YEAR 10 – SIMILARITY...

@whisto_maths

Congruence, similarity & enlargement

What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Congruent: the same size and shape
Corresponding: items that appear in the same place in two similar situations
Parallel: straight lines that never meet (equal gradients)

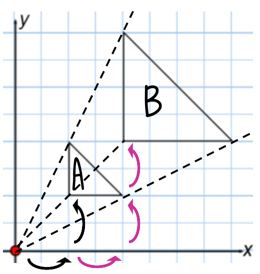
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

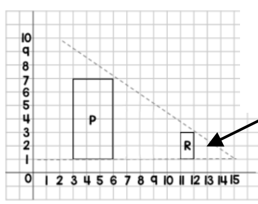
The distance from the point enlarges by 2



Fractional scale factors R

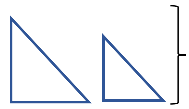
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



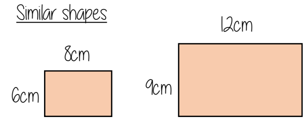
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

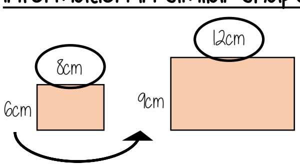


Scale Factor: Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 → 2 : 3   8 : 12 → 2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

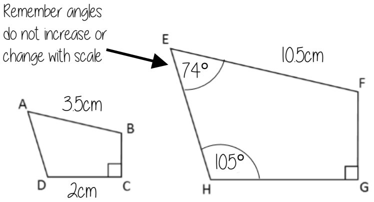
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

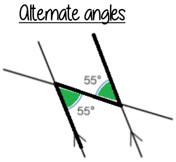
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines R

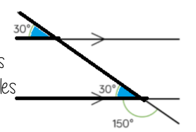
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

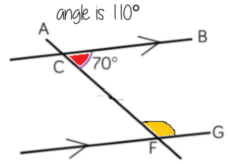
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

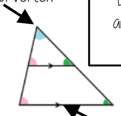
Because co-interior angles have a sum of 180° the highlighted angle is 110°



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Similar triangles

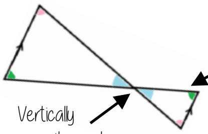
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

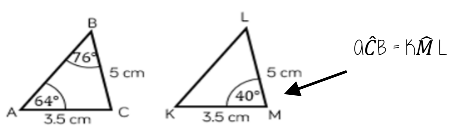


All the angles in both triangles are the same and so similar

Vertically opposite angles

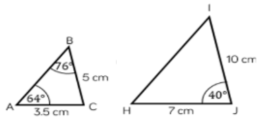
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 10 – SIMILARITY...

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Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Constant: a value that remains the same

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.

Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

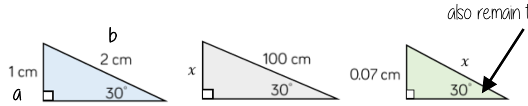
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

Inverse: function that has the opposite effect.

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

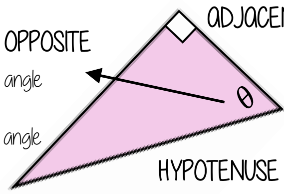
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

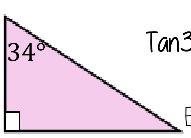
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



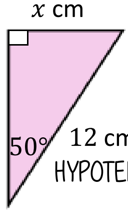
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

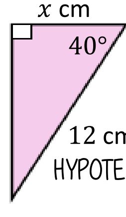
$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio



$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

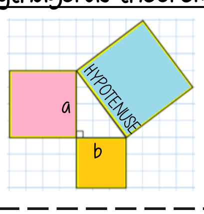
Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

R

$\text{Hypotenuse}^2 = a^2 + b^2$

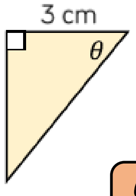


This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

$\text{Tan}\theta = \frac{3}{4}$

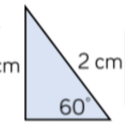
$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

Key angles

This side could be calculated using Pythagoras

Because trig ratios remain the same for similar shapes you can generalise from the following statements



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

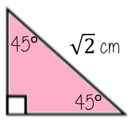
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$

$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$

$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

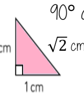
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$

$\text{Sin}90 = 1$

$\text{Cos}0 = 1$

$\text{Cos}90 = 0$

YEAR 10 – DEVELOPING ALGEBRA…

Representing solutions of equations and inequalities

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What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet

Equation: an equation says that two things are equal – it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: On equation where both sides have variables that cause the same answer includes ≡

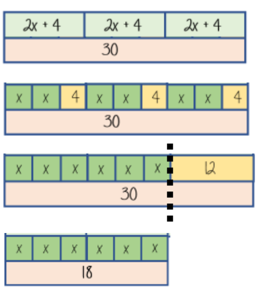
Linear: an equation or function that is the equation of a straight line

Intersection: the point that two lines meet

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations (R)

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

-12 -12

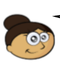
Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$

$\div 6$ $\div 6$

$x = 3$

Form and solve inequalities (R)



Two more than treble my number is greater than 11

Form $x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$3x + 2 > 11$

Solve $x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

Solutions on a number line

$x < 1$ $x \leq 1$ $x > 1$ $x \geq 1$

Both represent values less than 1

Includes the value 1

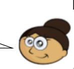
Both represent values more than 1

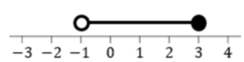
Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1





$-1 < x \leq 3$

This includes the integer values 0,1,2,3

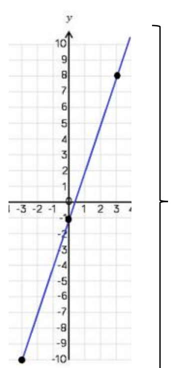
Plotting straight line graphs (R)

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	−10	−1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

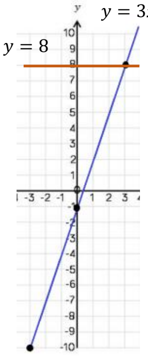
Remember to join the points to make a line

Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



$3x - 1 = 8$

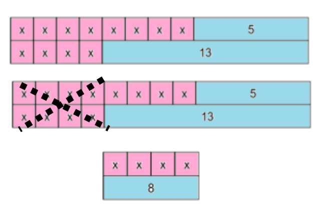
Remember equation of a line format is y = mx + c

The solution is the point the two lines meet

(3,8)

Equations: unknown on both sides (R)

$8x + 5 = 4x + 13$



$8x + 5 = 4x + 13$

$-4x$ $-4x$

$4x + 5 = 13$

-5 -5

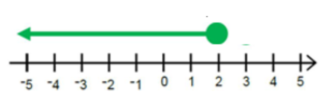
$4x = 8$

$\div 4$ $\div 4$

$x = 2$

Inequalities: unknown on both sides

$8x + 5 \leq 4x + 13$ → $x \leq 2$



Any value 2 or less will satisfy this inequality

YEAR 10 – DEVELOPING ALGEBRA…

Simultaneous Equations

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What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

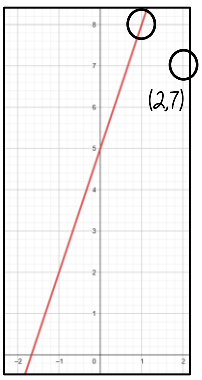
$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5



Substituting known variables

A line has the equation 3x + y = 14

Two different variables, two solutions

Stephanie knows the point x = 4 lies on that line. Find the value for y

$x = 4$

$3x + y = 14$

$3(4) + y = 14$

$12 + y = 14$
$-12 \qquad -12$

$y = 2$

Substituting in an expression

x = 2y

x + y = 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

$x = 2y \qquad x + y = 30$

$3y = 30$

$3y = 30$
$\div 3 \qquad \div 3$
$y = 10$

$x = 2y$

$x = 20$

Solve graphically

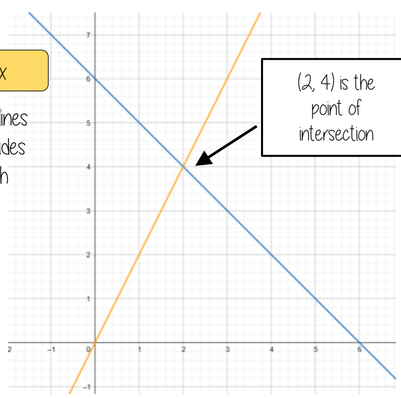
x + y = 6

y = 2x

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

$x = 2$ and $y = 4$



(2, 4) is the point of intersection

Solve by subtraction

$3x + 2y = 18$
$- \quad x + 2y = 10$

$2x = 8$
$\div 2 \qquad \div 2$
$x = 4$

$x + 2y = 10$
$(4) + 2y = 10$
$-4 \qquad -4$
$2y = 6$
$\div 2 \qquad \div 2$
$y = 3$

$x = 4$
$y = 3$

= 18
= 10

= 18
= 10

= 8
= 4
= 3

Solve by addition

Addition makes zero pairs

$3x + 2y = 16$
$+ \quad 6x - 2y = 2$

$9x = 18$
$\div 9 \qquad \div 9$
$x = 2$

$3x + 2y = 16$
$3(2) + 2(y) = 16$
$6 + 2y = 16$
$-6 \qquad -6$
$2y = 10$
$y = 5$

= 16
= 2
= 18
= 2
= 5

Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values

12
29

$2h + 2j = 24$
$2h + 2j = 29$

24
29

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$
$5x - 2y = -7$

= 39
= −7

Use LCM to make equivalent x OR y values.
Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

Now solve by addition

= 78
= −21

Addition makes zero pairs