YEAR 10 – DELVING INTO DATA...

@whisto_maths

Collecting, representing and interpreting

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon, two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

Keywords

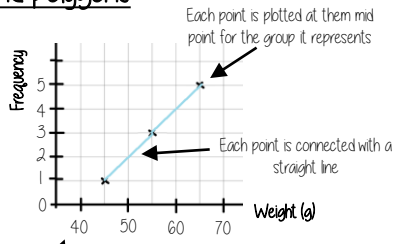
Population: the whole group that is being studied
Sample: a selection taken from the population that will let you find out information about the larger group
Representative: a sample group that accurately represents the population
Random sample: a group completely chosen by change. No predictability to who it will include.
Bias: a built-in error that makes all values wrong by a certain amount
Primary data: data collected from an original source for a purpose.
Secondary data: data taken from an external location. Not collected directly.
Outlier: a value that stands apart from the data set

Frequency tables and polygons

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5

We do not know from grouped data where each value is placed so have to use an estimate for calculations

MID POINTS
Mid-points are used as estimated values for grouped data. The middle of each group



Mid-point
Start point + End point
2

Two way tables R

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

Extract information to input to the two-way table

Subgroups each have their own heading

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Needs subgroup totals

Overall total

Draw and interpret Pie Charts R

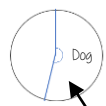
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw. This is 192°

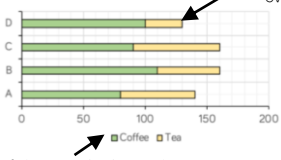
Multiple method
60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts:
You NEED the overall frequency to make any comparisons

Bar and line charts

Compare the bars green compared to yellow. The size of each bar is the frequency. Overall total easily comparable

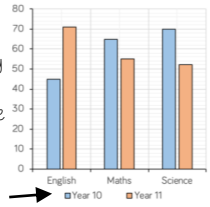
Composite bar charts



Categories clearly indicated

Dual bar charts

Bars are compared side by side
Easier to compare subgroups



Categories clearly indicated

Averages from a table R

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5

Mid Point	MP x Freq
45	45
65	195
65	325

Overall Frequency: 9
Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Averages from lists R

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

$55 \div 5$

Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order: 4, 8, 8, 11, 24

Find the value in the middle: 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group – which group has the highest frequency

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Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4 Means 14 years old

Stem and leaf diagrams:
Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

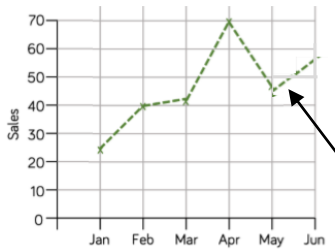
15 | 3,
Means 153 cm tall

Back to back stem and leaf diagrams
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

R

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

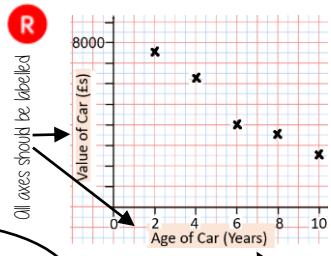
Mean, mode, median – allows for a comparison about more or less average
Range – allows for a comparison about reliability and consistency of data

Draw and interpret a scatter graph

R

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship



All axes should be labelled

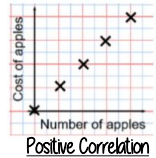
The axis should fit all the values on and be equally spread out

"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

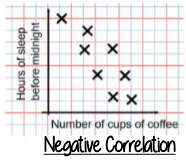
Linear Correlation

R



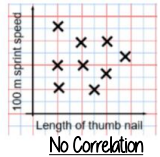
Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases



No Correlation

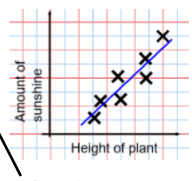
There is no relationship between the two variables

The line of best fit

R

The Line of best fit is used to make estimates about the information in your scatter graph

Things to know:
- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



It is only an estimate because the line is designed to be an average representation of the data

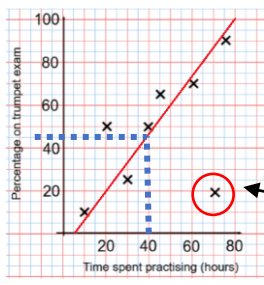
It is always a **straight line.**

Using a line of best fit

R

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45.



Extrapolation is where we use our line of best fit to predict information outside of our data.
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "**outlier**"
It is an outlier because it doesn't fit this model and stands apart from the data

YEAR 10 – USING NUMBER...

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Non-calculator methods

What do I need to be able to do?

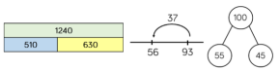
By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Credit: money that goes into a bank account
Debit: money that leaves a bank account
Profit: the amount of money after income - costs
Tax: money that the government collects based on income, sales and other activities
Balance: The amount of money in a bank account
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value

Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
–	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

3584 ÷ 7 = 512

Short division

$$7 \overline{)\,3\,{}^{3}5\;8\;{}^{1}4}\quad 5\;1\;2$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using factors

Division with decimals

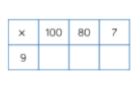
The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

24 ÷ 0.02 → 24 ÷ 0.2 → 240 ÷ 2

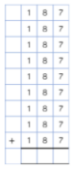
All give the same solution as represent the same proportion
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

Long multiplication (column)



Grid method



Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

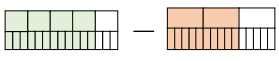
Perform multiplications as integers
e.g. 0.2 x 0.3 → 2 x 3

Make **adjustments** to your answer to match the question: 0.2 x 10 = 2
0.3 x 10 = 3
Therefore 6 ÷ 100 = 0.06

Four operations with fractions

Addition and Subtraction

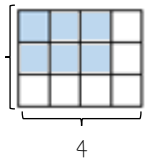
$$\frac{4}{5} - \frac{2}{3}$$



$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Multiplication

$$\frac{3}{4} \times \frac{2}{3}$$



$$= \frac{6}{12} = \frac{1}{2}$$

Division

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

$$= \frac{8}{15}$$

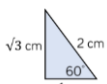
Exact Values

Leave in terms of π

$$= \frac{120}{360} \times 36\pi$$

$$= \frac{1}{3} \times 36\pi = 12\pi$$

Leave as a surd



$\tan 30 = \frac{1}{\sqrt{3}}$

Rounding

2.46192 (to 12dp) - Is this closer to 246 or 247

246 ← 247

2.46 | 192

This shows the number is closer to 246

Significant Figures

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

Estimation

Round to 1 significant figure to estimate

21.4 x 3.1 ≈ 20 x 3 ≈ 60

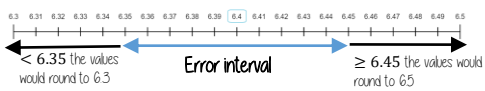
The equal sign changes to show it is an estimation

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors

Limits of accuracy

A width w has been rounded to 6.4cm correct to 1dp

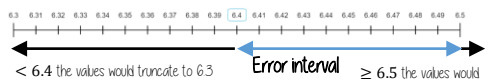


< 6.35 the values would round to 6.3 | Error interval | ≥ 6.45 the values would round to 6.5

The error interval: $6.35 \leq w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width w has been truncated to 6.4cm correct to 1dp



< 6.4 the values would truncate to 6.3 | Error interval | ≥ 6.5 the values would truncate to 6.5

$6.4 \leq w < 6.5$

Any value within these limits would **truncate** to 6.4 to 1dp

YEAR 10 – USING NUMBER...

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Types of number & sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the nth term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number
Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

This list continues and doesn't end

$3x$, $6x$, $9x$ ...

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2

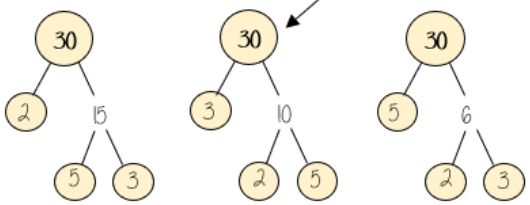
The first prime number
The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7 , 11, 13, 17, 19, 23, 29...

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

e.g. 60	30 x 2	2 x 3 x 5 x 2
150	30 x 5	2 x 3 x 5 x 5

Finding the HCF and LCM

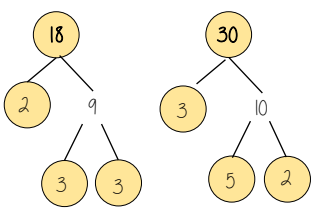
HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18
30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share

HCF = 6



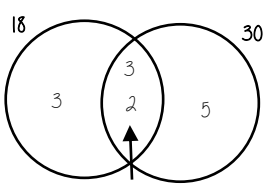
LCM – Lowest common multiple

LCM of 18 and 30

18: 18, 36, 54, 72, 90
30: 30, 60, 90

The first time their multiples match

LCM = 90



LCM = 90

HCF = 6

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term.

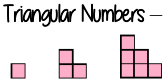
Position to term rule – take the rule and substitute in a position to find a term. E.g. Multiply the position number by 3 and then add 2.

Other sequences

Fibonacci Sequence
1, 1, 2, 3, 5, 8 ...

Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 ...

Square Numbers – look at the formation

1, 4, 9, 16...

Sequences are the repetition of a pattern

Finding the nth term

This is the 4 times table → 4, 8, 12, 16, 20....

4n

This has the same constant difference – but is 3 more than the original sequence

7, 11, 15, 19, 22

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 10 – USING NUMBER...

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Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Coefficient: The number used to multiply a variable

Square and cube numbers

R

Square numbers

1, 4, 9, 16...

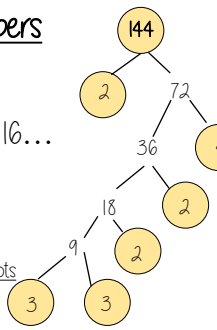
144 = 2 x 2 x 2 x 2 x 3 x 3

(2 x 2 x 3) x (2 x 2 x 3)

12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$



Cube numbers

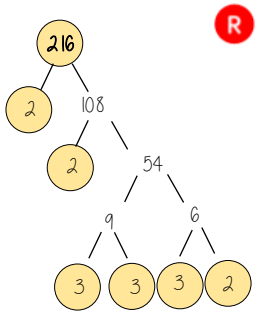
1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

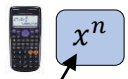
(2 x 3) x (2 x 3) x (2 x 3)

6 x 6 x 6

$\sqrt[3]{216} = 6$

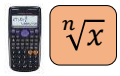


Higher powers and roots



x^n ← n – power (number of times multiplied by itself)

x – the base number.



$\sqrt[n]{x}$ ← Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$

$= 9 \times 100$

$= 900$

Standard form

R

Any integer

Any number between 1 and less than 10

$A \times 10^n$

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers in standard form with negative powers will be less than 1

$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated. Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Standard form calculations

R

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end.

$6 \times 10^5 + 8 \times 10^5$

Method 1

= 600000 + 800000

= 1400000

$= 1.4 \times 10^5$

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$

This is not the final answer

$= 1.4 \times 10^1 \times 10^5$

$= 1.4 \times 10^5$

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations