

R

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares → 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator

S⇔D

This will give you the answer in the simplest form

Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount

R

Find $\frac{3}{5}$ of £60

£60

£12	£12	£12	£12	£12

£36

Remember $\frac{3}{5}$ = 60%

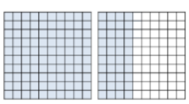
10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember
$\frac{3}{5}$ = 60% = 0.6

60% of £60
= 0.6 x 60
= £36

Convert FDP < and > 100%

100 hundredths
10 tenths
100%



40 hundredths
4 tenths
40%

140 hundredths
14 tenths
140%

100%+40%
1 + 0.40
= 1.40

Percentage decrease: Multipliers

100%

42%

Decrease by 58%

100% - 58% = 42%
1.00 – 0.58 = 0.42

Multiplier
Less than 1

Percentage increase: Multipliers

100% 12%

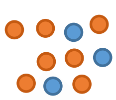
Increase by 12%

100% + 12% = 112%
1.00 + 0.12 = 1.12

Multiplier
More than 1

Express as a % - Non-calculator

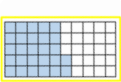
Percent – per hundred



7 per every 10 are orange
$\frac{7}{10}$

This means that 70 per every 100 are orange
$\frac{70}{100}$

70%



27 per every 50 shaded
$\frac{27}{50}$

54 per every 100 shaded
$\frac{54}{100}$

54%

Denominator 100

Equivalent fractions

Express as a % - Calculator

Rosie

$\frac{13}{30}$ → ■/□ → $\frac{13}{30}$ → × 100 → 43.3333....%
43%

Can't use equivalence easily to find 'per hundred'

This the same as 13 ÷ 30

Decimal percentages are still a percentage

Percentage change

I bought a phone for £200. A year later sold it for £125

100%

£200
£125

All values of change compare to the ORIGINAL value

Percentage loss
$\frac{75}{200}$ × 100 = 37.5%

$\frac{\text{Difference in value}}{\text{Original value}}$ × 100

I bought a house for £180,000, I later sold it for £216,000

100%

£180,000

Percentage profit

Money made (profit value) $\frac{36000}{180000}$ × 100 = 20%

Choose appropriate method

The language and wording of the question is the key

Have you represented the question in a bar model?
Can you use a calculator?

YEAR 8 - DEVELOPING NUMBER...

Standard Form

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What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard from
- Multiply/ Divide with standard form
- Use a calculator with standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example

3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Negative powers of 10

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	•	10^{-1}	10^{-2}	10^{-3}
0	0	•	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	•	10^{-1}	10^{-2}	10^{-3}
0	•	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	•	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= 0.5 \times 10^3$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$$6 \times 10^5 + 8 \times 10^5$$

Method 1

$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^5$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$
$= 14 \times 10^5$
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^5$

This is not the final answer

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press $\boxed{\times 10^x}$ Then press 5 (for the power)
Press $\boxed{\times}$
Input 3.9 and press $\boxed{\times 10^x}$ Then press 3 (for the power)
Press $\boxed{=}$

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press $\boxed{\text{SHIFT}}$ $\boxed{\text{SETUP}}$ and then press 7 for sci mode

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

YEAR 8 - DEVELOPING NUMBER...

Number Sense

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Round numbers to powers of 10 and 1 sf
- Round numbers to any dp
- Estimate solutions
- Calculate using order of operations
- Calculate with money, units of measurement and time

Keywords

Significant: Place value of importance
Round: Making a number simpler but keeping its value close to what it was.
Decimal: Place holders after the decimal point.
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value.
Metric: A system of measurement.
Balance: The amount of money in a bank account
Deposit: Putting money into a bank account.

Round to powers of 10 and 1 sig figure

R

If the number is halfway between we "**round up**"

5495 to the nearest 1000

5000 — 6000

5475 to the nearest 100

5400 — 5500

5475 to the nearest 10

5470 — 5480

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

Round to decimal places

2.46192 ← Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal
"To 2dp" – to two numbers after the decimal

2.46192 (to 1dp) - Is this closer to 2.4 or 2.5

2.4 — 2.5

2.4 | 6192 This shows the number is closer to 2.5

2.46192 (to 12dp) - Is this closer to 2.46 or 2.47

2.46 — 2.47

2.46 | 192 This shows the number is closer to 2.46

Estimate the calculation

Round to 1 significant figure to estimate

$4.2 + 6.7 \approx 4 + 7 \approx 11$ This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

$21.4 \times 3.1 \approx 20 \times 3 \approx 60$ This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Order of operations

R

Brackets Operations in brackets are calculated first

Other operations e.g. powers, roots,

Multiplication/ Division

They are carried out in the order from left to right in the question

Addition/ Subtraction

They are carried out in the order from left to right in the question

Calculations with money

Debit - You have £0 or more in an account

Credit - You have less than £0 in an account

Money calculations are to 2dp

Using a calculator – ensure you are working in the correct units.

£1.30 + 50p = 130 + 50 (in pence)
= 1.30 + 0.50 (in pounds)

£1 = 100p

Units are important:

Useful Conversions

$mm \xrightarrow{\times 10} cm \xrightarrow{\times 100} m \xrightarrow{\times 1000} km$; $km \xrightarrow{\div 1000} m \xrightarrow{\div 100} cm \xrightarrow{\div 10} mm$

$g \xrightarrow{\times 1000} kg \xrightarrow{\times 1000} T$; $T \xrightarrow{\div 1000} kg \xrightarrow{\div 1000} g$

$ml \xrightarrow{\times 1000} L$; $L \xrightarrow{\div 1000} ml$

Metric measures of length

Kilo = 1000 x meter

Centi - $\frac{1}{100}$ x meter

Milli - $\frac{1}{1000}$ x meter

Units of weight/ capacity

Weight = g, kg, t
Capacity (volume of liquid) = ml, L

Time and the calendar

1 Year – the amount of time it takes Earth to go around the sun 365 (and a quarter) **days**

Leap Year – **366 days** (every 4 years)

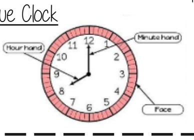
12 Months = one year = 52 weeks
31 days – **Jan, March, May, July Aug, Oct, Dec**
30 days – **April, June, Sept, Nov**
28 days – **Feb** (29 leap year)

1 week – 7 days
Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

1 day – 24 hours
1 hour - 60 minutes
1 minute – 60 seconds

Use a number line for time calculations!

Analogue Clock



12-hour clock
- Use am (morning) and pm (afternoon)
- Only use hour times up to 12

Digital Clock (24-hour times)

24-hour clock
- 0-11 (morning hours)
- 12-23 (afternoon hours)