5 MATRICES – Further Maths

Section 5.1

Q1. $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ Work out the matrix **AB**. **(2 marks)**

Q2. Work out $\begin{pmatrix} 5 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$ **(2 marks)**

Q3. Work out $3\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix}$ Give your answer as a single matrix. **(3 marks)**

Q4. Given that $\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a+1 \end{pmatrix}$ work out the values of a and b. **(5 marks)**

Q5. $4\begin{pmatrix} 1-2a \\ a \end{pmatrix} = \begin{pmatrix} b \\ 12 \end{pmatrix}$ Work out the values of a and b. **(3 marks)**

Q6. $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} s \\ -5 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} -1 \\ t \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 2 & 1 \\ 7 & u \end{pmatrix}$ s, t and u are constants.

(a) **AB** = **C**
Work out the values of s and t. **(3 marks)**

(b) **AD** = **I**
Work out the value of u. **(1 mark)**

Q7. $\begin{pmatrix} 7 & a^2 \\ b & -5 \end{pmatrix}\begin{pmatrix} 2 \\ a \end{pmatrix} = \begin{pmatrix} 78 \\ 12 \end{pmatrix}$ Work out the values of a and b. **(3 marks)**

Section 5.2

Q1. $\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ 0 & 0.4 \end{pmatrix} = k\mathbf{I}$ where k is a constant and **I** is the identity matrix.

Work out the values of a and b. **(4 marks)**

Q2. $\begin{pmatrix} m & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ -2 & -1 \end{pmatrix} = \mathbf{I}$ where **I** is the identity matrix.

Work out the value of m. **(2 marks)**

Q3. **M** = $\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$ Show that $\mathbf{M}^3 = \mathbf{I}$ **(4 marks)**

Section 5.3 – 5.4

Q1. $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Describe geometrically the single transformation represented by **A**. **(1 mark)**

Q2. Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **(3 marks)**

Q3. Under the transformation represented by $\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}$, the image of point $P(a, 2)$ is point Q.

Can point Q be the same as point P? You **must** show your working. **(4 marks)**

Q4. The transformation matrix $\begin{pmatrix} 2a & b \\ -b & -a \end{pmatrix}$ maps the point (3, 4) onto the point (8, −7)

Work out the values of a and b. **(5 marks)**

Q5. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y-axis.

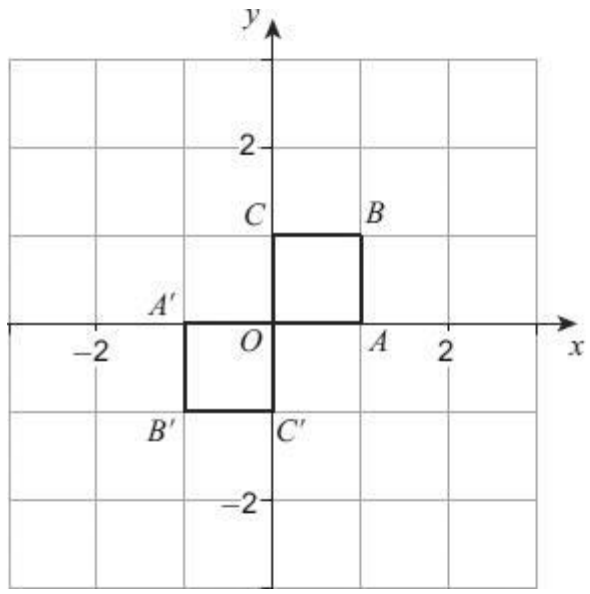
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$

Work out the matrix that represents a reflection in the y-axis followed by a reflection in the line $y = x$ **(2 marks)**

Q6. The unit square $OABC$ has vertices

$$O(0, 0) \quad A(1, 0) \quad B(1, 1) \quad C(0, 1)$$

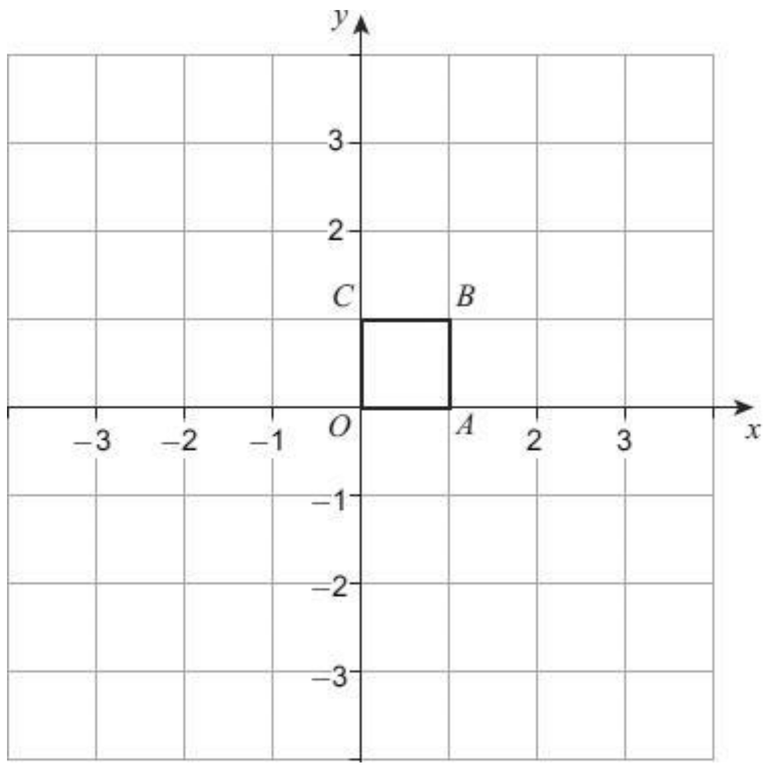
(a) $OABC$ is mapped to $OA'B'C'$ under transformation matrix **M**.



Work out matrix **M**. **(2 marks)**

(b) $OABC$ is mapped to $OA''B''C''$ under transformation matrix $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

Draw and label $OA''B''C''$ on the diagram below.



(3 marks)

Q7.

$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Describe geometrically the single transformation represented by $\mathbf{B}^2$

(2 marks)

Q8.

The transformation matrix **Q** is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The transformation matrix **R** is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Describe fully the **single** transformation represented by the matrix **QR**. **(2 marks)**

Q9.

The transformation matrix **P** represents a 90° anti-clockwise rotation about the origin.

Describe fully the **single** transformation represented by the matrix $\mathbf{P}^3$ **(2 marks)**

Q10.

Shape *A* maps to shape *B* by an enlargement, scale factor 3, centre the origin.
Shape *B* maps to shape *C* by a rotation through 180°, centre the origin.

Shape *A* can be mapped to shape *C* by a **single** transformation.

Use matrices to show that the single transformation is an enlargement, centre the origin.
State the scale factor of the enlargement. **(5 marks)**

Q11.

The transformation matrix **M** represents a 90° clockwise rotation about the origin.

(a) Write down the matrix **M**.

$\mathbf{M} = \begin{pmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{pmatrix}$

(1 mark)

(b) Describe fully the **single** transformation represented by $\mathbf{M}^2$.

(2 marks)

(c) Write down the matrix for the **single** transformation represented by $\mathbf{M}^2$.

$\mathbf{M}^2 = \begin{pmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{pmatrix}$

(1 mark)