

# Using Percentages

## What do I need to be able to do?

By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

## Keywords

- Percent:** parts per 100 – written using the % symbol  
**Decimal:** a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.  
**Fraction:** a fraction represents how many parts of a whole value you have.  
**Equivalent:** of equal value.  
**Reduce:** to make smaller in value.  
**Growth:** to increase/ to grow.  
**Integer:** whole number, can be positive, negative or zero.  
**Invest:** use money with the goal of it increasing in value over time (usually in a bank).  
**Multiplier:** the number you are multiplying by  
**Profit:** the income take away any expenses/ costs

## FDP Equivalence

Percentage  
100% = a whole = 100 hundredths

One Whole = 1

10 hundredths  
10 out of 100  
10%

One hundredth  
(one whole split into 100 equal parts)

$\frac{10}{100} = \frac{1}{10} = 0.10$

ones	tenths	hundredths
	.	

## Converting FDP

70/100

This also means 70 - 100

70 out of 100 squares  
70 "hundredths"  
= 7 "tenths"  
0.7

Using a calculator

Convert to a decimal

$\times 100$  converts to a percentage

Be careful of recurring decimals  
eg  $\frac{1}{3} = 0.3333333$   
 $\frac{1}{3} = 0.\dot{3}$   
The dot above the 3

## Percentage Increase/ Decrease

Decrease

100%

42%

Decrease by 58%

Multiplier Less than 1

$100 - 0.58 = 0.42$

Increase

100%

Increase by 12%

Multiplier More than 1

$100\% + 12\% = 112\%$   
 $100 + 0.12 = 1.12$

## Percentage change

I bought a phone for £200  
A year later sold it for £125.

All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

## Reverse Percentages

40% of my number is 16  
What am I thinking of?

Original Number (100%)

16

40% = 16  
10% = 4  
100% = 40

140% of my number is 84. What is the original number?

Original Number (100%)

84

140% = 84  
10% = 6  
100% = 60

Try to scale down to 10% or 1% and then scale back up to 100%

## Difference in values

$\frac{\text{Difference in values}}{\text{Original value}} \times 100$

I bought a house for £180,000, I later sold it for £216,000

Percentage profit

Money made (profit value)  $\rightarrow \frac{36000}{180000} \times 100 = 20\%$



# Straight Line Graphs

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use  $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

## Keywords

**Gradient:** the steepness of a line

**Intercept:** where two lines cross. The y-intercept: where the line meets the y-axis

**Parallel:** two lines that never meet with the same gradient

**Co-ordinate:** a set of values that show an exact position on a graph

**Linear:** linear graphs (straight line) – linear common difference by addition/ subtraction

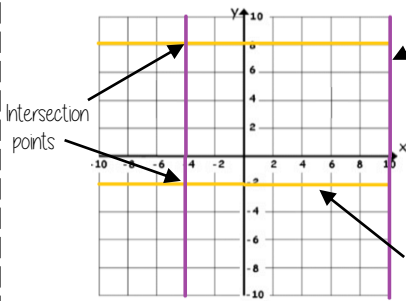
**Asymptote:** a straight line that a graph will never meet

**Reciprocal:** a pair of numbers that multiply together to give 1

**Perpendicular:** two lines that meet at a right angle

## Lines parallel to the axes

R



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form  $x = a$  and are vertical

Lines parallel to the x axis take the form  $y = a$  and are horizontal

All the points on this line have a y coordinate of -2 eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

## Plotting $y = mx + c$ graphs

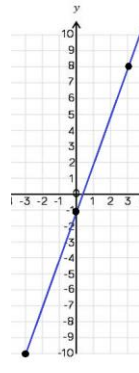
R

$y = 3x - 1$  → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

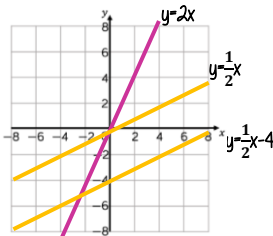
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

## Compare Gradients

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient – the steeper the line

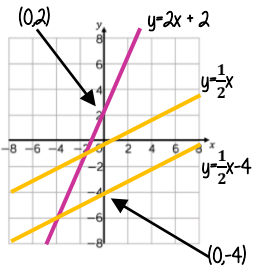
Positive gradients

Parallel lines have the same gradient

Negative gradients

## Compare Intercepts

$y = mx + c$  ← The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line

$y = mx + c$  ← The value of c is the point at which the line crosses the y-axis Y intercept  
y and x are coordinates

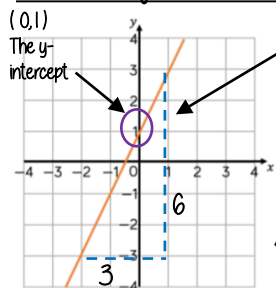
The equation of a line can be rearranged. Eg

$y = c + mx$

$c = y - mx$

Identify which coefficient you are identifying or comparing

## Find the equation from a graph



The Gradient  $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

## Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

## Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost. The gradient shows the price per pen.

The y-intercept shows the minimum charge. The gradient represents the price per mile

## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

## Keywords

- Credit:** money being placed into a bank account
- Debit:** money that leaves a bank account
- Balance:** the amount of money in a bank account
- Expense:** a cost/ outgoing
- Deposit:** an initial payment (often a way of securing an item you will later pay for)
- Multiplier:** a number you are multiplying by (Multiplier more than 1 = increasing, less than 1 = decreasing)
- Per Annum:** each year
- Currency:** the type of money a country uses
- Unitary:** one – the cost of one.

## Bills and Bank Statements

**Bills** – tell you the amount items cost and can show how much money you need to pay

Some can include a total  
Look for different units  
(Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

### Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19 <sup>th</sup> Sept	Salary	£1500		£1500
19 <sup>th</sup> Sept	Mortgage		£600	£900
25 <sup>th</sup> Sept	Bday Money	£15		£915

## Simple Interest

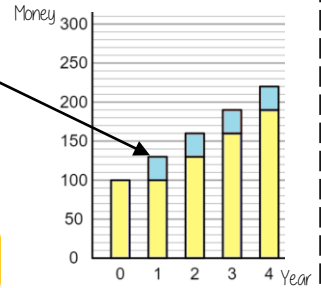
For each year of investment the interest remains the same

$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account  
e.g Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = \text{£}120$$

This account earned **£120** interest.  
At the end of year 4 they have **£220**



## Compound Interest

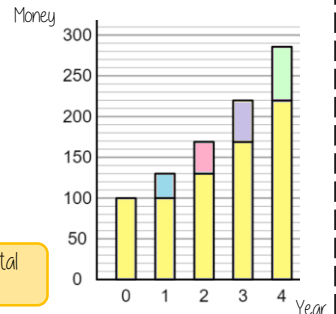
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = \text{£}285.61$$

This account has **£285.61** in total at the end of the 4 years.



## Value Added Tax (VAT)

VAT is payable to the government by a business in the UK VAT is 20% and added to items that are bought

Essential items such as food do not include VAT.

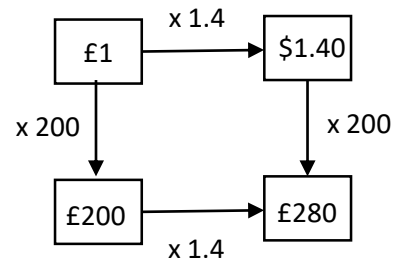
## Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:  
Time and a half – means 1.5 times their hourly rate  
Double – 2 times their hourly rate

## Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

### Common Currencies

United Kingdom	£	Pounds
United States of America	\$	Dollars
Europe	€	Euros

## Unit Pricing

4 Oranges £1	5 cupcakes £1.20
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$$\begin{aligned} 4 = \text{£}1.00 & \div 2 & 5 = \text{£}1.20 & \div 5 \\ 2 = \text{£}0.50 & & & \\ 1 = \text{£}0.25 & \div 2 & 1 = \text{£}0.20 & \end{aligned}$$

Cost per Unit

To calculate unit per cost you divide by the cost

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

# Transformations

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

## Keywords

**Rotate:** a rotation is a circular movement

**Symmetry:** when two or more parts are identical after a transformation

**Regular:** a regular shape has angles and sides of equal lengths

**Invariant:** a point that does not move after a transformation

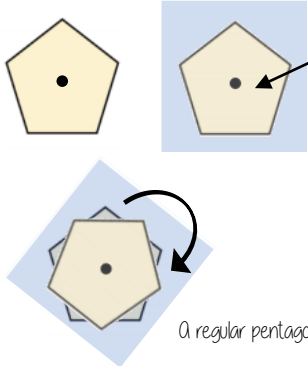
**Vertex:** a point two edges meet

**Horizontal:** from side to side

**Vertical:** from up to down

## Rotational Symmetry

Tracing paper helps check rotational symmetry



- 1 Trace your shape (mark the centre point)
- 2 Rotate your tracing paper on top of the original through  $360^\circ$
- 3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

## Translation and vector notation

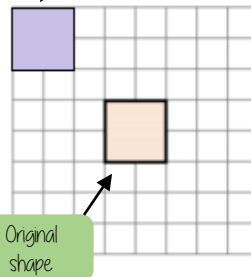
Vector Notation

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

How far left or right to move  
Negative value (left)  
Positive value (right)

How far up or down to move  
Negative value (down)  
Positive value (up)

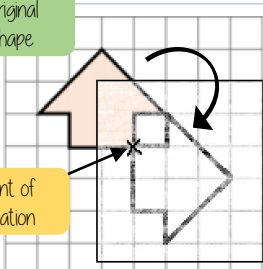
Translation  $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$



Every vertex has been translated by the same amount

## Rotate from a point (in a shape)

Original shape



Point of rotation

Image  $90^\circ$  clockwise

1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

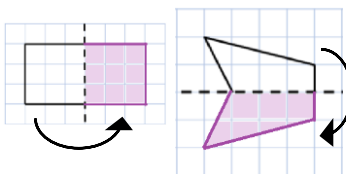
3 Draw the new shape



Clockwise

Anti-Clockwise

## Compare rotations and reflections



**R**

Reflections are a mirror image of the original shape

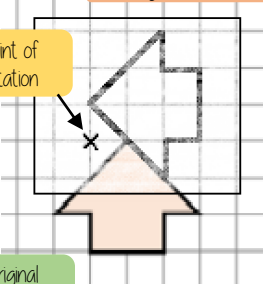
Information needed to perform a reflection:

- Line of reflection (Mirror line)

## Rotate from a point (outside a shape)

Image  $90^\circ$  anti-clockwise

Point of rotation



Original shape

1 Trace the original shape (mark the point of rotation)

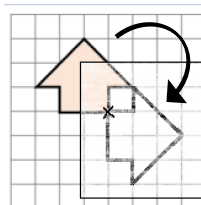
2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation



# Year 9 Transformations

## What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

## Keywords

**Mirror line:** a line that passes through the center of a shape with a mirror image on either side of the line

**Line of symmetry:** same definition as the mirror line

**Reflect:** mapping of one object from one position to another of equal distance from a given line.

**Vertex:** a point where two or more line segments meet

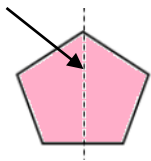
**Perpendicular:** lines that cross at  $90^\circ$

**Horizontal:** a straight line from left to right (parallel to the x axis)

**Vertical:** a straight line from top to bottom (parallel to the y axis)

## Lines of symmetry

Mirror line (line of reflection)



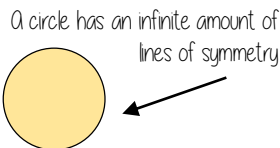
Shapes can have more than one line of symmetry...  
This regular polygon (a regular pentagon has 5 lines of symmetry)



Rhombus  
two lines of symmetry

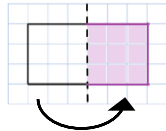
Parallelogram

No lines of symmetry

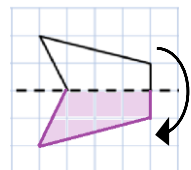


A circle has an infinite amount of lines of symmetry

## Reflect horizontally/ vertically (1)



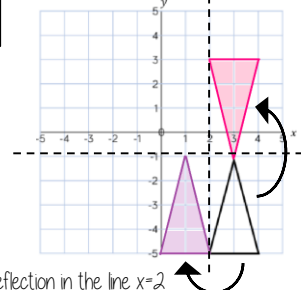
Reflection in a vertical line



Reflection in a horizontal line

Note a reflection doubles the area of the original shape

Reflection on an axis grid

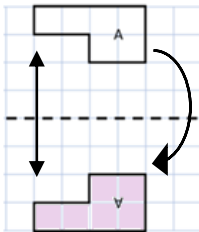


Reflection in the line  $y=2$

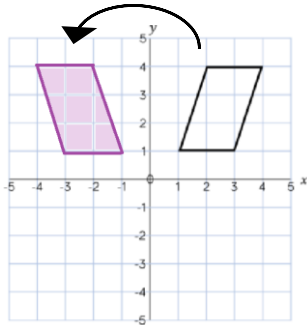
Reflection in the line  $x=2$

## Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



Reflection in the line  $y$  axis — this is also a reflection in the line  $x=0$



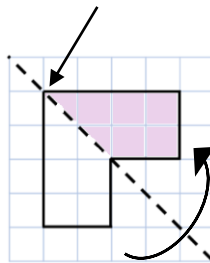
Lines parallel to the  $x$  and  $y$  axis

REMEMBER

Lines parallel to the  $x$ -axis are  $y = \dots$   
Lines parallel to the  $y$ -axis are  $x = \dots$

## Reflect Diagonally (1)

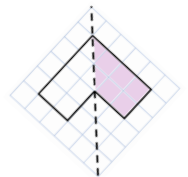
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

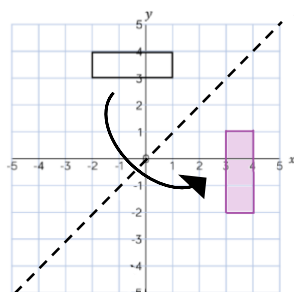


Drawing perpendicular lines

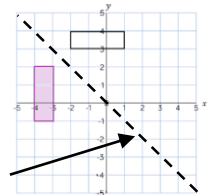
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

## Reflect Diagonally (2)

This is the line  $y = x$  (every  $y$  coordinate is the same as the  $x$  coordinate along this line)

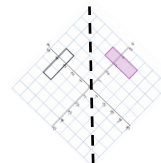


This is the line  $y = -x$   
The  $x$  and  $y$  coordinate have the same value but opposite sign



Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)



# YEAR 9 — REASONING WITH GEOMETRY... Enlargement & Similarity

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

## Keywords

**Similar Shapes:** shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

**Scale Factor:** the multiple describing how much a shape has been enlarged

**Enlarge:** to change the size of a shape (enlargement is not always making a shape bigger)

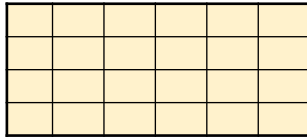
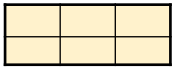
**Corresponding:** objects (or sides) that appear in the same place in two similar situations.

**Image:** the picture or visual representation of the shape

## Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

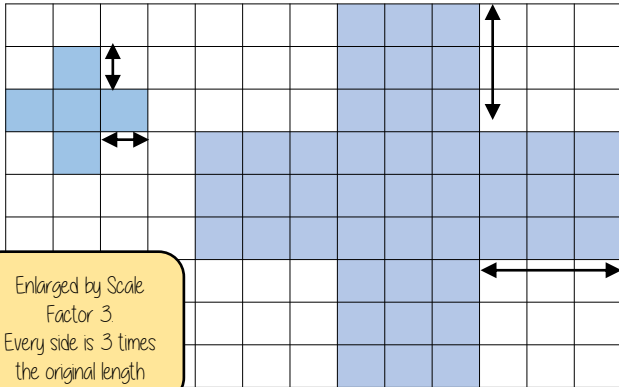
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

## Enlarge by a positive scale factor

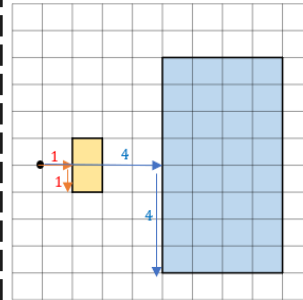
With a scale factor larger than 1 it makes the shape bigger



Enlarged by Scale Factor 3  
Every side is 3 times the original length

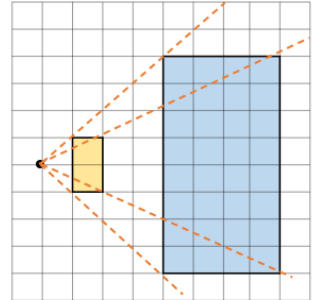
## Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

Rays method

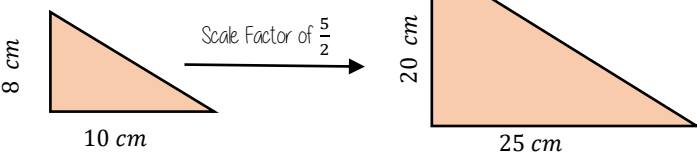
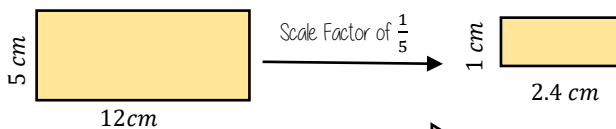


Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

## Enlarge by a positive scale factor

## Positive fractional scale factor

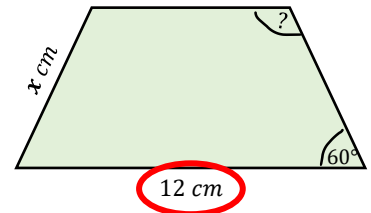
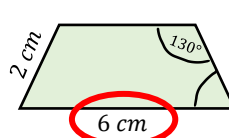
With a scale factor between 0 and 1 it makes the shape smaller



## Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes.

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor

$$\frac{12}{6} = 2$$

Scale Factor = 2

Calculate the missing side

Length (corresponding side)  $\times$  scale factor

$$2\text{ cm} \times 2$$

$$x = 4\text{ cm}$$

Enlargement does not change angle size

Calculate the missing angle

Corresponding angles remain the same  
 $130^\circ$