

# Constructing measuring

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

### Keywords

**Polygon** – A 2D shape made with straight lines  
**Scalene triangle** – a triangle with all different sides and angles  
**Isosceles triangle** – a triangle with two angles the same size and two sides the same size  
**Right-angled triangle** – a triangle with a right angle  
**Frequency** – the number of times a data value occurs  
**Sector** – part of a circle made by two radii touching the centre  
**Rotation** – turn in a given direction  
**Protractor** – equipment used to measure angles  
**Compass** – equipment used to draw arcs and circles

### Letter and labelling convention

The letter in the middle is the angle  
 The arc represents the angle

**Angle Notation:** three letters ABC  
 This is the angle at B = 113°

**Line Notation:** two letters EC  
 The line that joins E to C

### Draw and measure line segments

Conversions 1cm = 10mm, 1m = 100cm

The line segment is 3.9cm  
 Which is 39mm

AB is a line segment (part of the line)

Make sure the start of the line is at 0.

### Angles as measures of turn

**Clockwise**  
**Anti-Clockwise**

East to South is a quarter turn clockwise

**Quarter Turn** 90° Clockwise  
**Half Turn** 180°  
**Three-quarter Turn** 270° Anti-Clockwise  
**Full Turn** 360°

### Classify angles

**Acute Angles**  
 $0^\circ < \text{angle} < 90^\circ$

**Obtuse**  
 $90^\circ < \text{angle} < 180^\circ$

**Reflex**  
 $180^\circ < \text{angle} < 360^\circ$

**Right Angles**  
 $90^\circ$

Right angle notation

**Straight Line**  
 $180^\circ$

### Measure angles to 180°

This is the angle being measured

The base line follows the line segment

Make sure the cross is at the point the two lines meet

Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90° and 180°

### Draw angles up to 180°

Draw a 35° angle

Make a mark at 35° with a pencil. And join to the angle point (use a ruler)

Make sure the cross is at the end of the line (where you want the angle).

The angle

### Parallel and Perpendicular lines

**Parallel lines**  
 Straight lines that never meet (Have the same gradient)

**Perpendicular lines**  
 Straight lines that meet at 90°

### Angles over 180°

360° - smaller angle = reflex angle

Use your knowledge of straight lines 180° and angles around a point 360°

Measure the smaller angle first (less than 180°)

### Properties of Quadrilaterals

**Parallelogram**  
 Opposite sides are parallel  
 Opposite angles are equal  
 Co-interior angles

**Square**  
 All sides equal size  
 All angles 90°  
 Opposite sides are parallel

**Rectangle**  
 All angles 90°  
 Opposite sides are parallel

**Trapezium**  
 One pair of parallel lines

**Kite**  
 No parallel lines  
 Equal lengths on top sides  
 Equal lengths on bottom sides  
 One pair of equal angles

**Rhombus**  
 All sides equal size  
 Opposite angles are equal

### Draw Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192^\circ$

Use a protractor to draw. This is 192°

**Polygons**

3	- Triangle	5	- Pentagon	8	- Octagon
4	- Quadrilateral	6	- Hexagon	9	- Nonagon
		7	- Heptagon	10	- Decagon

### SAS, SSS, ASA constructions

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side

If all the sides and angles are the same, it is a **regular polygon**

# Geometric reasoning

## What do I need to be able to do?

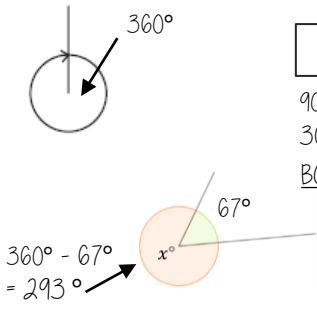
- By the end of this unit you should be able to:
- Understand/use the sum of angles at a point
  - Understand/use the sum of angles on a straight line
  - Understand/use equality of vertically opposite angles
  - Know and apply the sum of angles in a triangle
  - Know and apply the sum of angles in a quadrilateral

## Keywords

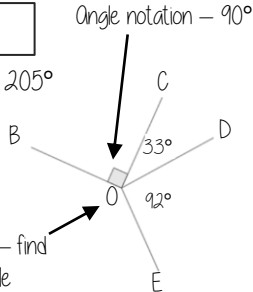
- Vertically Opposite:** angles formed when two or more straight lines cross at a point
- Interior Angles:** angles inside the shape
- Sum:** total, add all the interior angles together
- Convex Quadrilateral:** a four-sided polygon where every interior angle is less than  $180^\circ$
- Concave Quadrilateral:** a four-sided polygon where one interior angle exceeds  $180^\circ$
- Polygon:** a 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isoceles triangle:** a triangle with two angles the same size and two angles the same size
- Right-angled triangle:** a triangle with a right angle

## Sum of angles at a point

The sum of angles around a point is  $360^\circ$

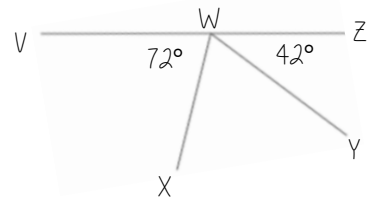


**Find angle BOE**  
 $90^\circ + 33^\circ + 92^\circ = 205^\circ$   
 $360^\circ - 205^\circ$   
**BOE =  $155^\circ$**



## Sum of angles on a straight line

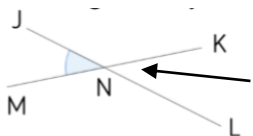
Adjacent angles that share a common point on a line add up to  $180^\circ$



**Find angle XWY**

$72^\circ + 42^\circ = 114^\circ$   
 $180^\circ - 114^\circ = \underline{66^\circ}$

## Vertically opposite angles

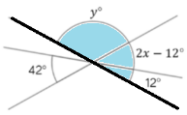


Angle JNM is vertically opposite to angle KNL

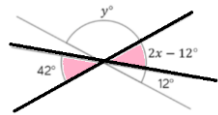
$JNM = KNL$

**Vertically opposite angles are the same**

Other angle rules still apply  
 Look for straight line sums and angles around a point

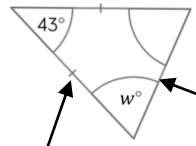


Form equations with information from diagrams  
 $2x - 12 = 42$   
 $2x = 54$   
 $x = 27^\circ$



## Sum of angles in triangles

Sum of interior angles in a triangle =  $180^\circ$



The two base angles will be the same size

Look at triangle notation  
 This indicates an isosceles triangle  
 $\therefore 180 - 43 = 137$   
 $137 \div 2 = 68.5^\circ$

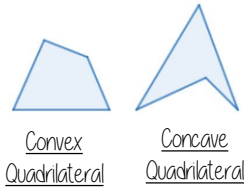
A triangle can only have **ONE** right angle



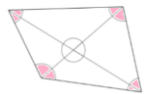
Have a go!  
 Tearing the corners from triangles forms a straight line which is therefore  $180^\circ$

## Sum of angles in quadrilaterals

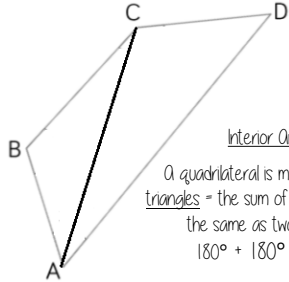
Sum of interior angles in a quadrilateral =  $360^\circ$



Convex Quadrilateral  
 Concave Quadrilateral



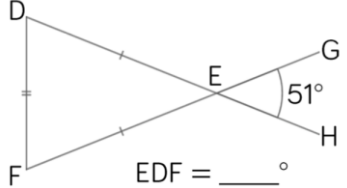
Interior angles are those that make up the perimeter (outline) of the shape



Interior Angles  
 A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles  
 $180^\circ + 180^\circ = 360^\circ$

## Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



- Angle DEF =  $51^\circ$  because it is a vertically opposite angle DEF = GEH
- Triangle DEF is isosceles (triangle notation)  $\therefore EDF = EFD$  and the sum of interior angles is  $180^\circ$   
 $180^\circ - 51^\circ = 129^\circ$        $129^\circ \div 2 = 64.5^\circ$
- Angle EDF =  $64.5^\circ$

Keep working out clear and notes together

# YEAR 7

# number sense

## What do I need to be able to do?

- By the end of this unit you should be able to:
- Know and use mental addition/ subtraction
  - Know and use mental multiplication/ division
  - Know and use mental arithmetic for decimals
  - Know and use mental arithmetic for fractions
  - Use factors to simplify calculations
  - Use estimation to check mental calculations
  - Use number facts
  - Use algebraic facts

## Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Dividend:** the number being divided
- Divisor:** the number we divide by
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Equation:** a mathematical statement that two things are equal
- Quotient:** the result of a division

## Mental methods for addition/ subtraction

Addition is commutative

$$6 + 3 = 3 + 6$$

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

The order of addition does not change the result

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

## Mental methods for multiplication/ division

Multiplication is commutative

$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

$$24 \times 6 = 20 \times 6 + 4 \times 6$$

$$= 120 + 24$$

$$= 144$$

Division is not associative

Chunking the division can help  $4000 \div 25$   
 "How many 25's in 100" then how many chunks of that in 4000.

## Mental methods for decimals

Multiplying by a decimal  $< 1$  will make the original value smaller e.g  $0.1 = \div 10$

Methods for multiplication  $12 \times 0.03$

$12 \times 3 = 36$	$12 \times 3 = 36$
$12 \times 0.3 = 3.6$	$\div 10 \downarrow + 100 \div 1000 \downarrow$
$12 \times 0.03 = 0.36$	$12 \times 0.03 = 0.36$

Methods for addition  $2.3 + 2.4$

$2 + 2 = 4$
$0.3 + 0.4 = 0.7$
$4 + 0.7 = 4.7$

Methods for division  $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

$$\begin{array}{r} 1.5 \div 0.05 \\ \times 100 \downarrow \quad \times 100 \downarrow \\ 150 \div 5 = 30 \end{array}$$

## Mental methods for fractions

Use bar models where possible

I've spent  $\frac{2}{5}$  of my money I have £21 left

How much did they have to begin with?

$$\frac{3}{3} + \frac{2}{3}$$

What is  $\frac{5}{3}$  of £15?

## Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative  
 Factors can be multiplied in any order

## Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach  $300 + 900$ .

The correct estimation would be  $200 + 900 = 1100$ .

## Number facts

Use  $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 50$$

For division you must consider the impact of the divisor becoming smaller or bigger.  
 Smaller — the answer will be bigger (it is being shared into less parts)  
 Bigger — the answer will be smaller (it is being shared into more parts)

## Algebraic facts

$$2a + 2b = 10 \quad \text{Everything } \times 2$$

$$0.1a + 0.1b = 0.5 \quad \text{Everything } \div 10$$

$$a + b = 5$$

$$a + b + 2 = 7 \quad \text{Add 2 to the total}$$

The unknown quantity isn't changing but the variables change what is done to give the result

# Sets and probability

### What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

### Keywords

**Set:** collection of things  
**Element:** each item in a set is called an element  
**Intersection:** the overlapping part of a Venn diagram (AND  $\cap$ )  
**Union:** two ellipses that join (OR  $\cup$ )  
**Mutually Exclusive:** events that do not occur at the same time  
**Probability:** likelihood of an event happening  
**Bias:** a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice  
**Fair:** there is zero bias, and all outcomes have an equal likelihood  
**Random:** something happens by chance and is unable to be predicted

### Identify and represent sets

The **universal set** has this symbol  $\xi$  - this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things - you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$   
 $A = \{1, 4, 9, 16, 25, 36, 49\}$

All the numbers in set A are square number and between 1 and 50

### Interpret and create Venn diagrams

**Mutually exclusive sets**  
 The two sets have nothing in common  
 No overlap

**Union of sets**  
 The two sets have some elements in common - they are placed in the intersection

**Subset**  
 All of set B is also in Set A so the ellipse fits inside the set

The box  
 Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

### Intersection of sets

Elements in the intersection are in set A AND set B

The notation for this is  $A \cap B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$   
 $A = \{\text{Multiples of 5}\}$     $B = \{\text{Multiples of 3}\}$

The element in  $A \cap B$  is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

### Union of sets

Elements in the union could be in set A OR set B

The notation for this is  $A \cup B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$   
 $A = \{\text{Multiples of 5}\}$     $B = \{\text{Multiples of 3}\}$

The elements in  $A \cup B$  are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

This Venn shows the **number of elements** in each set

### Sample space - for single events

A sample space represents a possible outcome from an event

They can be interpreted in a variety of ways because they do not tell you the probability

A sample space for rolling a six-sided dice is  $S = \{1, 2, 3, 4, 5, 6\}$

A sample space for this spinner is  $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

### Probability of a single event

Probability =  $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

$P(\text{Blue}) = \frac{4}{10}$  ← There are 4 blue sectors  
 ← There are 10 sectors overall

Probability notation  $P(\text{event})$

$= \frac{2}{5}$

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

Probability is always a value between 0 and 1

### The probability scale

Impossible 0 or 0%      Even chance 0.5,  $\frac{1}{2}$  or 50%      Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is  $\frac{1}{5}$

### Sum of probabilities

Probability is always a value between 0 and 1

The probability of getting a blue ball is  $\frac{1}{5}$   
 ∴ The probability of **NOT** getting a blue ball is  $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$

# Prime numbers and Proof

## What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

## Keywords

**Multiples:** found by multiplying any number by positive integers

**Factor:** integers that multiply together to get another number.

**Prime:** an integer with only 2 factors

**Conjecture:** a statement that might be true (based on reasoning) but is not proven

**Counterexample:** a special type of example that disproves a statement

**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

**HCF:** highest common factor (biggest factor two or more numbers share)

**LCM:** lowest common multiple (the first time the times table of two or more numbers match)

## Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

$x$  could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

45 is not a multiple of 3 because it is  $3 \times 15$

Not an integer

## Factors

••••• Arrays can help represent factors •••••

$5 \times 2$  or  $2 \times 5$

**Factors of 10**  
1, 2, 5, 10

$10 \times 1$  or  $1 \times 10$

Factors and expressions

$x \ x \ x \ x \ x \ x$

The number itself is always a factor

**Factors of  $6x$**   
 $6, x, 1, 6x, 2x, 3, 3x, 2$

$6x \times 1$  OR  $6 \times x$

$x \ x$

$x \ x$

$2x \times 3$

$x \ x \ x$

$x \ x \ x$

$3x \times 2$

## Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number  
The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

## Square and triangular numbers

Square numbers



Representations are useful to understand a square number  $n^2$

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful - an extra counter is added to each new row



Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

## Common factors and HCF

Common factors are factors two or more numbers share

**HCF - Highest common factor**

**HCF of 18 and 30**

18 | 1, 2, 3, 6, 9, 18

30 | 1, 2, 3, 5, 6, 10, 15, 30

Common factors  
(factors of both numbers)  
1, 2, 3, 6

**HCF = 6**

6 is the biggest factor they share

## Common multiples and LCM

Common multiples are multiples two or more numbers share

**LCM - Lowest common multiple**

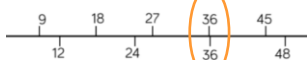
**LCM of 9 and 12**

9 | 9, 18, 27, 36, 45, 54

12 | 12, 24, 36, 48, 60

**LCM = 36**

The first time their multiples match



Comparing fractions

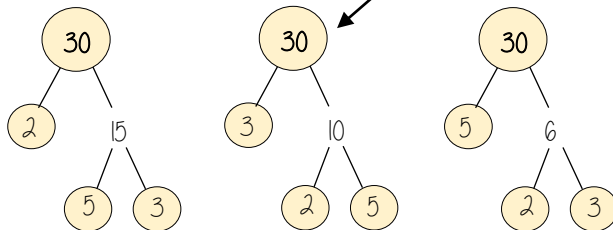
$\frac{3}{5}$  and  $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$  and  $\frac{7}{10}$

## Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

**$30 = 2 \times 3 \times 5$**

Multiplication of prime factors

Using prime factors for predictions

e.g. 60 |  $30 \times 2$  |  $2 \times 3 \times 5 \times 2$

150 |  $30 \times 5$  |  $2 \times 3 \times 5 \times 5$

## Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples



This sequence isn't doubling it is adding 2 each time

Only one counterexample is needed to disprove a conjecture