YEAR 8

Multiplying and Dividing Fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned

Keywords

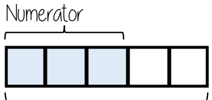
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Denominator: the number below the line on a fraction. The number represent the total number of parts.
Whole: a positive number including zero without any decimal or fractional parts.
Commutative: an operation is commutative if changing the order does not change the result.
Unit Fraction: a fraction where the numerator is one and denominator a positive integer.
Non-unit Fraction: a fraction where the numerator is larger than one.
Dividend: the amount you want to divide up.
Divisor: the number that divides another number.
Quotient: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient
Reciprocal: a pair of numbers that multiply together to give 1

Representing a fraction

$\frac{\text{Numerator}}{\text{Denominator}}$

$\frac{3}{5}$

Number of parts represented
Numerator



Number of parts to make up the whole
Denominator

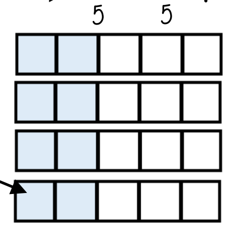
ALL PARTS of a fraction are of equal size

Repeated addition = multiplication by an integer

$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

Integer (Whole number)

Each part represents $\frac{1}{5}$

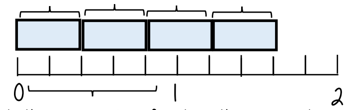


$\frac{\text{How many parts are shaded?}}{\text{What each part represents}}$

Revisit: When adding fractions with the same denominator = add the numerators

$= \frac{8}{5} \leftarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

$= 1\frac{3}{5}$



Each whole is split into the same number of parts as the denominator

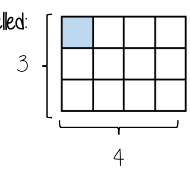
Multiplying unit fractions

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Multiplying non-unit fractions

$\frac{3}{4} \times \frac{2}{3}$

Shade in 3 parts

Repeat it on this many rows

This many columns

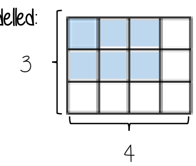
This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Quick Multiplying and Cancelling down

$\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{9}_{3}}$

The 3 and the 9 have a common factor and can be simplified

Quick Solving
Multiply the numerators
Multiply the denominators

$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is $\frac{1}{3}$ and vice versa

Reciprocals for division

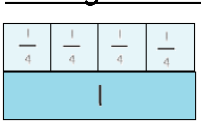
e.g.

$5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

Dividing an integer by an unit fraction



$1 \div \frac{1}{4} = 4$

How many quarters are in 1?

"There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes"

$5 \div \frac{1}{4} = 20$

Dividing any fractions

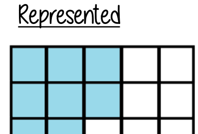
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 8

Ratio and Scale

@whistonmaths

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved.

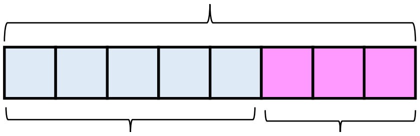
Keywords

Ratio: a statement of how two numbers compare
Equal Parts:: all parts in the same proportion, or a whole shared equally
Proportion: a statement that links two ratios
Order: to place a number in a determined sequence
Part: a section of a whole
Equivalent: of equal value
Factors: integers that multiply together to get the original value
Scale: the comparison of something drawn to its actual size.

Representing a ratio

"For every 5 boys there are 3 girls"

This is the "whole" – boys and girls together

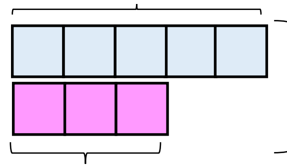


This represents the 5 boys   This represents the 3 girls

5:3

Double Number Line

This represents the 5 boys



This is the "whole" – boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat. ✕

Simplifying a ratio

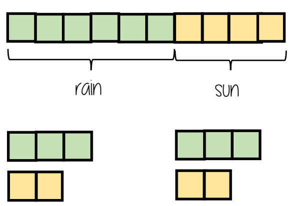
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2



rain   sun

Find the biggest common factor that goes into all parts of the ratio

For **6** and **4** the biggest factor (number that multiplies into them is **2**)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

4 : 20

1 : 5

The question states that **this part has to be 1 unit** Therefore **Divide by 4**

This side has to be divided by 4 too – to keep in **proportion**

H the n part does not have to be an integer for this type of question

Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions

mm →(x10) cm →(x100) m →(x1000) km; km →(÷1000) m →(÷100) cm →(÷10) mm

g →(x1000) kg; kg →(÷1000) g

ml →(x1000) L; L →(÷1000) ml

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4

James
Lucy
£350

Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

☐ = one part = £50

Put back into the question

James: Lucy
x 50 3 : 4 x 50
£150:£200

James = 3 x £50 = £150
£350
Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

☐ = one part = 10 pens

Blue pens
Red pens

One unit = 10 pens

Put back into the question

Blue : Red
x 10 5 : 1 x 10
50: 10

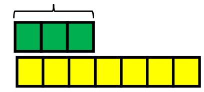
Blue pens = 5 x 10 = 50 pens
Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

Ratio as a fraction

Trees: Flowers
3 : 7

Trees



Flowers

Ratio

There are **3** parts for trees

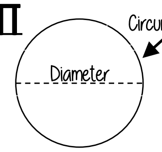
Fraction of trees

$$\frac{\text{Number of parts of in group}}{\text{Total number of parts}} \rightarrow \frac{3}{10}$$

Fraction

Tree parts **3** + Flower parts **7** = **10**

Pi π



The ratio of a circles circumference to its diameter

YEAR 8

Multiplicative Change

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

Keywords

Proportion: a statement that links two ratios
Variable: a part that the value can be changed
Axes: horizontal and vertical lines that a graph is plotted around
Approximation: an estimate for a value
Scale Factor: the multiple that increases/ decreases a shape in size
Currency: the system of money used in a particular country
Conversion: the process of changing one variable to another
Scale: the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 (× 0.5) → 2 cans of pop = £1.20 (× 0.5)

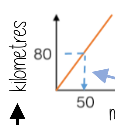
This multiplier is the same In the same way that this would be for ratio

4 cans of pop = £2.40 (× 3) → 12 cans of pop = £7.20 (× 3)

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

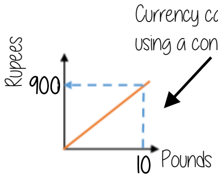
Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees (× 10) → £10 = 900 Rupees (× 10)

Currency can be converted using a conversion graph

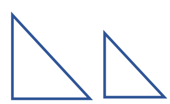


Convert 630 Rupees into Pounds

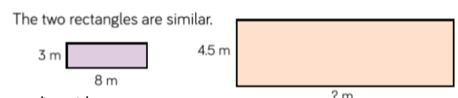
630 ÷ 90 = 7

£1 = 90 Rupees (× 7) → £7 = 630 Rupees (× 7)

Ratio between similar shapes



Angles in similar shapes do not change.
e.g. if a triangle gets bigger the angles can not go above 180^0



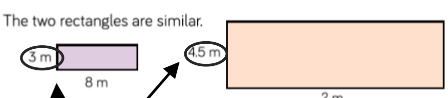
Corresponding sides

3m : 4.5m (÷ 3) → 1 m : 1.5 m (÷ 3)

8m : 12m (× 8) ← 1 m : 1.5 m (× 8)

Note
Simplify to the same ratio

Understand Scale Factor



3 x 1.5 = 4.5

This is a multiplicative change

Use corresponding sides to calculate a scale factor

Missing length
8 x 1.5 = 12m

Scale factor can also be calculated by:

$\frac{\text{Bigger corresponding side}}{\text{Smaller corresponding side}}$

Small corresponding side → × SF → Big corresponding side
Big corresponding side → ÷ SF → Small corresponding side

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image	:	Real life
1cm	:	30cm
10cm	:	300cm

(× 10)

The car in real life is 210cm

Image	:	Real life
1cm	:	30cm
7cm	:	210 cm

(× 7)

Interpret maps with scale factors

mm ÷ 10 → cm ÷ 100 → m ÷ 1000 → km

km × 1000 → m × 100 → cm × 10 → mm

1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m
1 cm : 25000cm

250 x 100 = 25000

For every 1cm on my map is 25000cm in real life.

YEAR 8 - ALGEBRAIC TECHNIQUES

What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Linear: the difference between terms increases or decreases (+ or -) by a constant value each time
Non-linear: the difference between terms increases or decreases in different amounts, or by x or ÷
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

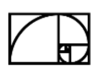
Linear Sequences – increase by addition or subtraction and the same amount each time
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

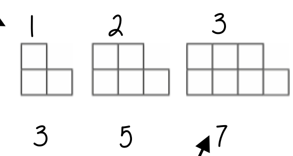
0 1 1 2 3 5 8 …

Each term is the sum of the previous two terms



Sequence in a table and graphically

Position: the place in the sequence



"The **term** in position 3 has 7 squares"

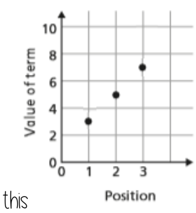
Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ → Substitute the number of the term you are looking for in place of 'n'

eg
1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$3n - 4 = 201$

Algebraic rule → ; ← Term to check

Solving this will find the position of the term in the sequence.
ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$2n^2$ – 2 times whatever n squared is

eg
1st term = $2 \times 1^2 = 2$
2st term = $2 \times 2^2 = 8$
100th term = $2 \times 100^2 = 2000$

$(2n)^2$ – 2 times n then square the answer

eg
1st term = $(2 \times 1)^2 = 4$
2st term = $(2 \times 2)^2 = 16$
100th term = $(2 \times 100)^2 = 40000$

$n(n + 5)$ ←

eg
1st term = 1 (1 + 5) = 6
2st term = 2 (2 + 5) = 14
100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20…

4n

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 8 - ALGEBRAIC TECHNIQUES

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

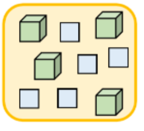
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Coefficient: The number used to multiply a variable
Simplify: To reduce a power to its lowest term
Product: Multiply

Addition/ Subtraction with indices

Coefficient → $5x^2 + 4x^4$ ← Power

$5x^2$: Term; $4x^4$: Term; $5x^2 + 4x^4$: Expression

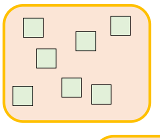
$5x^2 + 4x^4$ →



Each square represents x^2 and each cube represents x^4

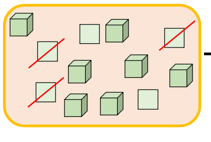
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →



→ $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →



→ $2x^2 + 7x^4$

Divide expressions with indices

$$\frac{24}{36} \rightarrow \frac{2 \times 2 \times 2 \times 3}{2 \times 3 \times 2 \times 3} \rightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{5 \times a \times a \times a \times b \times b}{3 \times 5 \times a \times b \times b \times b \times b \times b \times b} \rightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$3^5 \times 3^2$
$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$ → 3^7

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2$ → 3^3

$$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

YEAR 8

Representing Data

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and interpret scatter graphs
- Describe correlation and relationships.
- Identify different types of non-linear relationships.
- Design and complete an ungrouped frequency table.
- Read and interpret grouped tables (discrete and continuous data)
- Represent data in two way tables.

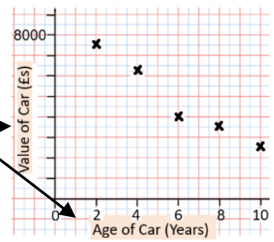
Keywords

Variable: a quantity that may change within the context of the problem.
Relationship: the link between two variables (items). E.g. Between sunny days and ice cream sales
Correlation: the mathematical definition for the type of relationship.
Origin: where two axes meet on a graph.
Line of best fit: a straight line on a graph that represents the data on a scatter graph.
Outlier: a point that lies outside the trend of graph.
Quantitative: numerical data
Qualitative: descriptive information, colours, genders, names, emotions etc.
Continuous: quantitative data that has an infinite number of possible values within its range.
Discrete: quantitative or qualitative data that only takes certain values.
Frequency: the number of times a particular data value occurs.

Draw and interpret a scatter graph.

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship



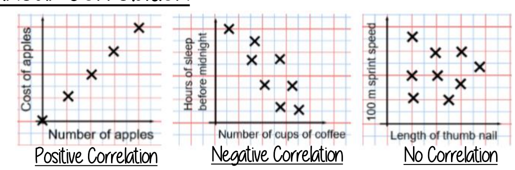
All axes should be labelled

The axis should fit all the values on and be equally spread out

"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

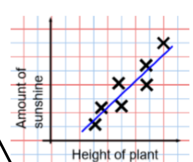
Linear Correlation



Positive Correlation	Negative Correlation	No Correlation
As one variable increases so does the other variable	As one variable increases the other variable decreases	There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

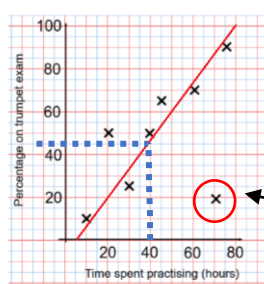
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "outlier"
It is an outlier because it doesn't fit this model and stands apart from the data

Ungrouped Data

The number of times an event happened

The table shows the number of siblings students have. The answers were
3, 1, 2, 2, 0, 3, 4, 1, 1, 2, 0, 2

Number of siblings	Frequency	
0	2	0
1	3	3
2	4	2 + 2 + 2 + 2 OR 2 x 4 = 8
3	2	3 + 3 OR 3 x 2 = 6
4	1	4

2 people had 0 siblings. This means there are 0 siblings to be counted here

2 people have 3 siblings so there are 6 siblings in total

Best represented by discrete data (Not always a number)

OVERALL there are
0 + 3 + 8 + 6 + 4
Siblings = 21 siblings

Grouped Data

If we have a large spread of data it is better to group it. This is so it is easier to look for a trend. Form groups of equal size to make comparison more valid and spread the groups out from the smallest to the largest value.

Discrete Data
The groups do not overlap

Cost of TV (£)	Tally	Frequency
101 – 150	~~IIII~~ II	7
151 – 200	~~IIII~~ ~~IIII~~ I	11
201 – 250	~~IIII~~	5
251 - 300	III	3

We do not know the exact value of each item in a group – so an estimate would be used to calculate the overall total (Midpoint)

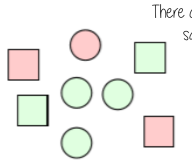
Continuous Data
To make sure all values are included inequalities represent the subgroups

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5

e.g. this group includes every weight bigger that 60Kg, up to and including 70Kg

Representing data in two-way tables

Two-way tables represent discrete information in a visual way that allows you to make conclusions, find probability or find totals of sub groups



There are 2 green squares

There are 5 green shapes

	Squares	Circles	Total
Green	2	3	5
Red	2	1	3
Total	4	4	8

There are 8 items in total

Using your two-way table

To find a fraction
e.g. What fraction of the items are red? **3 red items** but **8 items in total** = $\frac{3}{8}$

Interleaving: Use your fraction, decimal percentage equivalence knowledge

YEAR 8

Tables and Probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct a sample space diagram.
- Systematically list outcomes.
- Find the probability from two-way tables.
- Find the probability from Venn diagrams.

Keywords

Outcomes: the result of an event that depends on probability.
Probability: the chance that something will happen.
Set: a collection of objects.
Chance: the likelihood of a particular outcome.
Event: the outcome of a probability — a set of possible outcomes.
Biased: a built in error that makes all values wrong by a certain amount.
Union: Notation ‘∪’ meaning the set made by comparing the elements of two sets.

Construct sample space diagrams

Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation to list the outcomes S =

In between the { } are a;; the possible outcomes

S = { 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

What is the probability that an outcome has an even number and a tails?

This is the set notation that represents the question P

P (Even number and Tails) = $\frac{3}{12}$

In between the ( ) is the event asked for

There are three even numbers with tails

There are twelve possible outcomes

Numerator: the event

Denominator: the total number of outcomes

Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

P (Girl walk to school) = $\frac{21}{100}$

The event

The total in the set

The total number of items

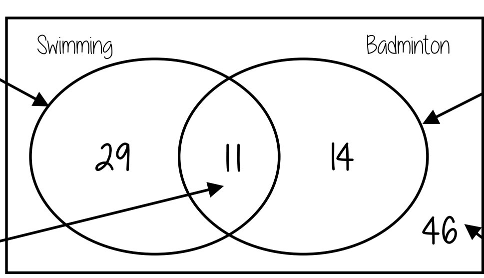
Product Rule

The number of items in event a × The number of items in event b

Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club.
40 went swimming, 25 went to badminton and 11 went to both.

This whole curve includes everyone that went swimming.
Because 11 did both we calculate **just** swimming by 40- 11



This whole curve includes everyone that went to badminton.
Because 11 did both we calculate **just** badminton by 25 - 11

The intersection represents both.
Swimming **AND** badminton

The number outside represents those that did **neither** badminton or swimming

100 — 29 — 11 — 14

P (Just swimming) = $\frac{29}{100}$