YEAR 9

Pythagoras' theorem

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

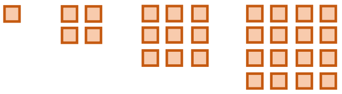
Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots



This can also be written as 6^2

1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

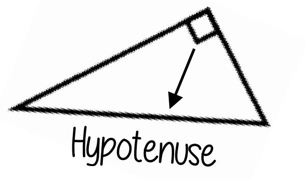
Square numbers

√ is the square root symbol

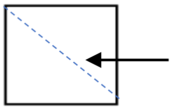
eg $\sqrt{64} = 8$

Because 8 × 8 = 64

Identify the hypotenuse

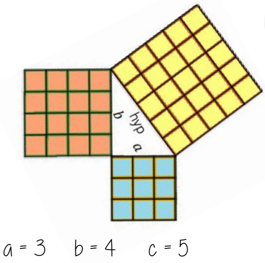


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

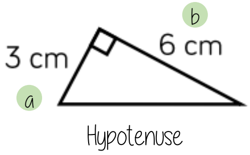
$$a^2 + b^2 = \text{hypotenuse}^2$$

eg $a^2 + b^2 = \textit{hypotenuse}^2$

$$\left.\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}\right\}$$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

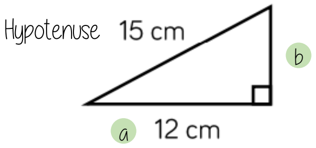
$$45 = \text{hypotenuse}^2$$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$$\sqrt{45} = \text{hypotenuse}$$

$$\mathbf{6.71}\boldsymbol{cm} = \text{hypotenuse}$$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1. Substitute in the values you are given

$$\underset{-144}{144} + b^2 = \underset{-144}{225}$$

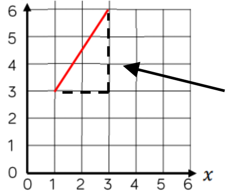
Rearrange the equation by subtracting the shorter square from the hypotenuse squared

Square root to find the length of the side

$$\left\{\begin{aligned} b^2 &= 111 \\ b &= \sqrt{111} = 10.54\ cm \end{aligned}\right.$$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$$\underbrace{a^2 + b^2}_{} = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

YEAR 9

Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

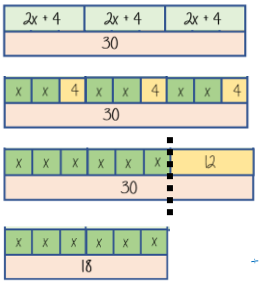
Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

R



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$\div 6 \quad \div 6$

$\underline{x = 3}$

Form and solve inequalities

R

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \quad +3x$

$2 > 17 + 3x$

$-17 \quad -17$

$-15 > 3x$

$\div 3 \quad \div 3$

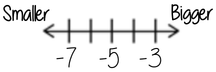
$\underline{-5 > x}$

x is true for any value smaller than -5

✓ CHECK IT!

$2 - 3(-6) = 20$

TRUE/ CORRECT



Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \quad -2$

$-3x > 15$

$\div -3 \quad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true...

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

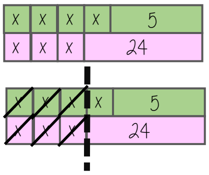
$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

$-5 \quad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

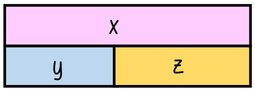
✓ -20 IS smaller than -18

Formulae and Equations

Substitute in values

Formulae – all expressed in symbols → **Equations** – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$y \rightarrow +z \rightarrow x$

$y \leftarrow -z \leftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging...

Change the subject

Make XXX the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \quad +3$

$4x = 12$

$\div 4 \quad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \quad +s$

$xy = a + s$

$\div y \quad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$ Gradient $= \dfrac{4}{2} = 2$

YEAR 9

Solving ratio & proportion problems

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40
$\times 0.5$ ↓ 2 cans of pop = £1.20 $\times 0.5$

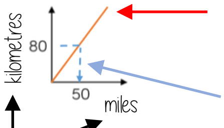
This multiplier is the same In the same way that this would be for ratio

4 cans of pop = £2.40
$\times 3$ ↓ 12 cans of pop = £7.20 $\times 3$

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.

Using a ruler helps for accuracy

Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2 × 4

T	1	2	8
G	40	20	5

× 2 ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

	Shop A	Shop B
	4 cans for £1.20	3 cans for 93p
	£1.20 ÷ 4	£0.93 ÷ 3
Cost per item	1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

	Shop A	
	4 cans for £1.20	3 cans for 93p
	4 ÷ £1.20	3 ÷ £0.93
Cost per pound	£1 buys 3.333 cans of pop	£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

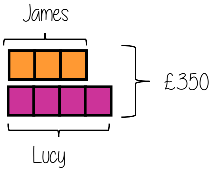
Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole: £350

7 parts to share between

(3 James, 4 Lucy)

£350 ÷ 7 = £50

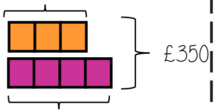
☐ = one part = £50

Put back into the question

James: Lucy

$$\begin{tikzcd}
3 \arrow[d, bend right, "\times 50"'] & : & 4 \arrow[d, bend left, "\times 50"] \\
£150 & : & £200
\end{tikzcd}$$

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

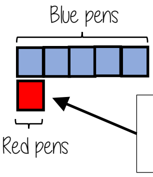
Inside a box are blue and red pens in the ratio 5:1.
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

☐ = one part = 10 pens



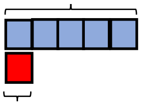
One unit = 10 pens

Put back into the question

Blue : Red

$$\begin{tikzcd}
5 \arrow[d, bend right, "\times 10"'] & : & 1 \arrow[d, bend left, "\times 10"] \\
50 & : & 10
\end{tikzcd}$$

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 9

3D Shapes

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g. length and width

3D: three dimensions to the shape e.g. length, width and height

Vertex: a point where two or more line segments meet

Edge: a line on the boundary joining two vertex

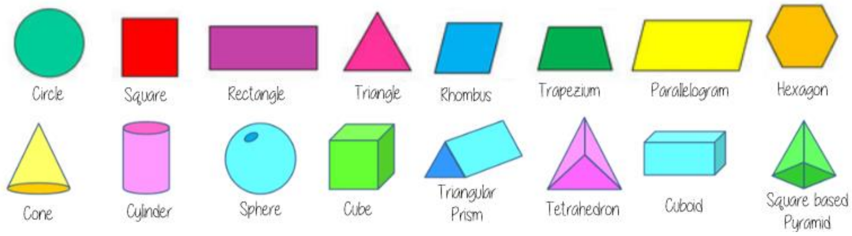
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

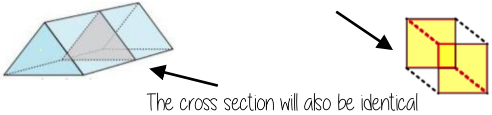
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes

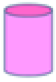


Recognise prisms

A solid object with two identical ends and flat sides

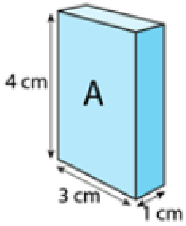


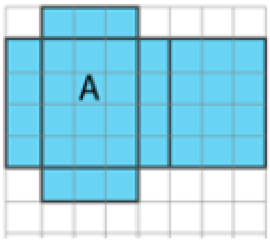
The cross section will also be identical to the end faces.



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

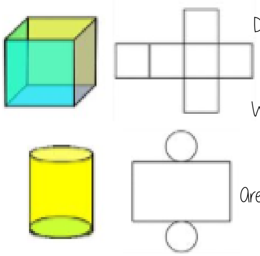




1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

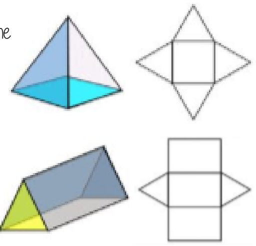
Sketch and recognise nets



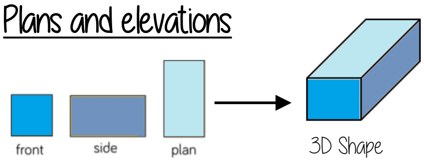
Do they have the same number of faces?

Where do the edges join?

Are the shapes of the faces correct?



Plans and elevations

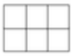


The direction you are considering the shape from determines the front and side views

Area of 2D shapes

Rectangle

Base x Height

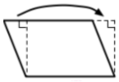


Triangle

½ x Base x Perpendicular height

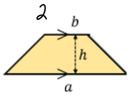
Parallelogram/ Rhombus

Base x Perpendicular height



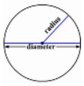
Area of a trapezium

$\frac{(a+b) \times h}{2}$



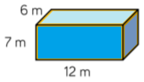
Area of a circle

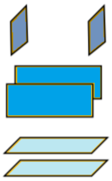
π x radius^2



Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area





Sides 6 x 7

6 x 7

Front and back 12 x 7

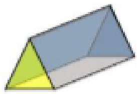
12 x 7

Top and Bottom 12 x 6

12 x 6

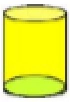
Sum of all sides is surface area

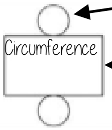
For cubes and cuboids you can also find one of each face and double it



For other shapes = not all the sides are the same, so calculate the individually

Surface area - cylinders





The area of the circle

π x radius^2

The width of this face is the same as the circumference

π x diameter x height

$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$

Volumes

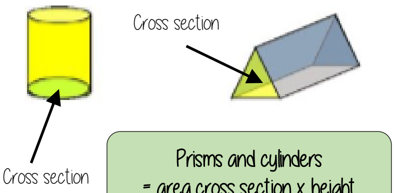
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Prisms and cylinders
= area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 9

Numbers

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What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative

Rational: a number that can be made by dividing two integers

Irrational: a number that cannot be made by dividing two integers

Inverse operation: the operation that reverses the action

Quotient: the result of a division

Product: the result of a multiplication

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

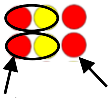
The square root of a negative is not a real number and cannot be found

Directed number R

🔴 = −1
🟡 = 1

Addition

2 + - 4 = -2



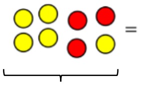
Zero pair (-1 + 1 = 0)

Two " – 1 " left = =2

Generalisation

+ – = –

Subtraction



= 2

Representation for calculation

Generalisation

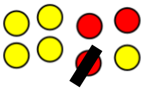
– – = +

"Subtract" – means take away or remove

2 - - 1 = 3

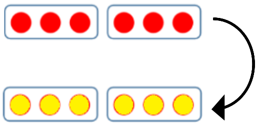
Take away one 🔴

Start with the representation of 2



= 3

Multiplication



-2 x -3 = 6

The act of making counters into their negative is turning them over

Divisions are the inverse operations

a = 5

b = -4

Brackets around negative substitutions helps remove calculation errors

2a – b = 2 x 5 – (-4) = 10 + 4 = 14

HCF/LCM R

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Standard form R

$A \times 10^{n}$

Any number between 1 and less than 10

Any integer

$6 \times 10^5 + 8 \times 10^5$

= 600000 + 800000

= 1400000

= 1.4×10^5

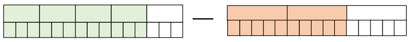
$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \quad \times \quad 10^5 \div 10^3$

= 5×10^2

Addition/ Subtraction of fractions R

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplication/ Division of fractions R

$\frac{3}{4} \times \frac{2}{3}$

Shade in 3 parts

Repeat it on this many rows

This many columns

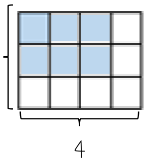
This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



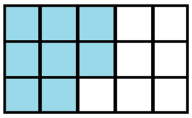
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$