YEAR 10 – SPRING TERM...

Simultaneous Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Substitute: replace a variable with a numerical value

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Eliminate: to remove

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Coordinate: a set of values that show an exact position.

Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

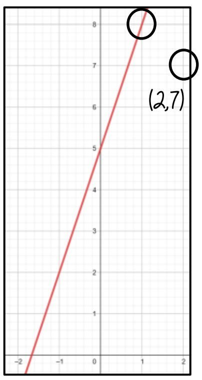
$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5



Substituting known variables

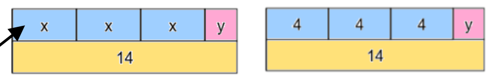
A line has the equation 3x + y = 14

Two different variables, two solutions

Stephanie knows the point x = 4 lies on that line. Find the value for y

$x = 4$

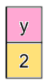
$3x + y = 14$ $\quad 3(4) + y = 14$



$12 + y = 14$

$-12 \quad -12$

$y = 2$



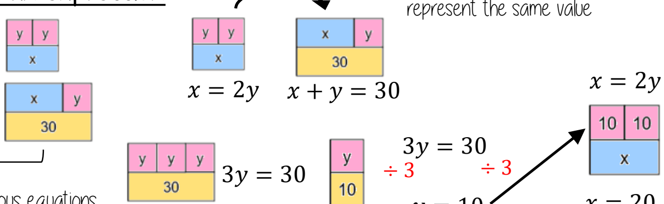
Substituting in an expression

x = 2y

x + y = 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value



$x = 2y \quad x + y = 30$

$3y = 30$

$3y = 30$

$\div 3 \quad \div 3$

$y = 10$

$x = 2y$

$x = 20$

Solve graphically

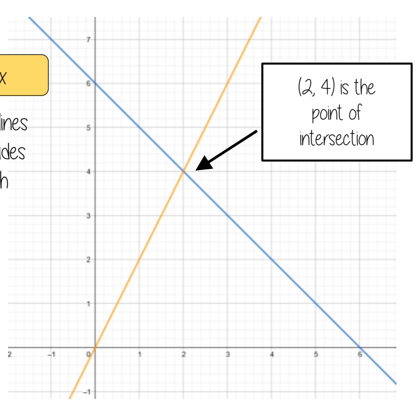
x + y = 6

y = 2x

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations

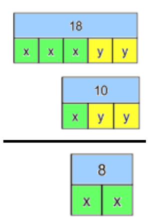
The solution that satisfies both equations is

$x = 2$ and $y = 4$



(2, 4) is the point of intersection

Solve by subtraction



$x = 4$

$y = 3$

$3x + 2y = 18$

$-\quad x + 2y = 10$

$2x = 8$

$\div 2 \quad \div 2$

$x = 4$

$x + 2y = 10$

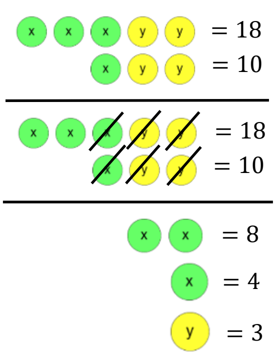
$(4) + 2y = 10$

$-4 \quad -4$

$2y = 6$

$\div 2 \quad \div 2$

$y = 3$



Solve by addition

$3x + 2y = 16$

$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \quad \div 9$

$x = 2$

$3x + 2y = 16$

$3(2) + 2(y) = 16$

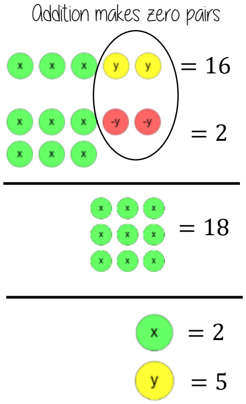
$6 + 2y = 16$

$-6 \quad -6$

$2y = 10$

$y = 5$

Addition makes zero pairs

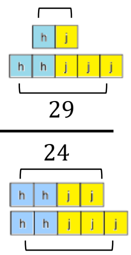


Solve by adjusting one

$h + j = 12$

$2h + 2j = 29$

No equivalent values



$2h + 2j = 24$

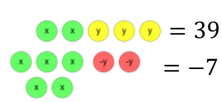
$2h + 2j = 29$

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$

$5x - 2y = -7$

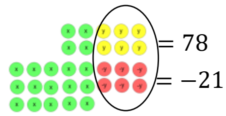


Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$

$15x - 6y = -21$

Now solve by addition



Addition makes zero pairs

YEAR 10 – SPRING TERM...

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Ratios and fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

Keywords

Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Integer: whole number, can be positive, negative or zero.
Fraction: represents how many parts of a whole.
Denominator: the number below the line on a fraction. The number represent the total number of parts.
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Origin: (0,0) on a graph. The point the two axes cross
Gradient: The steepness of a line

Compare with ratio R

"For every dog there are 2 cats"

Dogs: Cats
1:2

The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction R

Trees: Flowers
3 : 7

Trees
Flowers

Ratio

Fraction of trees

$\frac{\text{Number of parts of in group}}{\text{Total number of parts}}$ → $\frac{3}{10}$

Fraction

Sharing a whole into a given ratio R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4

James
Lucy
£350

Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

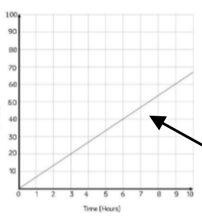
☐ = one part = £50

Put back into the question

James: Lucy
x 50 3 : 4 x 50
£150:£200

James = 3 x £50 = £150
£350
Lucy = 4 x £50 = £200

Ratio and graphs R



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale R

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life
x 10 1cm : 30cm x 10
10cm : 300cm

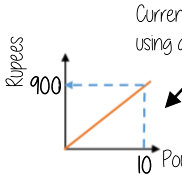
Conversion between currencies R

For every £1 I have 90 Rupees

£1 = 90 Rupees ← Currency is directly proportional

x 10 £1 = 90 Rupees x 10
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

x 7 £1 = 90 Rupees x 7 ← 630 ÷ 90 = 7
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Best buys

You could work out how much 40 pens are and then compare

	4 pens costs £2.60	10 pens costs £6.00
"1 pen costs..."	£2.60 ÷ 4 = £0.65	£6.00 ÷ 10 = £0.60
"1-pound buys..."	4 ÷ 2.60 = 1.54 pens	10 ÷ 6 = 1.67 pens

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

The best value means £1 buys you more pens

Combining ratios

B R G

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green

10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

YEAR 10 – SPRING TERM...

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Percentages and Interest

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

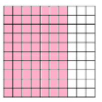
Keywords

Exponent: how many times we use a number in multiplication. It is written as a power
Compound interest: calculating interest on both the amount plus previous interest
Depreciation: a decrease in the value of something over time.
Growth: where a value increases in proportion to its current value such as doubling
Decay: the process of reducing an amount by a consistent percentage rate over time.
Multiplier: the number you are multiplying by
Equivalent: of equal value.

Compare FDP R

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator

$\frac{\blacksquare}{\square}$ → S⇆D Convert to a decimal

This will give you the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount R

Find $\frac{3}{5}$ of £60

£60
| £12 | £12 | £12 | £12 | £12 |
£36

Remember $\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5}$ = 60% = 0.6

60% of £60
= 0.6 x 60
= £36

Percentage increase/decrease R

100%

42%

Decrease by 58%

100% - 58% = 42%

100 – 0.58 = 0.42 ← Multiplier Less than 1

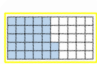
100% 12%

Increase by 12%

100% + 12% = 112%

100 + 0.12 = 1.12 ← Multiplier More than 1

Express as a percentage R



27 per every 50 shaded $\frac{27}{50}$

54 per every 100 shaded $\frac{54}{100}$

54%

$\frac{13}{30}$ → $\frac{\blacksquare}{\square}$ → $\frac{13}{30}$ → × 100

43.3333...%

Can't use equivalence easily to find 'per hundred' → 43%

Decimal percentages are still a percentage

Simple and compound interest

Simple Interest

James invests £2000 at 5% simple interest

100% 5%
| £2000 | £100 |

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 Original amount: £100
10%
£100 £10 Y1: £110
10%
£110 £11 Y2: £121
10%
£121 £12.1 Y3: £132.10

The multiplier 1.10 repeats each year

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 1.10 x 1.10 x 1.10

£100 x 1.10 3

Original amount

Repeated multiplier

Number of occurrences

Depreciation

Depreciation calculations use multipliers **less** than 1

Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

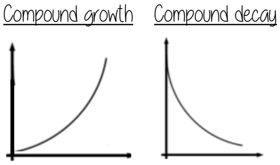
eg Increase of 10% then a reduction of 10%

x 1.10 x 0.9

x 0.99 The multiplier

Growth and decay

Compound growth Compound decay



Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test.

Original x 0.6 = 24

60%
24

24 ÷ 0.6 = 40 marks

10% = 6
100% = 40

Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?

100% 20%

£3000

Original x 1.2 = 3000

120% = £3000
10% = £250
100% = £2500

YEAR 10 – SPRING TERM

Collecting, representing and interpreting data

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon. two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

Keywords

Population: the whole group that is being studied
Sample: a selection taken from the population that will let you find out information about the larger group
Representative: a sample group that accurately represents the population
Random sample: a group completely chosen by change. No predictability to who it will include.
Bias: a built-in error that makes all values wrong by a certain amount
Primary data: data collected from an original source for a purpose.
Secondary data: data taken from an external location. Not collected directly.
Outlier: a value that stands apart from the data set

Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4 Means 14 years old

Stem and leaf diagrams:
Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

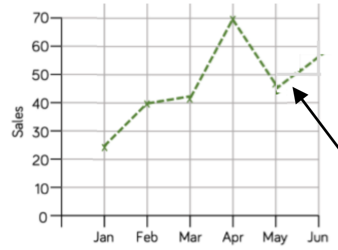
Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

15 | 3
Means 153 cm tall

Back to back stem and leaf diagrams
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency

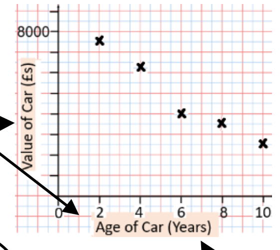
Mean, mode, median – allows for a comparison about more or less average
Range – allows for a comparison about reliability and consistency of data

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled

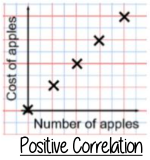


"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

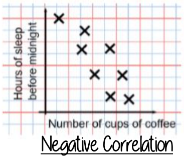
The axis should fit all the values on and be equally spread out

Linear Correlation



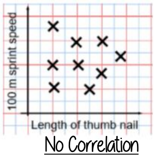
Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases

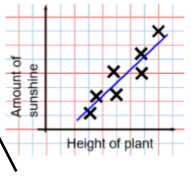


No Correlation

There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

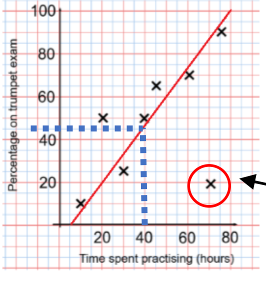
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "**outlier**"
It is an outlier because it doesn't fit this model and stands apart from the data

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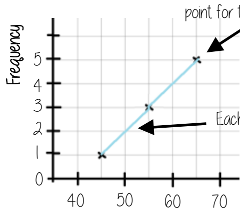
Frequency tables and polygons

x Weight(g)	Frequency
$40 < x \le 50$	1
$50 < x \le 60$	3
$60 < x \le 70$	5

We do not know from grouped data where each value is placed so have to use an estimate for calculations

MID POINTS
Mid-points are used as estimated values for grouped data. The middle of each group

Each point is plotted at them mid point for the group it represents



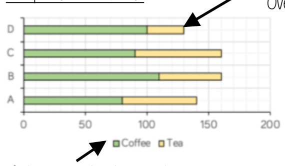
Each point is connected with a straight line

The data about weight starts at 40. So the axis can start at 40

Mid-point
$\frac{\text{Start point + End point}}{2}$

Bar and line charts

Composite bar charts

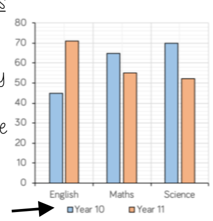


Compare the bars green compared to yellow. The size of each bar is the frequency
Overall total easily comparable

Categories clearly indicated

Dual bar charts

Bars are compared side by side
Easier to compare subgroups



Categories clearly indicated

Averages from a table R

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \le 50$	1	45	45
$50 < x \le 60$	3	65	195
$60 < x \le 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Two way tables R

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant

Extract information to input to the two-way table.

Subgroups each have their own heading

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Needs subgroup totals

Overall total

Draw and interpret Pie Charts R

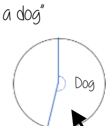
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw This is 192°

Multiple method
As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts:
You NEED the overall frequency to make any comparisons

Averages from lists R

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)
55
Divide the overall total by how many pieces of data you have
$55 \div 5$ Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order 4, 8, 8, 11, 24
Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group – which group has the highest frequency

YEAR 10 – SPRING TERM

Types of number & sequences

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the nth term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number
Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number

R

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

This list continues and doesn't end

$3x$, $6x$, $9x$ …

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

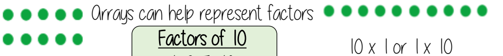
4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

R

Arrays can help represent factors



5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

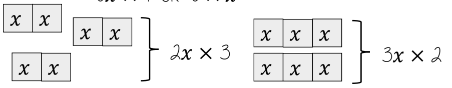
The number itself is always a factor

Factors and expressions

x	x	x	x	x	x

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2



Prime numbers

2

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

Learn or how-to quick recall…

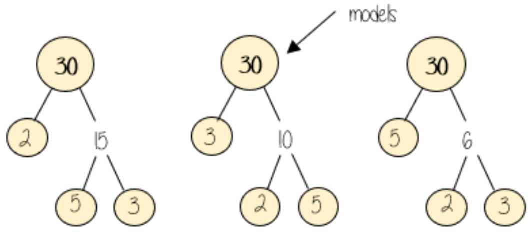
2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

R

Product of prime factors

R

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5

Multiplication of prime factors

Using prime factors for predictions

eg **60**	30 x 2	2 x 3 x 5 **x 2**
150	30 x 5	2 x 3 x 5 **x 5**

Finding the HCF and LCM

R

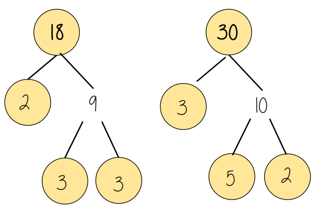
HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share

HCF = 6



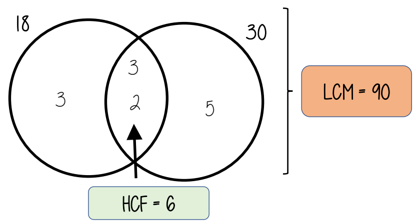
LCM – Lowest common multiple

LCM of 18 and 30

18	18, 36, 54, 72, 90
30	30, 60, 90

The first time their multiples match

LCM = 90



Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term

Position to term rule – take the rule and substitute in a position to find a term. Eg. Multiply the position number by 3 and then add 2

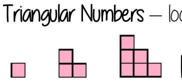
Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 …

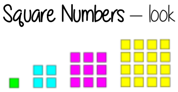
Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 …

Square Numbers – look at the formation



1, 4, 9, 16…

Sequences are the repetition of a patten

Finding the nth term

R

This is the 4 times table → 4, 8, 12, 16, 20…

4n

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence