YEAR 7 – ALGEBRAIC THINKING...

Sequences

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

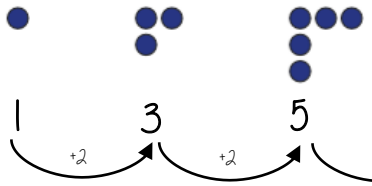
- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Rule: instructions that relate two variables
Linear: the difference between terms increases or decreases by the same value each time
Non-linear: the difference between terms increases or decreases in different amounts
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

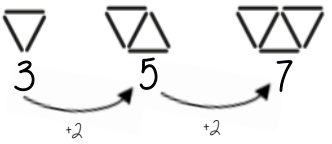
Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image



What will the next number be? Can you draw this?

Predict and check terms



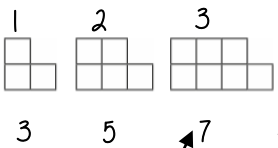
CHECK – draw the next terms

Predictions:
Look at your pattern and consider how it will increase.

e.g. How many lines in pattern 6?
Prediction - 13
If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

Position: the place in the sequence



"The **term** in **position** 3 has 7 squares"

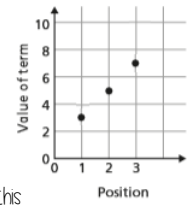
Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.

Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?
It increases by adding 4 to each term.
How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference).
How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence.

Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?
It increases by multiplying the previous term by 2 – this is a geometric sequence because the constant is multiply by 2
How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference).
How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence

Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation
Use key maths language – doubles, halves, multiply by two, add four to the previous term etc.

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term. The sequence begins at 4.

4, 12, 36, 108...
x3 x3 x3
First term

YEAR 7 – ALGEBRAIC THINKING...

Algebraic notation

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What do I need to be able to do?

By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions
- Represent functions graphically

Keywords

Function: a relationship that instructs how to get from an input to an output.
Input: the number/ symbol put into a function.
Output: the number/ expression that comes out of a function.
Operation: a mathematical process
Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Commutative: the order of the operations do not matter.
Substitute: replace one variable with a number or new variable.
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Evaluate: work out
Linear: the difference between terms increases or decreases by the same value each time
Sequence: items or numbers put in a pre-decided order

Single function machines

INPUT → [ ] → OUTPUT

The number that goes IN → The number that comes out

This box gives the calculation instruction

To find the input from the output
Use the **INVERSE** operation

Using letters to represent numbers

$5 + 5 + 5$
3×5
5×3

$y + y + y + y$
$y \times 4$
$4 \times y$
$4y$ ← 4 lots of 'y'

Addition and multiplication can be done in any order
Commutative calculations

$20 \div h$
$\frac{20}{h}$ ← 20 shared into 'h' number of groups

Single function machines (algebra)

INPUT		OUTPUT
a	x 10	$10a$
$3c$		$30c$

÷ 10

To find the input from the output
Use the **INVERSE** operation

Find functions from expressions

INPUT $7x$ → [?] → OUTPUT $14x$

Find the relationship between the input and the output

Sometimes there can be a number of possible functions e.g. **+7x** or **x 2** could both be solutions to the above function machine

Substitution into expressions

$4y$ ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4×7 **OR** $7 + 7 + 7 + 7$ **OR** 7×4 = 28

e.g. $y - 2$
$= 7 - 2 = 5$

Two step function machines

INPUT → [ ] → [ ] → OUTPUT

Calculate the value at the end of each operation

For the input use the **INVERSE** operations

Two step function machines (algebra)

b → x 5 → (5b) → + 4 → 5b + 4
(÷ 5 / − 4)

c → + 4 → (c+4) → x 5 → 5(c + 4) = 5c + 20
(− 4 / ÷ 5)

IMPORTANT
Calculate the value at the end of each operation

NOTE:
The whole first output is multiplied by 5

Find functions from expressions

f → x 5 → + 3 → $\frac{f + 5}{3}$

f add 5 then divide by 3

f → ÷ 3 → + 5 → $\frac{f}{3} + 5$

f divided by then add 5

NOTE: the difference in the two expressions

Sometimes it helps to try to explain the expression in word – and consider what has happened to the input

Substitution into an expression

$2(x + 3)$

Add 3 to the input then times 2

Put the expression into a function machine

INPUT → + 3 → x 2 → OUTPUT

If x = 10
10 + 3 = 13.... 13 x 2 = 26

Forming a sequence

$2(x + 3)$

INPUT	1	2	3
OUTPUT	8	10	12

← The substitution is the 'input' value
The OUTPUT becomes the sequence

Representing functions graphically

Take the function and generate a sequence $2(x + 3)$

INPUT → + 3 → x 2 → OUTPUT

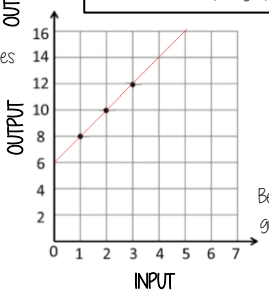
To represent graphically the input becomes x co-ordinates and the output becomes y co-ordinates

$y = 2(x + 3)$

INPUT (x)	1	2	3
OUTPUT (y)	8	10	12

This becomes a co-ordinate pair (2, 10) to plot on a graph

Not all graphs will be linear only those with an integer value for x.
Powers and fractions generate differently shaped graphs.



NOTE:
Because this is a linear graph you can predict other values

YEAR 7 — ALGEBRAIC THINKING...

Equality and Equivalence

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What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

Keywords

Equality: two expressions that have the same value
Equation: a mathematical statement that two things are equal
Equals: represented by '=' symbol — means the same
Solution: the set or value that satisfies the equation
Solve: to find the solution
Inverse: the operation that undoes what was done by the previous operation (The opposite operation)
Term: a single number or variable
Like: variables that are the same are 'like'
Coefficient: a multiplicative factor in front of a variable e.g. 5x (5 is the coefficient, x is the variable)
Index: the power
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equality

$2 + 14 = 5 + 5 + 6$

16 = 16

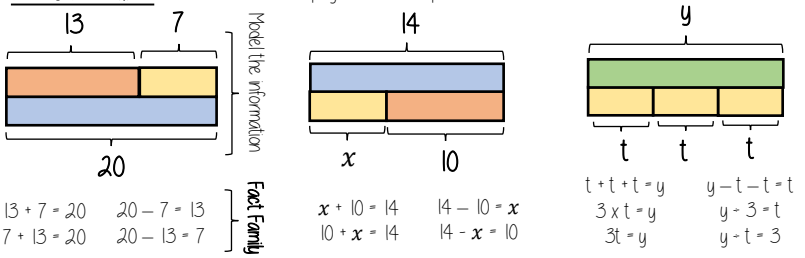
"Is equal to"

The sum on the left has the same result as the sum on the right

Saying it out loud sometimes helps you to understand equality

Fact Families

Use a bar model to display the relationships between terms and numbers



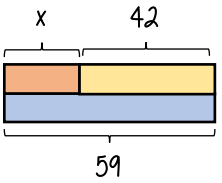
Solve one step equations (+/-)

There is more to this than just spotting the answer

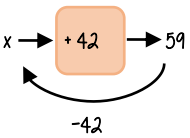
$x + 42 = 59$

$x + 42 = 59$
$42 + x = 59$

$59 - x = 42$
$59 - 42 = x$



Don't forget you know how to use function machines

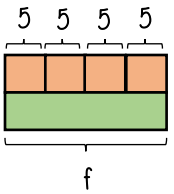


Solve one step equations (x/÷)

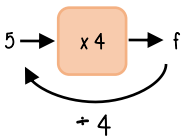
$\frac{f}{4} = 5$

$f \div 4 = 5$
$f \div 5 = 4$

$5 \times 4 = f$
$4 \times 5 = f$

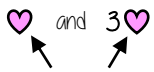


Don't forget you know how to use function machines



Like and unlike terms

Like terms are those whose variables are he same



are **like** terms

the variable is the same

are **unlike** terms

the variables are NOT the same

Examples and non-examples

Like terms	Un-like terms
y, 7y	y, 7x
$2x^2$, x^2	$2x^2$, $2c^2$
ab, 10ba	ab, 10a
5, -2	5, -2t

Note here ab and ba are commutative operations, so are still like terms

Equivalence

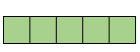
Check equivalence by substitution
e.g. m= 10

5m	2 x 2m	7m − 3m
5 x 10	2 x (2x 10)	(7x 10) − (3x 10)
= 50	= 2 x 20	= 70 − 30
	= 40	= 40

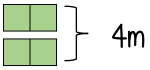
Equivalent expressions

Repeat this with various values for m to check

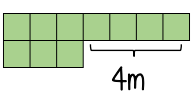
5m



2 x 2m



7m − 3m



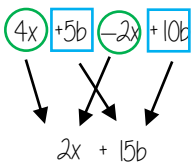
Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms

Only **like terms** can be combined

$4x + 5b - 2x + 10b$



$2x + 15b$

Common misconceptions

$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x2 and x terms are un-like terms so can not be collected

YEAR 7 – PLACE VALUE AND PROPORTION

@whisto_maths

Ordering integers and decimals

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand place value and the number system including decimals
- Understand and use place value for decimals, integers and measures of any size
- Order number and use a number line for positive and negative integers, fractions and decimals;
- use the symbols =, ≠, ≤, ≥
- Work with terminating decimals and their corresponding fractions
- Round numbers to an appropriate accuracy
- Describe, interpret and compare data distributions using the median and range

Keywords

Approximate: To estimate a number, amount or total often using rounding of numbers to make them easier to calculate with

Integer: a whole number that is positive or negative

Interval: between two points or values

Median: A measure of central tendency (middle, average) found by putting all the data values in order and finding the middle value of the list

Negative: Any number less than zero; written with a minus sign.

Place holder: We use 0 as a place holder to show that there are none of a particular place in a number

Place value: The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Range: The difference between the largest and smallest numbers in a set

Significant figure: A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal fraction is the first non-zero number after the decimal point.

Integer Place Value

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9

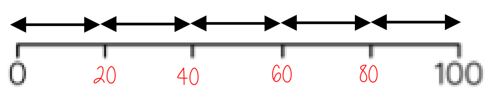
Placeholder

Three billion, one hundred and forty eight million, thirty three thousand and twenty nine

1 billion 1, 000, 000, 000

1 million 1 000, 000

Intervals on a number line

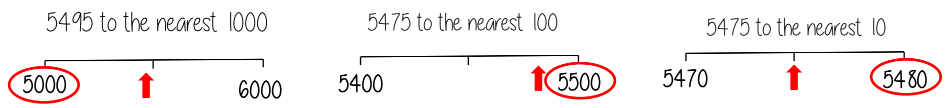


Divide the difference by the number of intervals (gaps).

Eg 100 ÷ 5 = 20

Rounding to the nearest power of ten

If the number is halfway between we "round up"



Compare integers using <, >, =, ≠

< less than

> greater than

= equal to

≠ not equal to

Two and a half million	=	2 500 000
300 000 000	=	Three billion
Six thousand and eighty	<	68 000

Range *Spread of the values*

Difference between the biggest and smallest

3 9 8 12

Range: Biggest value – Smallest value

12 – 3 = 9

Range = 9

Median *The middle value*

Example 1

4 3 9 8 12

Median: put the in order 3 4 8 9 12

find the middle number 3 4 (8) 9 12

Example 2

150 154 148

137 160 158

Median: put the in order

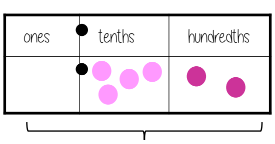
137 148 150 154 158 160

There are 2 middle numbers

Find the midpoint

152

Decimals



We say "nought point five two"

Five tenths and two hundredths

0 ones, 5 tenth and 2 hundredths

0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01

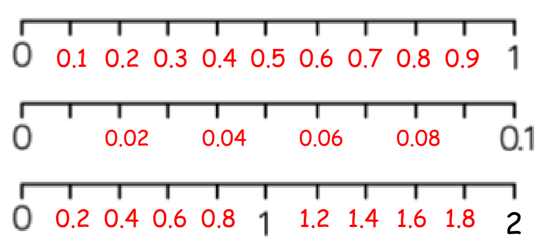
= 0 + 0.5 + 0.02

= 0.52

Decimal intervals on a number line

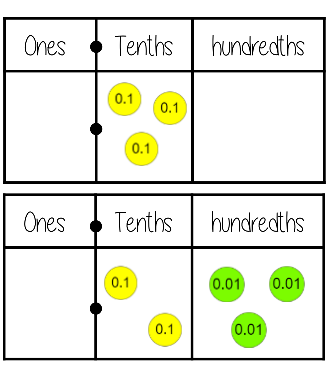
One whole spit into 10 parts makes tenths = 0.1

One tenth split into 10 parts makes hundredths = 0.01



Comparing decimals

Which the largest of 0.3 and 0.23?



0.3 > 0.23

"There are more counters in the furthest column to the left"

0.30

0.23

Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

Round to 1 significant figure

370 to 1 significant figure is 400

37 to 1 significant figure is 40

3.7 to 1 significant figure is 4

0.37 to 1 significant figure is 0.4

0.00000037 to 1 significant figure is 0.0000004

Round to the first non zero number

YEAR 7 – PLACE VALUE AND PROPORTION...

FDP equivalence

@whisto_maths

What do I need to be able to do?

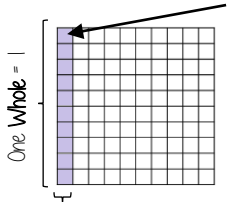
By the end of this unit you should be able to:

- Convert fluently between fractions, decimals & percentages

Keywords

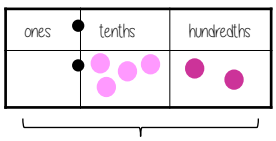
Fraction: how many parts of a whole we have
Decimal: a number with a decimal point used to separate ones, tenths, hundredths etc.
Percentage: a proportion of a whole represented as a number between 0 and 100
Place value: the numerical value that a digit has decided by its position in the number
Placeholder: a number that occupies a position to give value
Interval: a range between two numbers
Tenth: one whole split into 10 equal parts
Hundredth: one whole split into 100 equal parts
Sector: a part of a circle between two radius (often referred to as looking like a piece of pie)
Recurring: a decimal that repeats in a given pattern

Tenths and hundredths



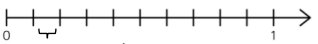
One **hundredth** (one whole split into 100 equal parts) = $\frac{1}{100}$ = 0.01

One **tenth** (one whole split into 10 equal parts) = $\frac{1}{10}$ = 0.1



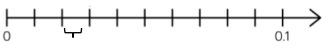
0 ones, 5 tenth and 2 hundredths
0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01
= 0 + 0.5 + 0.02
= **0.52**

On a number line



One whole – split into 10 equal parts

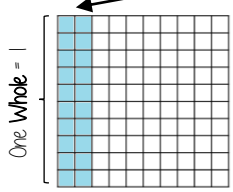
One tenth = $\frac{1}{10}$ = 0.1

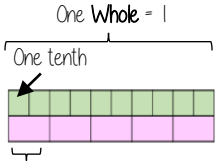


One tenth – split into 10 equal parts

One hundredth = $\frac{1}{100}$ = 0.01

Fifths

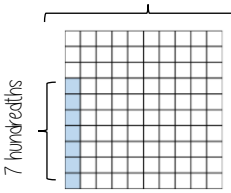




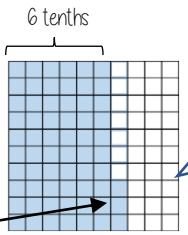
One **fifth** (one whole split into 5 equal parts) = $\frac{1}{5}$ = 0.2

Percentages on a hundred grid

100% = a whole = 100 hundredths



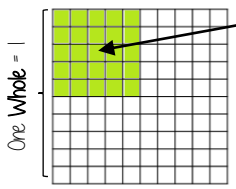
7 hundredths
7 out of 100
7%

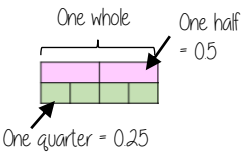


6 tenths and 3 hundredths
63 hundredths
63%

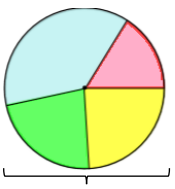
Quarters

One **quarter** (one whole split into 4 equal parts) = $\frac{1}{4}$ = 0.25





Simple pie charts



A pie chart has 360° so all FDP calculations are out of 360

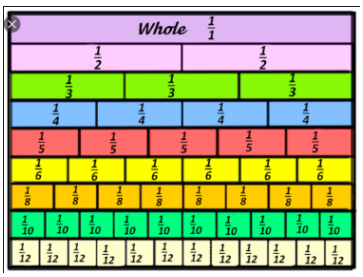
Split into 10 parts = 10% = 36°

Split into 2 parts = 50% = 180°

Split into 5 parts = 20% = 72°

Equivalent fractions

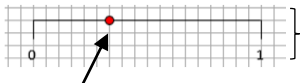
Represent equivalence with fraction walls



Fractions – on a diagram

The denominator is represented by EQUALLY sized parts – this is split into quarters

Fractions – on a number line



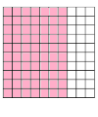
This point is at the 6th part
6 is the **numerator**

One whole split into 18 equal parts
18 is the **denominator**

$\frac{6}{18} \leftarrow \frac{3}{9} \leftarrow \frac{1}{3}$

Convert FDP

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares → 70 "hundredths" = 7 "tenths" = 0.7



→ 70 hundredths = 70%

Using a **calculator**

→ S⇌D → Convert to a decimal

This will give you the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals
e.g $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3