## 1 NUMBER - Further Maths

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## Section 1.1

## Mark schemes

Q1.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $1: 2: 5$ | B3 | B2 For any ratio that is one step <br> away from the answer <br> e.g. $\sqrt{ } 12: 2 \sqrt{ } 12: 5 \sqrt{ } 12$ <br> $\sqrt{ } 1: \sqrt{ } 4: \sqrt{ } 25$ <br> $2: 4: 10$ <br>  |
|  | B1 For at least two of the three <br> terms in their simplest form <br> i.e. two of $2 \sqrt{ } 3: 4 \sqrt{ } 3: 10 \sqrt{ } 3$ <br> B1 For any correct equivalent <br> ratio <br> e.g. $\sqrt{ } 2: \sqrt{ } 8: \sqrt{ } 50$ <br> $\sqrt{3}: \sqrt{ } 12: \sqrt{ } 75$ |  |

Q2.

| Answer | Mark | Comments |
| :---: | :---: | :--- |
| 29 and 23 identified | B2 | B1 $1(n+9)(n+3)$ or 667 or 29 or <br> 23 |

Q3.

| Answer | Mark | Comments |
| :---: | :---: | :---: |


| $1.5 t$ | M1 | $\begin{aligned} \hline \text { oe eg1 } & t+\frac{50}{100} t \\ \text { eg2 } & 2 x=3 t \\ \text { eg3 } & x: t=3: 2 \end{aligned}$ |
| :---: | :---: | :---: |
| 0.9w | M1 | $\begin{aligned} \text { oe eg1 } & w-\frac{10}{100} w \\ \text { eg2 } & 10 y=9 w \\ \text { eg3 } & w: y=10: 9 \end{aligned}$ |
| Their $1.5 t=$ their $0.9 w$ | M1dep | Dep on at least one M mark gained <br> oe eg1 $\frac{\text { their } 0.9}{\text { their } 1.5}$ $\begin{aligned} & \operatorname{eg} 215 t=9 w \\ & \operatorname{eg} 3 w(: x): t=10(: 9): 6 \end{aligned}$ |
| 0.6 | A1ft | ft from M1 M0 M1 or M0 M1 M1 SC2 1.6 or 1.67 <br> SC1 ${ }^{\frac{5}{3}}$ oe fraction |

Alternative method
\(\left.$$
\begin{array}{|l|c|l|}\hline \begin{array}{l}\text { Chooses an appropriate pair } \\
\text { of values for } x \text { and } t \\
\text { eg } x=90 \text { and } t=60\end{array} & \mathrm{M} 1 & \begin{array}{l}\text { Chooses an appropriate pair of } \\
\text { values for } y \text { and } w \\
\text { eg } y=180 \text { and } w=200\end{array} \\
\hline \text { Their } 90=0.9 w \quad(w=100) & \mathrm{M} 1 & \text { Their } 180=1.5 t \quad(t=120) \\
\hline \frac{\text { their } 60}{\text { their } 100} & \text { M1dep } & \begin{array}{l}\text { Dep on at least one M mark } \\
\text { gained } \\
\text { their } 120\end{array}
$$ <br>

their 200\end{array}\right]\)| At from M1 M0 M1 or M0 M1 M1 |
| :--- |
| SC2 1.6 or 1.67 |
| 0.6 |

Q4.
(a)

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $\frac{2}{5} n$ or $0.4 n$ | B1 | oe |

(b)

| $(10 m=) 10 \times$ their $\frac{2}{5} n(=4 n)$ | M 1 | $10 \times 2(=20)$ and $3 \times 5(=$ <br> $15)$ |
| :--- | :--- | :--- |
| $4: 3$ | A1ft | oe numerical ratio of integers <br> ft their $\frac{2}{5} n$ if used |

Q5.

| Answer | Mark | Comments |
| :---: | :---: | :---: |
| Alternative method 1 |  |  |
| $1.2 \text { or } \frac{6}{5}$ | M1 | oe could be seen in calculation ( $120 \%$ is not M1 - something needs to have been done with it) <br> 5 <br> $\overline{6}$ if used correctly could be an oe. Don't award just for $\frac{5}{6}$ seen |
| $\begin{aligned} & 1.2 x+1.2=x+6 \\ & \text { or } 0.2 x+1.2=6 \\ & \text { or } 0.2 x=4.8 \end{aligned}$ | M1dep | oe but must have expanded brackets missing brackets need to be recovered |
| 24 | A1 |  |

Alternative method 2

| $(x+1)+\frac{(x+1)}{5}$ | M1 | oe |
| :--- | :--- | :--- |
| $\frac{(x+1)}{5}=5$ |  |  |
| or $(x+1)=25$ | M1dep | oe <br> eg could be written as $20 \%$ of $(x$ <br> $+1)=5$ |
| 24 | A1 |  |


| Additional Guidance |  |
| :--- | :--- |
| $20 \%=5$ or $100 \%=25$ |  |
| $1.2(x+1)=x+6$ then $1.2 x+1=x+6$ would not gain | SC 1 |

Q6.

| Answer | Mark | Comments |
| :---: | :---: | :---: |
| $2: 1$ | B2 | B1Ratio equivalent to $2: 1$ <br> or |
|  | $1: 2$ <br> SC1 1 <br> Ratio seen that is correctly <br> converted to simplest form |  |


| Additional Guidance |  |
| :--- | :---: |
| Equivalent ratios may involve decimals or fractions eg $1.8: 0.9$ | B1 |
| Equivalent ratios must be a pair of values or a pair of single term <br> expressions in the same variable |  |
| eg1 $36: 18$ | B1 |
| eg2 | $6 b: 3 b$ |
| eg3 $20-2: 9$ | B1 |
| For B1 equivalent ratios to $2: 1$ can be seen as fractions <br> $\frac{18}{9}$ | eg |

Q7.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $\pm 25$ or $\pm 15$ seen | B1 |  |
| Using $80 \%$ or $20 \%$ in a <br> correct calculation <br> eg <br> $0.8 \times(16--9)$ or $0.2 \times 15$ or <br> $\frac{4}{5} \times 15$ or $\frac{1}{5} \times 25$ or $\frac{80}{100} \times 25$ | oe |  |
| $(11,6)$ | A2 | A1 for each <br> answers of 5 or 3 or 20 or 12 <br> seen is evidence of a correct <br> calculation |

Q8.

| Answer | Mark | Comments |
| :---: | :---: | :---: |


| Alternative method $\mathbf{1}$ |  |  |
| :--- | :---: | :--- |
| $(0.8 \times 5 x)$ or $4 x$ | M1 | oe |
| $(1.3 \times 2 x)$ or $2.6 x$ | M1 | oe |
| their $4 x-$ their $2.6 x=35$ | M1dep | oe <br> dep on at least M1 M0 or M0 M1 |
| 25 | A1 |  |


| Alternative method 2 |  |  |
| :--- | :---: | :--- |
| Two numbers in the ratio $5: 2$ <br> and one correctly evaluated <br> increase or decrease | M1 |  |
| Both increase and decrease <br> calculations correctly <br> evaluated | M1dep | dep on the first M1 |
| Trial seen with 125 red and <br> 50 <br> blue | M1dep | dep on both previous M marks |
| or 100 red and 65 blue seen |  |  |
| 25 | A1 |  |

Q9.

| Answer | Mark | Comments |
| :--- | :---: | :---: |
| Always true |  | B1 for each correct answer |
| Sometimes true | B4 |  |
| Never true <br> Sometimes true |  |  |


| Additional Guidance |
| :--- |
| More than one box selected in a row is B0 for that row |
| Allow any unambiguous indication of a selection in a row |
| eg uses crosses instead of ticks |
| Ignore working seen and mark the boxes |

Q10.

| Answer | Mark | Comments |
| :---: | :---: | :---: |


| $b(a-11)$ or $-b(11-a)$ | M1 | Implied by square numbers $>1$ <br> used <br> eg1 4(36-11) <br> eg2 $9(16-11)$ |
| :--- | :---: | :--- |
| $a=36$ and $b=$ square <br> number $>1$ <br> with working for M1 seen | A1 | Must be in correct order <br> Allow unprocessed squares <br> eg $a=6^{2}$ and $b=5^{2}$ <br> SC1 $a=36$ and $b=$ square <br> number > 1 <br> without working for M1 seen |

## Additional Guidance

| $b(a-11)=0$ or $b(a-11)$ with further work | M 1 |
| :--- | :---: |
| Answer line takes precedence over working lines |  |
| Embedded answer eg $81(36-11)$ | M 1 A 0 |

Q11.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| Alternative method 1 |  | M1 |
| $1.25 \times 4 x$ or $5 x$ | M1 | oe |
| $0.6 \times 7 x$ or $4.2 x$ | M1dep | oe eg their $5 x=$ their $4.2 x+28$ <br> dep upon at least one of <br> previous M marks earned |
| their $5 x-$ their $4.2 x=28$ |  |  |
| or $0.8 x=28$ |  |  |$\quad$ A1 $\quad$|  |
| :--- |
| $x=35$ |


| Alternative method 2 |  |  |
| :--- | :---: | :--- |
| two numbers in the ratio 4:7 | M1 |  |
| correct increase by 25\% and <br> decrease by 40\% calculations <br> and comparison with 28 | M1dep | If difference is not 28, then first <br> numbers must be clearly rejected |
| second trial with correct <br> calculations and comparison | M1dep | correct first trial means 2nd and <br> 3rd M marks scored <br> automatically |


| $x=35$ | A 1 |  |
| :--- | :--- | :--- |

## Additional Guidance

Mark the better of their two versions if they try both methods.
In alt 2 ... for the 2nd M1 (dep on 1st M1) ... the \% calculations must be correct. If the difference is not 28 they must reject them. Attempting another two \% calculations is sufficient evidence of this.

In alt $2 \ldots$ for the 3rd M1 (dep on the first two M1s) ... the difference must be closer than their first attempt. They can have more than one attempt at this so as to eventually score the 3rd M1. To score this mark they need to indicate clearly that this further attempt is better than their first attempt.

In alt 2 ... if it isn't clear in which order they have done their attempts (eg very untidy working written all over the page) and they do not indicate which is the better attempt, then they can score a maximum of 2 marks.

Q12.

| Answer | Mark | Comments |
| :---: | :---: | :---: |
| $3(x) 455$ or $5(x) 273$ or $7(x)$ 195 or $13(x) 105$ or $15(x) 91$ or $21(x) 65$ or $35(x) 39$ or $3(x) 5(x) 7(x) 13$ | M1 | oe eg $1365 \div 5=273$ <br> Any order <br> Must be integers <br> May be seen in a factor tree or repeated division |
| 3 5 91  <br> or 3 7 65 <br> or 3 13 35 <br> or 5 7 39 <br> or 5 13 21 <br> or 7 13 15 | A1 | Any order <br> Must be integers |


| Additional Guidance |  |
| :--- | :---: |
| If using division the correct answer must be seen for M1 |  |
| Correct answer can be implied by working lines <br> eg $3(x) 5(x) 91$ with blank answer line | M1A1 |
| Answer line correct | M1A1 |
| Allow inclusion of 1 for M1 eg $1(x) 3(x) 455$ |  |
| M1 |  |

Q13.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $5 m \times(1-0.4)$ or $5 m \times 0.6$ <br> or $3 m$ | M1 | oe eg $5 m-0.4 \times 5 m$ or $5 m-2 m$ <br> may be seen in an equation <br> eg $3 m=m+1$ |
| $\frac{1}{2}$ or 0.5 | A1 |  |


| Additional Guidance |  |
| :--- | :---: |
| $2 m$ only | M0 |
| $2 m=1$ | M1 |

Q14.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $\frac{1}{3} \pi r^{2} \times \frac{5 r}{3}=\frac{320}{9} \pi$ | M1 | oe eg $\frac{5}{9} \pi r^{3}=\frac{320}{9} \pi$ |
| $r^{3}=\frac{320}{5}$ or $r^{3}=64$ |  |  |
| or $\sqrt[3]{64}$ | M1dep | oe eg $r^{3}=\frac{\frac{320 \pi}{9}}{\frac{5 \pi}{9}}$ |
| 4 |  |  |

Q15.

| Answer | Mark | Comments |
| :--- | :---: | :---: |
| $\sqrt{\frac{130-49}{130+39}}$ or $\sqrt{\frac{81}{169}}$ | M1 |  |
| $\pm \frac{9}{13}$ | A1 |  |

Q16.

| Answer | Mark | Comments |
| :---: | :---: | :---: |


| $\frac{1}{5} \times 3 a=\frac{35}{100} \times(a+6)$ | M1 | oe |
| :--- | :--- | :--- |
| $60 a=35 a+210$ <br> or <br> $\frac{3 a}{5}-\frac{35 a}{100}=\frac{210}{100}$ | oe eg $25 a=210$ <br> expands brackets and eliminates <br> fractions <br> or <br> expands brackets and collects <br> terms |  |
| $\frac{42}{5}$ or 8.4 | A1 | oe value |

## Section 1.2

Mark schemes

Q1.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| 240 | B2 | B1 $2 \times 5 \times 4 \times 3 \times 2$ or $2 \times 120$ <br> or $2 \times 5!$ or 240 seen <br> SC1 answer 120 or 360 or 480 <br> or 600 or 720 |


| Additional Guidance |  |
| :--- | :---: |
| lgnore $\times 1$ for B1 |  |
| 240 in working lines with 60 on answer line | B1 |
| 720 in working lines with 1440 on answer line | Zero |
| Allow dots for multiplication but do not allow addition |  |

Q2.

| Answer | Mark | Comments |
| :---: | :---: | :---: |
| $5 \times \ldots$ | M1 | oe eg listing the 5 possible first |


|  |  | digits |
| :--- | :---: | :--- |
| $5 \times 5 \times 4 \times 3$ | M1dep |  |
| 300 | A1 | SC1 $(6 \times 5 \times 4 \times 3=) 360$ |

Q3.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $6 \times 5 \times 4 \times 3 \times 2$ or 720 | M1 | oe eg 6! <br> 6 digit numbers |
| $2 \times 5 \times 4 \times 3 \times 2$ or 240 | M1 | oe <br> 5 digit numbers |
| 960 | A1 |  |

Q4.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| 18 | B3 | B2 identifies there are 3 choices <br> for first digit and 3 choices for <br> second digit <br> B1 identifies there are 3 choices <br> for first digit <br> or <br> identifies there is 1 choice for <br> last digit |

## Section 1.3

## Mark schemes

## Q1.

| Answer | Mark | Comments |
| :---: | :---: | :--- |
| $1: 2: 5$ | B3 | B2 For any ratio that is one step <br> away from the answer <br>  |
|  |  | e.g. $\sqrt{ } 12: 2 \sqrt{ } 12: 5 \sqrt{ } 12$ <br> $\sqrt{ } 1: \sqrt{ } 4: \sqrt{ } 25$ <br>  |
|  |  | $2: 4: 10$ |


|  | B1 For at least two of the three <br> terms in their simplest form <br> i.e. two of $2 \sqrt{ } 3: 4 \sqrt{ } 3: 10 \sqrt{ } 3$ <br> B1 For any correct equivalent <br> ratio <br> e.g. $\sqrt{ } 2: \sqrt{ } 8: \sqrt{ } 50$ <br> $\sqrt{3}: \sqrt{ } 12: \sqrt{ } 75$ |
| :--- | :--- | :--- |

Q2.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $\frac{5 \sqrt{2}(3 \sqrt{6}+7)}{(3 \sqrt{6}-7)(3 \sqrt{6}+7)}$ | M1 |  |
| Numerator $=15 \sqrt{ } 2 \sqrt{ } 6+35 \sqrt{ } 2$ | M1dep | oe eg $15 \sqrt{ } 12+35 \sqrt{ } 2$ or $5 \sqrt{ } 2 \times$ <br> $3 \sqrt{6}+35 \sqrt{ } 2$ <br> dep on the first M1 |
| Denominator $=54-49$ | M1dep | dep on the first M1 |
| $3 \sqrt{ } 12+7 \sqrt{ } 2$ | A1 | oe eg $6 \sqrt{ } 3+7 \sqrt{ } 2$ |
| $\sqrt{108}+\sqrt{98}$ | A1 |  |

Q3.

| Answer | Mark | Comments |
| :--- | :---: | :---: |
| $\sqrt{ } 500=10 \sqrt{ } 5$ or $\sqrt{ } 45=3 \sqrt{ } 5$ | M1 | or for $5 \sqrt{ } 4 \sqrt{ } 5$ and $2 \sqrt{ } 9 \sqrt{ } 5$ |
| $4 \sqrt{ } 5$ | A1 |  |

Q4.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $(y=) \frac{8}{\sqrt{3-1}}$ | M1 | oe |
| $(y=) \frac{8}{(\sqrt{3}-1)} \times \frac{(\sqrt{ } 3+1)}{(\sqrt{3}+1)}$ | M1 |  |
| $(y=) \frac{8 \sqrt{ } 3+8}{3-1}$ | A1 |  |


| $(y=) 4 \sqrt{ } 3+4$ | A1 | $2 \sqrt{ } 3+2$ from $\frac{8 \sqrt{ } 3+8}{3+1}$ |
| :--- | :--- | :---: |
| and $\sqrt{ } 3+1$ from $\frac{8 \sqrt{ } 3+8}{9-1}$ |  |  |
| both score SC3 |  |  |


| Alternative method 1 |  |  |
| :--- | :--- | :--- |
| $y \sqrt{ } 3=8+y$ <br> and <br> $3 y^{2}=64+16 y+y^{2}$ | M 1 | Re-arrange and square both <br> sides <br> Allow one error |
| $y^{2}-8 y-32=0$ <br> or <br> $2 y^{2}-16 y-64=0$ <br> and <br> $(y=) \frac{8 \pm \sqrt{\left.8^{2}-4(1)(-32)\right\}}}{2(1)}$ | M1 | Allow one substitution or sign <br> error |
| or |  |  |
| $(y=) \frac{16 \pm \sqrt{\left\{16^{2}-4(2)(-64)\right\}}}{2(2)}$ |  | A1 |
| $(y=) 4 \pm 4 \sqrt{3}$ | A1 | Solution with negative sign must <br> be discounted |
| $(y=) 4+4 \sqrt{3}$ |  |  |

Alternative method 2

| $(a+b \sqrt{ } 3)(\sqrt{ } 3-1)(=8)$ | M 1 |  |
| :--- | :---: | :--- |
| $a \sqrt{ } 3+3 b-a-b \sqrt{ } 3$ | M 1 |  |
| $a=b$ | A 1 |  |
| $(y=) 4+4 \sqrt{ } 3$ | A 1 |  |

Q5.

| Answer | Mark | Comments |
| :---: | :---: | :--- |
| $\sqrt{125}=5 \sqrt{5}, \sqrt{20}=2 \sqrt{5}$ | M1 | allow one error <br> any two of these correct seen |


| and $\sqrt{80}=4 \sqrt{5}$ |  | anywhere in the working |
| :--- | :--- | :--- |
| $(\sqrt{x}=) 3 \sqrt{5}$ | A1 |  |
| 45 | A1 |  |

Q6.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| Alternative method 1 |  | B1 |
| $10^{\sqrt{3}}$ | M1 | oe |
| $\frac{(24-\text { their } 10 \sqrt{ } 3)(4 \sqrt{ } 3+5)}{(4 \sqrt{ } 3-5)(4 \sqrt{ } 3+5)}$ |  |  |
| $96 \sqrt{3}-120+120-50 \sqrt{ } 3$ | M1dep | allow one sign error |
| $48-25$ or 23 | M1 |  |
| $2^{\sqrt{3}}$ | A1 |  |


| Alternative method 2 |  |  |
| :--- | :---: | :--- |
| $(24-\sqrt{ } 300)(4 \sqrt{ } 3+5)$ <br> $(4 \sqrt{ } 3-5)(4 \sqrt{ } 3+5)$ | M1 |  |
| $96 \sqrt{ } 3+120-4 \sqrt{ } 900-5 \sqrt{ } 300$ | M1dep | allow one sign error |
| $96 \sqrt{ } 3-120+120-50 \sqrt{ } 3$ | M1 |  |
| $48-25$ or 23 | M1 |  |
| $2^{\sqrt{3}}$ | A1 |  |

## Additional Guidance

For the 1st M1, multiplying numerator and denominator by $(4 \sqrt{ } 3+5)$ could legitimately be replaced by $-4 \sqrt{ } 3-5$... almost identical working ... it just changes all the signs on the next lines of working

Q7.

| Answer | Mark | Comments |
| :---: | :---: | :---: |
| $(-2)^{3}$ or -8 seen | B1 |  |
| $-\sqrt{x}=$ (their -8$)-3$ or | M1 |  |


| $-\sqrt{x}-11$ <br> or $\sqrt{x}=11$ |  |  |
| :--- | :--- | :--- |
| 121 | A1 |  |

Additional Guidance
$-2^{3}$ (no brackets) is B0 unless -8 seen
For M1 it must say $\sqrt{x}=\ldots . . .$. or $-\sqrt{x}=\ldots . .$. Note: $\ldots$ (their -8 )
cannot be -2
... and it must be correct manipulation from their -8

$$
\begin{aligned}
& \text { eg } 3^{-\sqrt{x}=(-2)^{3} \text { or } 3^{-\sqrt{x}}=-8 \quad \mathrm{~B} 1} \begin{array}{l}
\sqrt{x}=-11 \mathrm{M} 0 \text { (error in manipulating terms) } \\
\text { working) }
\end{array} \begin{array}{l}
x=121 \quad \mathrm{AO} \text { (correct answer from wrong }
\end{array}
\end{aligned}
$$

Q8.

| Answer | Mark | Comments |
| :---: | :---: | :--- |
| $4-\sqrt{5}+8 \sqrt{5}-2 \sqrt{5} \sqrt{5}$ | M1 | oe <br> allow one incorrect term in a four <br> term expansion |
| $-6+7 \sqrt{5}$ | A1 |  |

## Additional Guidance

Any incorrect further work loses the A mark, so they can only score M1 A0

Q9.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $7+12 \sqrt{5}+6(9-2 \sqrt{5})$ | M1 | oe $\quad$ eg $7+6 \times 9$ or $7+54$ |
| or |  | or $6 \times-2=-12$ |
| $12 \sqrt{5}+6(-2 \sqrt{5})=0$ |  | allow <br> or <br> or <br> $12 \sqrt{5} \div 2 \sqrt{5}=6$ |


| or <br> states that need to add 6 lots <br> of <br> $(9-2 \sqrt{5})$ <br> or <br> 7th term | with $n=7$ | allow $7+12 \sqrt{5}+n(9-2 \sqrt{5})$ <br> with $n=6$  |
| :--- | :--- | :--- |


| Additional Guidance |  |
| :--- | :---: |
| 61 in working lines with 7(th) on answer line | M1 A0 |
| If repeatedly adding $(9-2 \sqrt{5})$ they must stop after adding 6 <br> lots or clearly select the relevant one |  |
| Answer 6 or 6th term with M1 not seen | M0 A0 |
| Ignore any conversions to decimals |  |
| Beware $(9-2 \sqrt{5})(9+2 \sqrt{5})=61$ | M0 A0 |

Q10.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $\frac{(21-11 \sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$ | M1 | $-3-\sqrt{5}$ |
| condone missing final bracket of |  |  |
| $3+\sqrt{5}$ if written in this form. |  |  |
| Brackets not needed if written as |  |  |
| two separate fractions |  |  |$|$| Denominator of 4 | M1 | allow three terms correct in a 4 <br> term expansion. If error appears <br> in 2 or 3 term simplification and 4 <br> term expansion not seen award <br> M0 <br> Numerator <br> $63-33 \sqrt{5}+21 \sqrt{5}-55$ <br> $8-12 \sqrt{5}$ |
| :--- | :--- | :--- |
| or | expansion could be seen in a <br> grid |  |
| $2-3 \sqrt{5}$ or $-3 \sqrt{5}+2$ | A1 | only if M1 awarded for correct <br> product |

## Additional Guidance

Correct first A mark and M1dep mark would assume first M mark correct if not seen.

Q11.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| Alternative method 1 |  | M1 |
| $\frac{\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$ or(3- 3 3) can still gain full marks if <br> recovered but doesn't gain M1 if <br> the second M mark isn't awarded |  |  |
| $\frac{\sqrt{3}}{3+\sqrt{3}} \times \frac{\sqrt{3}-3}{\sqrt{3}-3}$ | M1dep | oe eg $\frac{3 \sqrt{3}-\sqrt{3} \sqrt{3}}{9+3 \sqrt{3}-3 \sqrt{3}-\sqrt{3} \sqrt{3}}$ |
| $\frac{3 \sqrt{3}-3}{9-3}$ | A1 | or $\frac{3 \sqrt{3}-\sqrt{9}}{9+3 \sqrt{3}-3 \sqrt{3}-\sqrt{9}}$ or to something fully simplified $\frac{3 \sqrt{3}}{6}-\frac{3}{6}$ |
| $\frac{\sqrt{3}-1}{2}$ | eg $\frac{1}{2}-\frac{1}{2}$ or $\frac{1-\sqrt{3}}{-2}$ |  |

## Alternative method 2

| $\frac{\sqrt{3}}{3+\sqrt{3}}=\frac{1}{\sqrt{3}+1}$ | M1 |  |
| :--- | :--- | :--- |
| $\frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ or | M1dep | oe <br> $\times(\sqrt{3}-1)$ can still gain full marks <br> if recovered but doesn't gain M1 <br> if the A mark isn't awarded |
| $\frac{1}{\sqrt{3}+1} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$ | A1 | oe eg $\frac{\sqrt{3}}{2}-\frac{1}{2}$ or $\frac{1-\sqrt{3}}{-2}$ |
| $\frac{\sqrt{3}-1}{2}$ |  |  |

## Alternative method 3

| $\frac{\sqrt{3}}{3+\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{3}{3 \sqrt{3}+3}$ | M1 |  |
| :--- | :--- | :--- |
| $\frac{3}{3 \sqrt{3}+3} \times \frac{3 \sqrt{3}-3}{3 \sqrt{3}-3}$ or | M1dep | oe |
| $\frac{3}{\times(3 \sqrt{3}-3) \text { can still gain full }}$marks if recovered but doesn't <br> gain M1 if the A mark isn't <br> awarded |  |  |
| $\frac{3-3 \sqrt{3}+3}{3-3 \sqrt{3}-1} 2$ | A1 | oe eg $\frac{\sqrt{3}}{2}-\frac{1}{2}$ or $\frac{1-\sqrt{3}}{-2}$ |


| Additional Guidance |  |
| :--- | :--- |
| Penalise further incorrect working |  |

Q12.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| Multiplies numerator and <br> denominator by $\sqrt{3}-1$ | M1 |  |
| (denominator $=$ ) 2 | A1 |  |
| (numerator $=$ ) $3-\sqrt{3}-7 \sqrt{3}+7$ | M1dep |  |
| or $10-8 \sqrt{3}$ |  |  |
| $5-4 \sqrt{3}$ | A1 |  |

Q13.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| Alternative method 1 | M1 |  |
| $\sqrt{225}+\sqrt{144}$ |  |  |
| or |  |  |
| $15+12$ | A1 |  |
| 27 |  |  |


| Alternative method 2 |  |  |
| :--- | :--- | :--- |
| $5 \sqrt{3}+4 \sqrt{3}$ or $9 \sqrt{3}$ M1  <br> or   <br> $9 \times 3$   <br> or   <br> $15+12$ A1  <br> 27   |  |  |

Q14.

| Answer | Mark | Comments |
| :--- | :---: | :--- |
| $10 \sqrt{6}$ or $3 \sqrt{6}$ or $2 \sqrt{6}$ | M1 |  |
| or $\frac{\sqrt{100 \times 6}-\sqrt{9 \times 6}}{\sqrt{4 \times 6}}$ |  |  |
| Two of <br> $10 \sqrt{6}$ and $3 \sqrt{6}$ and $2 \sqrt{6}$ | M1dep |  |
| $\frac{7}{2}$ or 3.5 | A1 | oe value |

## Q15.

| Answer | Mark | Comments |
| :---: | :---: | :---: |
| Alternative method 1 Works out $\frac{1}{2} \times(6+2 \sqrt{7})$ |  |  |
| $\frac{1}{2} \times(6+2 \sqrt{7}) \times A D$ | M1 | oe eg $(3+\sqrt{7}) \times A D$ or $(3+\sqrt{7}) \times A C \sin c$ may be implied |
| $\frac{13+3 \sqrt{7}}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}}$ | M1 | ft their $\frac{13+3 \sqrt{7}}{3+\sqrt{7}}$ <br> their denominator must have 2 terms |
| $\begin{aligned} & \text { (numerator }=\text { ) } \\ & 39-13 \sqrt{7}+9 \sqrt{7}-21 \end{aligned}$ <br> or $18-4 \sqrt{7}$ | M1dep | ft their numerator which must have 2 terms <br> oe <br> dep on 2nd M1 |
| (denominator =) 2 | M1dep | ft their denominator dep on 2nd M1 |
| $9-2 \sqrt{7}$ | A1 |  |


| Alternative method 2 Works out $2 \times(13+3 \sqrt{7})$ |  |  |
| :--- | :---: | :--- |
| $\frac{1}{2} \times(6+2 \sqrt{7}) \times A D$ | M 1 | oe eg $(6+2 \sqrt{7}) \times A D=(26+6$ <br> $\sqrt{7})$ <br> may be implied |


| $\frac{26+6 \sqrt{7}}{6+2 \sqrt{7}} \times \frac{6-2 \sqrt{7}}{6-2 \sqrt{7}}$ | M 1 | ft their $\frac{26+6 \sqrt{7}}{6+2 \sqrt{7}}$ <br> their denominator must have 2 <br> terms |
| :--- | :--- | :--- |
| (numerator $=$ ) <br> $156-52 \sqrt{7}+36 \sqrt{7}-84$ <br> or $72-16 \sqrt{7}$ | M1dep | ft their numerator which must <br> have 2 terms <br> oe <br> dep on 2nd M1 |
| (denominator $=$ ) 8 | M1dep | ft their denominator <br> dep on 2nd M1 |
| $9-2 \sqrt{7}$ | A1 |  |


| Alternative method 3 Using identities |  |  |
| :--- | :--- | :--- |
| $(6+2 \sqrt{7}) \times A D=(26+6 \sqrt{7})$ | M 1 | oe |
| $(6+2 \sqrt{7}) \times(a+b \sqrt{7})=(26+6$ <br> $\sqrt{7})$ | M 1 | oe |
| $6 a+14 b=26$ and $2 a+6 b=$ <br> 6 | M1dep | oe eg $3 a+7 b=13$ and $a+3 b$ <br> $=3$ |
| $a=9$ or $b=-2$ | M1dep |  |
| $9-2 \sqrt{7}$ | A1 |  |


| Additional Guidance |  |
| :--- | :---: |
| Alt1 $\frac{18-4 \sqrt{7}}{2}$ | M4 |
| Alt2 $\frac{72-16 \sqrt{7}}{8}$ | M4 |
| Omission of $\frac{1}{2}$ can score up to M0M1M1M1A0 | M0M1 |
| eg $\frac{13+3 \sqrt{7}}{6+2 \sqrt{7}} \times \frac{6-2 \sqrt{7}}{6-2 \sqrt{7}}$ | M1M1 |
| $\frac{78-26 \sqrt{7}+18 \sqrt{7}-42}{8}$ | A0 |
| $4.5-\sqrt{7}$ |  |
| (If the $\frac{1}{2}$ is recovered then all 5 marks are possible) |  |

