YEAR 11 — EXPANDING & FACTORISING

By the end of this unit you should be able to:	MathsWatch clip	Video tutorial
• Expand & factorise with a single bracket	93 134a 94	
• Expand binomials	134b	
• Factorise quadratic expressions		Corbett
• Factorise complex quadratic expressions (H)	192	Corbett
• Solve equations equal to 0		
• Solve quadratic equations by factorisation	157	Corbett
• Solve complex quadratic equations by factorisation (H)		
• Complete the square (H)	209a 209b	Corbett
• Solve quadratic equations using the quadratic formula (H)	191	Corbett

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Keywords

Expand: multiply out terms to remove brackets

Coefficient: the number in front of a letter in an algebraic term, such as $5x^3$

Quadratic: an expression in which the highest power is 2, such as $x^2 - 5x + 3$

Cubic: an expression in which the highest power is 3, such as $8 + x^3$

Estimate: read approximate values from a graph

Gradient: the steepness (or slope) of a line. A negative gradient means the line slopes downhill

Substitute: put numbers in place of letters to find the value of an expression

Reciprocal: a graph with an equation of the form $y = \dfrac{k}{x}$ where k is a number

Roots: the solutions when an equation equals zero (often the x-intercepts of a graph)

Exponential: a graph with an equation of the form $y = k^x$ where k is a number

Tangent: a straight line touching a curve which can be used to estimate the gradient of the curve at that point

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Some (but not all) key points:

Completing the square:

$$x^2 - 6x + 5$$

$$\Rightarrow (x-3)^2 - 3^2 + 5$$

$$\Rightarrow (x-3)^2 - 9 + 5$$

$$\Rightarrow (x-3)^2 - 4$$

Solve complex quadratics by factorising:

$$2x^2 - 7x + 3 = 0$$

$$(2x-1)(x-3) = 0 \quad \longleftarrow \text{Factorise}$$

One of the brackets must equal zero

$$2x - 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = 1 \qquad\qquad x = 3$$

$$x = \frac{1}{2} \qquad \longleftarrow \text{Solutions}$$

Solving a quadratic by using the quadratic formula

$a = 2 \qquad b = -8 \qquad c = -24$

$$2x^2 - 8x - 24 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \longleftarrow \text{The quadratic formula}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-24)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 - (-192)}}{4}$$

$$x = \frac{8 \pm \sqrt{256}}{4}$$

$$x = \frac{8 \pm 16}{4} = 2 \pm 4$$

$$x = 6, -2$$

Solutions (often these may be decimals)

YEAR 11 — GRADIENTS & LINES

By the end of this unit you should be able to:	MathsWatch clip	Video tutorial
• Find equations of lines parallel to the axis	05	
• Plot straight lines	96	
• Interpret y = mx + c		Corbett
• Find the equation of a straight line:		
• i) from a graph	159a	Corbett
• ii) given one point and a gradient	159b	
• iii) given two points	159b	Corbett
• Determine whether a point is on a line		
• Solve linear simultaneous equations graphically	140	MathsGenie
• Recognise when straight lines are perpendicular (H)	208	
• Find the equations of perpendicular lines (H)	208	MathsGenie
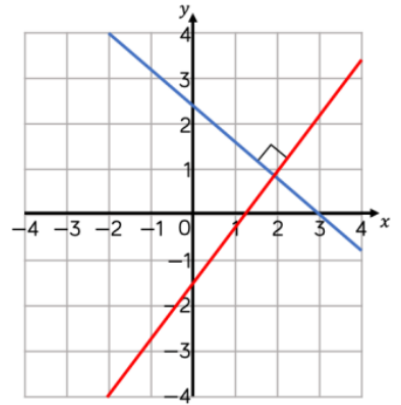
Keywords

Parallel: straight lines that never meet (equal gradients)

Horizontal: a straight line which goes from side to side, parallel to the x-axis

Vertical: a straight line which goes up and down, parallel to the y-axis

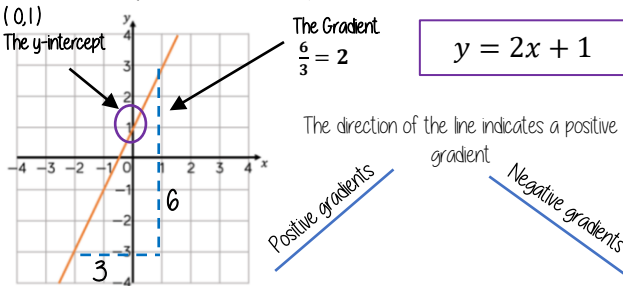**Intercept:** the point where a line crosses the axis of a graph

Gradient: the steepness (or slope) of a line. A negative gradient means the line slopes downhill

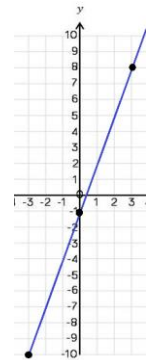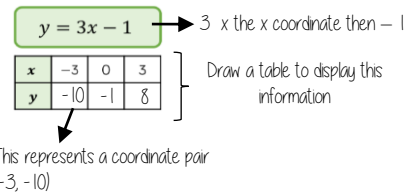**Substitute:** put numbers in place of letters to find the value of an expression

Reciprocal: the reciprocal of a number is 1 divided by that number.

Some (but not all) key points:

Find the equation from a graph

(0,1)
The y-intercept

The Gradient
$\frac{6}{3} = 2$

6

3

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then − 1

x	−3	0	3
y	−10	−1	8

Draw a table to display this information

This represents a coordinate pair
(−3, −10)

You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Lines parallel to the axes   **R**

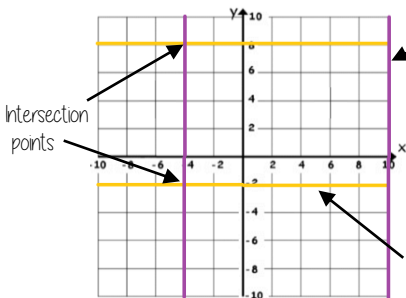All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Intersection points

Lines parallel to the **y axis** take the form **x** = **a** and are **vertical**

Lines parallel to the **x axis** take the form **y** = **a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g. (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

YEAR 11 — WORKING IN DIFFERENT DIMENSIONS....

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

Keywords

Circumference: the length around the outside of the circle – the perimeter
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
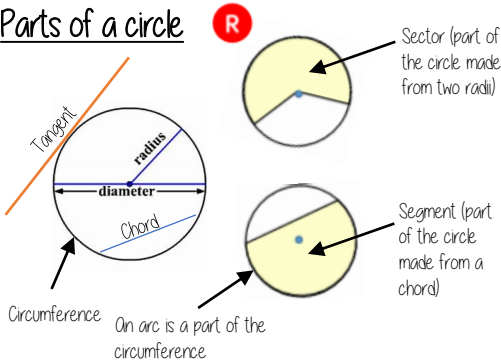**Tangent**: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
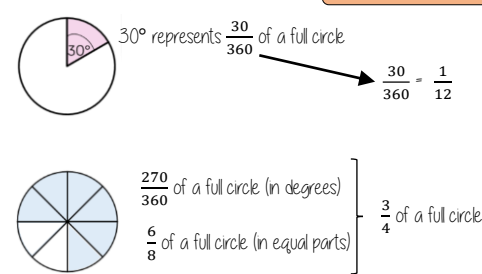**Frustrum**: a pyramid or cone with the top cut off
Hemisphere: half a sphere
Surface area: the total area of the surface of a 3D shape.
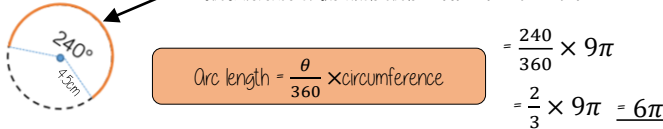
Parts of a circle

Sector (part of the circle made from two radii)

Tangent
radius
diameter
Chord
Circumference

Segment (part of the circle made from a chord)

An arc is a part of the circumference

Fractional parts of a circle

A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$

30° represents $\frac{30}{360}$ of a full circle

$$\frac{30}{360} = \frac{1}{12}$$

$\frac{270}{360}$ of a full circle (in degrees)

$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length

Remember an arc is part of the circumference
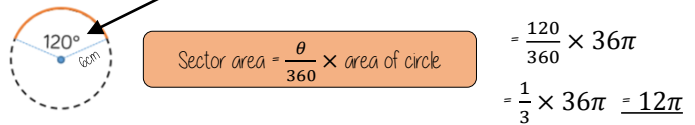Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$

240°
4.5cm

Arc length = $\frac{\theta}{360} \times$ circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi = \underline{6\pi}$

Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

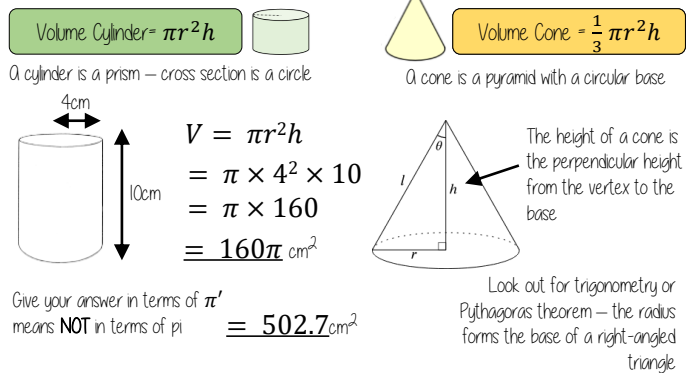Perimeter = $\frac{\theta}{360} \times$ circumference $+ 2r$

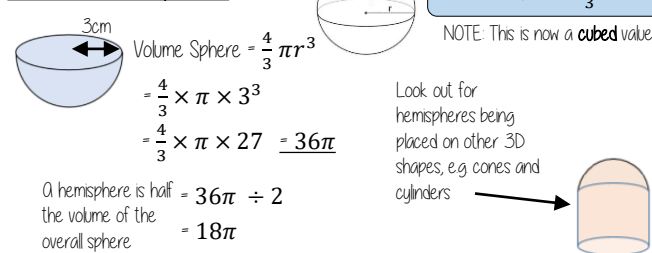$= \underline{6\pi + 9}$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$

120°
6cm

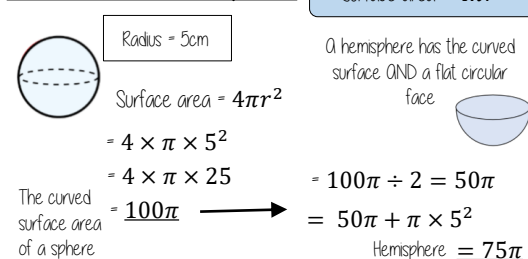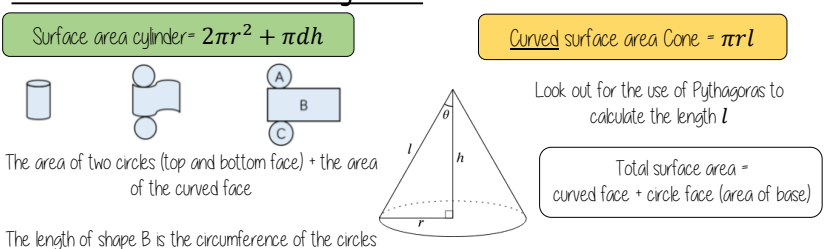Sector area = $\frac{\theta}{360} \times$ area of circle

$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = \underline{12\pi}$

Volume of a cone and a cylinder

Volume Cylinder= $\pi r^2 h$

A cylinder is a prism – cross section is a circle

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base

4cm
10cm

$$V = \pi r^2 h$$
$$= \pi \times 4^2 \times 10$$
$$= \pi \times 160$$
$$= \underline{160\pi} \text{ cm}^2$$

Give your answer in terms of π' means **NOT** in terms of pi $= \underline{502.7 \text{cm}^2}$

The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Volume of a sphere

Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value

3cm

Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

$= \frac{4}{3} \times \pi \times 27 = \underline{36\pi}$

A hemisphere is half the volume of the overall sphere
$= 36\pi \div 2$
$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders

Surface area of a sphere

Surface area = $4\pi r^2$

Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

$= \underline{100\pi}$

The curved surface area of a sphere

A hemisphere has the curved surface AND a flat circular face

$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= \underline{75\pi}$

Surface area of cones and cylinders

Surface area cylinder= $2\pi r^2 + \pi dh$

Curved surface area Cone = πrl

A
B
C

The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

YEAR 11 — MULTIPLICATIVE REASONING

By the end of this unit you should be able to:	MathsWatch clip	Video tutorial
• Use scale factors		
• Understand direct proportion		
• Construct complex direct proportion equations (H)	199	Corbett
• Calculate with pressure & density	142	Corbett Corbett
• Understand inverse proportion		
• Construct inverse proportion equations (H)	199	Corbett
• Solve ratio problems		
• Calculate with speed, distance and time		
• Use area and volume scale factors in similar shapes		
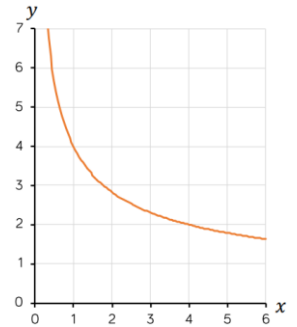
Keywords

Similar: same shape and angles, but a different size

Direct proportion: two quantities which remain in the same ratio at all times

Inverse proportion: a relationship in which one quantity increases as the other decreases

Linear: a direct proportion relationship — shown by a straight diagonal line on a graph
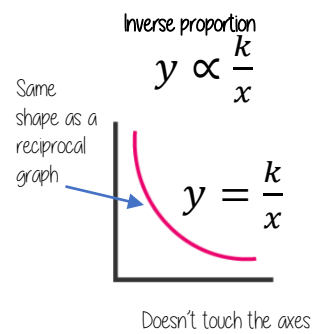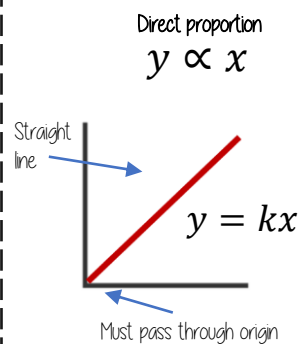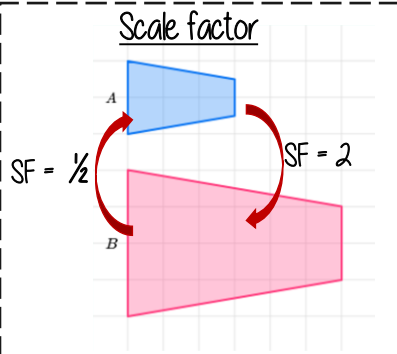
Varies directly: another was of saying 'direct proportion'

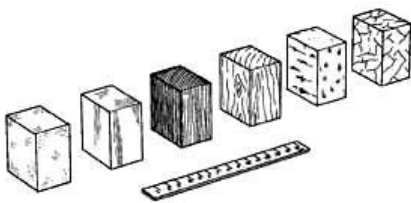**Constant of proportionality**: the ratio between two quantities that are in proportion

Density: how much matter is in a particular volume of space, calculated as mass ÷ volume

Pressure: the effect of an object's weight on a surface, calculated as force ÷ area

Some (but not all) key points:

Scale factor

SF = ½   SF = 2

Direct proportion

$$y \propto x$$

Straight line

$$y = kx$$

Must pass through origin

Inverse proportion

$$y \propto \frac{k}{x}$$

Same shape as a reciprocal graph

$$y = \frac{k}{x}$$

Doesn't touch the axes

The density of an object is calculated by:

mass ÷ volume

Density depends on what material the object is made from.

Units are usually g/cm^3 or kg/m^3.

y is <u>inversely proportional</u> to x. When $x=3$, then $y=6$. Find the value of y when $x=8$. **(H)**

$$y \propto \frac{1}{x}$$

Substitute $x = 3$ and $y = 6$

$$y = \frac{k}{x}$$ ← constant of proportionality

$$6 = \frac{k}{3}$$

$\times 3$ ( ) $\times 3$

$$18 = k$$

Find the value of k

Re-write the equation of proportionality, using the found value of k

$$y = \frac{18}{x}$$

When $x=8$: $y = \frac{18}{8}$

$$y = \frac{9}{4}$$

$$y = 2\tfrac{1}{4}$$