YEAR 10 – AUTUMN TERM...

@whisto_maths

Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Constant: a value that remains the same

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.

Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

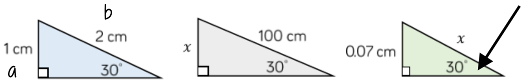
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

Inverse: function that has the opposite effect.

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

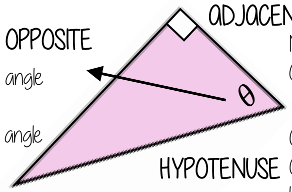
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

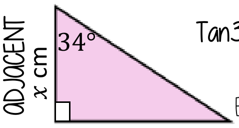
ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



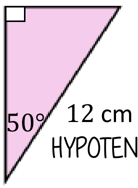
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

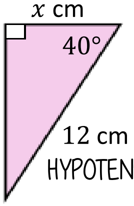
$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio



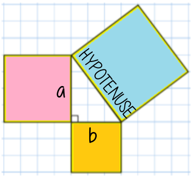
$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

R



$\text{Hypotenuse}^2 = a^2 + b^2$

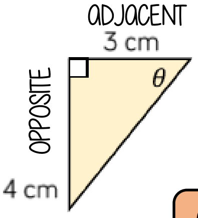
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = \text{Tan}^{-1}\ \frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \text{Sin}^{-1}\ \frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\ \frac{\text{adjacent side}}{\text{hypotenuse side}}$

$\text{Tan}\theta = \frac{3}{4}$

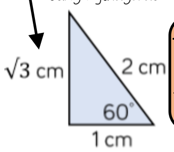
$\theta = \text{Tan}^{-1}\ \frac{3}{4}$

$\theta = 36.9°$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

This side could be calculated using Pythagoras



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

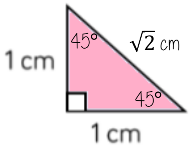
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$

$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$

$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = \mathbf{1}$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

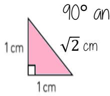
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

Tan0 = 0

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



Sin0 = 0

Sin90 = 1

Cos0 = 1

Cos90 = 0

YEAR 10 – AUTUMN TERM...

@whisto_maths

Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result.

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Square and cube numbers

Square numbers

1, 4, 9, 16...

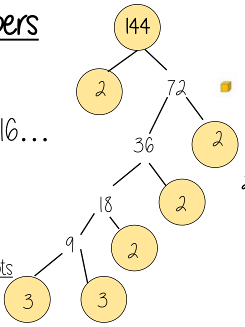
144 = 2 x 2 x 2 x 2 x 3 x 3

2 x 2 x 3 x 2 x 2 x 3

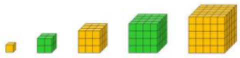
12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$



Cube numbers



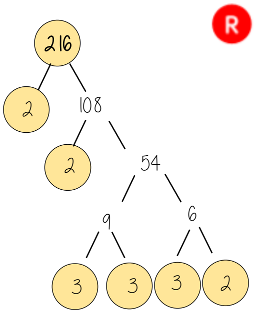
1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

2 x 3 x 2 x 3 x 2 x 3

6 x 6 x 6

$\sqrt[3]{216} = 6$



Higher powers and roots

x^n ← n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ ← Finding the nth root of any value

Other mental strategies for square roots

$$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$$
$$= 9 \times 100$$
$$= 900$$

Standard form

$$A \times 10^n$$

Any number between 1 and less than 10 → A

Any integer → n

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

= 32000

Non-example

$\bigcirc{0.8} \times 10^4$

$5.3 \times 10^{\bigcirc{0.7}}$

Numbers in standard form with negative powers will be less than 1

$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \underline{0.00032}$

Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$
$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$$2^2 = 4$$
$$2^1 = 2$$
$$2^0 = 1$$
$$2^{-1} = \frac{1}{2}$$
$$2^{-2} = \frac{1}{4}$$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = \underbrace{2^3 \times 2^3 \times 2^3 \times 2^3}$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = \underbrace{2x^3 \times 2x^3 \times 2x^3 \times 2x^3}$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Standard form calculations

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$$6 \times 10^5 + 8 \times 10^5$$

Method 1

$= 600000 + 800000$

$= 1400000$

$\underline{= 1.4 \times 10^5}$

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$

$= 1.4 \times 10^1 \times 10^5$

$\underline{= 1.4 \times 10^5}$

This is not the final answer

Multiplication and division

$$\frac{1.5 \times 10^5}{0.3 \times 10^3}$$

Division questions can look like this

$$(\bigcirc{1.5} \times 10^5) \div (\bigcirc{0.3} \times 10^3)$$

$$\boxed{1.5 \div 0.3} \times 10^5 \div 10^3$$

$$\underline{= 5 \times 10^2}$$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

YEAR 10 – AUTUMN TERM…

Equations and inequalities

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: On equation where both sides have variables that cause the same answer includes ≡

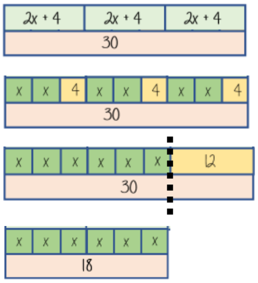
Linear: an equation or function that is the equation of a straight line

Intersection: the point that two lines meet

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations R

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \qquad -12$

$6x = 18$

$\div 6 \qquad \div 6$

Substitute to check your answer. This could be negative or a fraction or decimal

$x = 3$

Form and solve inequalities R

Two more than treble my number is greater than 11

Form

$x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$3x + 2 > 11$

Solve

$x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

Solutions on a number line

$x < 1$ $\quad x \leq 1$ $\quad x > 1$ $\quad x \geq 1$

Both represent values less than 1

Includes the value 1

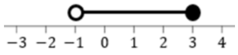
Both represent values more than 1

Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



$-1 < x \leq 3$

This includes the integer values 0,1,2,3

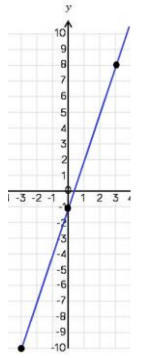
Plotting straight line graphs R

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

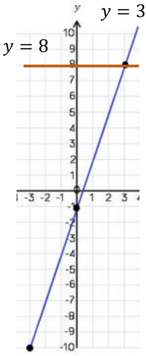
Remember to join the points to make a line

Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



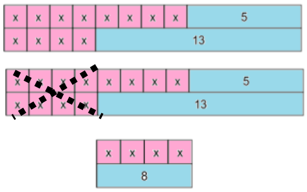
$3x - 1 = 8$

Remember equation of a line format is y = mx + c

The solution is the point the two lines meet (3,8)

Equations: unknown on both sides R

$8x + 5 = 4x + 13$



$$8x + 5 = 4x + 13$$
$$-4x \qquad -4x$$
$$4x + 5 = 13$$
$$-5 \qquad -5$$
$$4x = 8$$
$$\div 4 \qquad \div 4$$
$$x = 2$$

Inequalities: unknown on both sides

$8x + 5 \leq 4x + 13 \longrightarrow x \leq 2$

Only value 2 or less will satisfy this inequality

YEAR 10 – AUTUMN TERM...

@whisto_maths

Non-calculator methods

What do I need to be able to do?

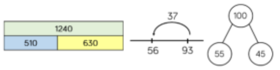
By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Credit: money that goes into a bank account
Debit: money that leaves a bank account
Profit: the amount of money after income - costs
Tax: money that the government collects based on income, sales and other activities.
Balance: The amount of money in a bank account
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value

Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
–	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

Short division

3584 ÷ 7 = 512

$$\begin{array}{r|l} & 5\ \ 1\ \ 2 \\ 7 & 3\ {}^{3}5\ \ 8\ {}^{1}4 \end{array}$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using factors

Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

$$\underbrace{24 \div 0.02 \longrightarrow 24 \div 0.2 \longrightarrow 240 \div 2}$$

All give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

	H	T	O
	1	8	7
×			9

Long multiplication (column)

×	100	80	7
9			

Grid method

	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
+	1	8	7

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers
e.g. 0.2 x 0.3 ⟶ 2 x 3

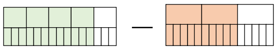
Make **adjustments** to your answer to match the question: 0.2 x 10 = 2
0.3 x 10 = 3

Therefore 6 ÷ 100 = 0.06

Four operations with fractions

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$

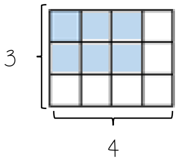


$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Multiplication

$$\frac{3}{4} \times \frac{2}{3}$$

$$= \frac{6}{12} = \frac{1}{2}$$



Division

$$\frac{2}{5} \div \frac{3}{4}$$

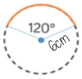
$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

$$= \frac{8}{15}$$

Exact Values

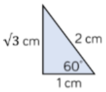
Leave in terms of π



$$= \frac{120}{360} \times 36\pi$$

$$= \frac{1}{3} \times 36\pi = \underline{12\pi}$$

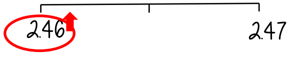
Leave as a surd



$$\text{Tan}30 = \frac{1}{\sqrt{3}}$$

Rounding

2.46192 (to 12dp) - Is this closer to 2.46 or 2.47



2.46 | 192

This shows the number is closer to 2.46

Significant Figures

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

Estimation

Round to 1 significant figure to estimate

21.4 x 3.1 ≈ 20 x 3 ≈ 60

The equal sign changes to show it is an estimation

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Limits of accuracy

A width w has been rounded to 6.4cm correct to 1dp.

< 6.35 the values would round to 6.3

Error interval

≥ 6.45 the values would round to 6.5

The error interval $6.35 \le w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width w has been truncated to 6.4cm correct to 1dp.

< 6.4 the values would truncate to 6.3

Error interval

≥ 6.5 the values would truncate to 6.5

$6.4 \le w < 6.5$

Any value within these limits would truncate to 6.4 to 1dp