6 GEOMETRY – Further Maths

Section 6.1 (Area & Volume)

Q1. A sphere has radius x centimetres. A hemisphere has radius y centimetres.

The shapes have equal volumes. Work out the value of $\frac{y}{x}$.

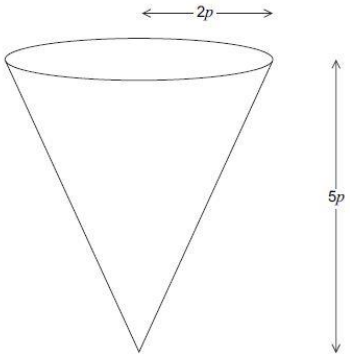
Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer. **(3 marks)**

Q2. A cone has base radius r cm, perpendicular height h cm and slant height l cm
The curved surface area is 60π cm^2 $l = 3r$

Work out the value of h.

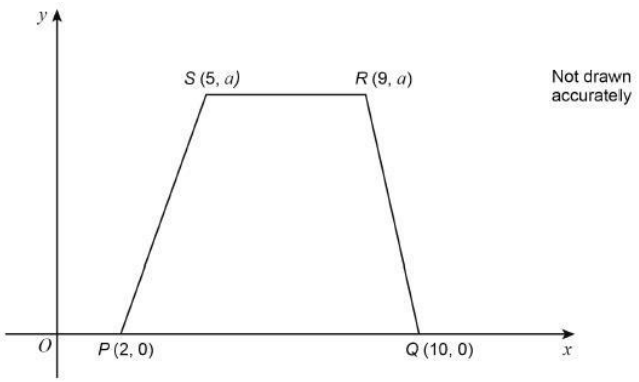
Give your answer in the form $a\sqrt{10}$ where a is an integer greater than 1 **(5 marks)**

Q3. This right circular cone has radius $2p$ and height $5p$. The dimensions are in centimetres.



The volume of the cone is 22500π cm^3 . Work out the value of p. **(4 marks)**

Q4. *PQRS* is a trapezium.

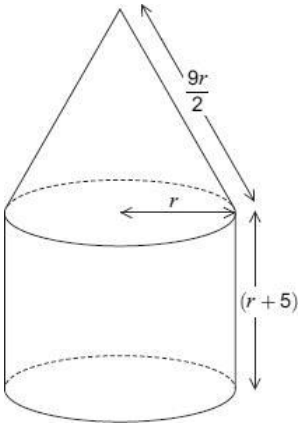


The area of the trapezium is 63 square units. Work out the value of a. **(2 marks)**

Q5. On this diagram all lengths are given in centimetres.

A cylinder and cone are joined together to make a solid. The cylinder has radius r and height $(r + 5)$

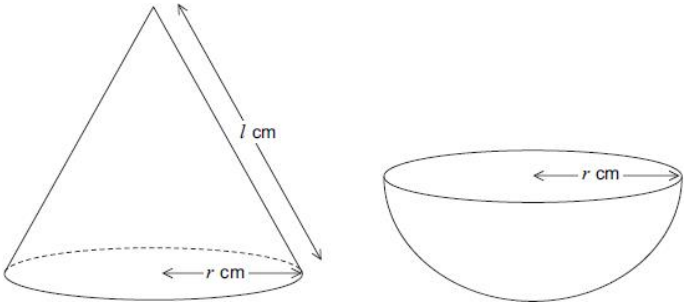
The cone has radius r and slant height $\frac{9r}{2}$



(a) Show that the **total** surface area of the solid, in cm^2, is $\frac{5\pi r}{2}(3r + 4)$ **(4 marks)**

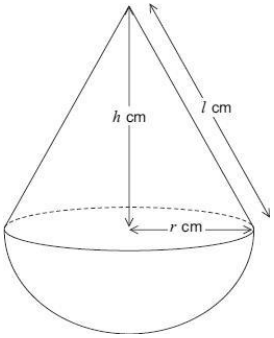
(b) The total surface area of the solid is 1200π cm^2 Work out the value of r. **(5 marks)**

Q6. A cone has base radius r cm and slant height l cm A hemisphere has radius r cm



(a) The curved surface area of the cone equals the curved surface area of the hemisphere.
Show that $l = 2r$ **(1 mark)**

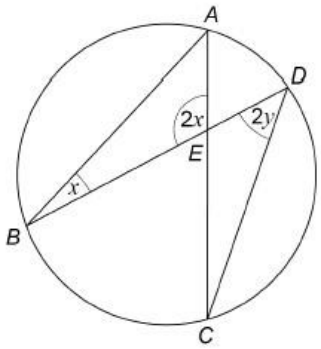
(b) The cone has vertical height h cm



Show that the volume of the shape can be written as $\frac{1}{3}\pi r^3(a+\sqrt{b})$ cm^3
where a and b are integers. **(4 marks)**

Section 6.1 – 6.2

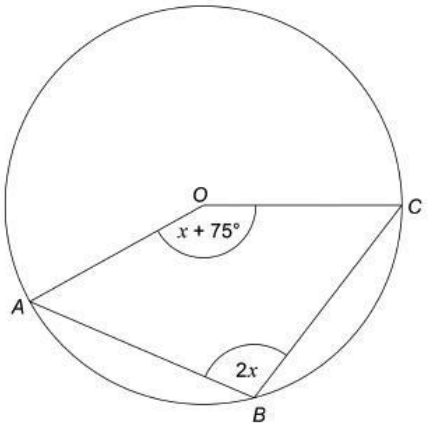
Q1. *A*, *B*, *C* and *D* are points on a circle. *AC* and *BD* intersect at *E*.



Not drawn accurately

Prove that $y = 90° - \frac{3}{2}x$ **(3 marks)**

Q2. Points *A*, *B* and *C* lie on a circle, centre *O*. Angle *AOC* = *x* + 75° Angle *ABC* = 2*x*



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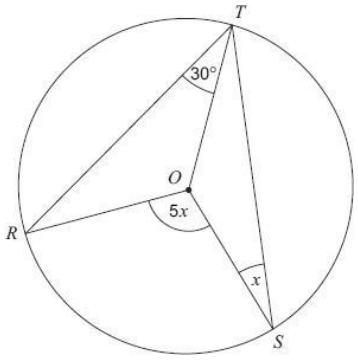
Work out the value of *x*. **(3 marks)**

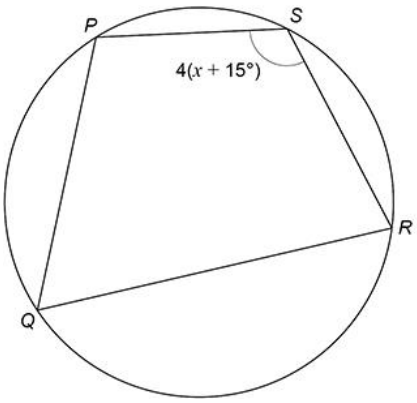
Q3. *R*, *S* and *T* are on the circumference of a circle, centre *O*.



(a) Give a reason why angle *OTS* = *x* **(1 mark)**

(b) Work out the value of *x*. **(3 marks)**

Q4. *PQRS* is a cyclic quadrilateral.



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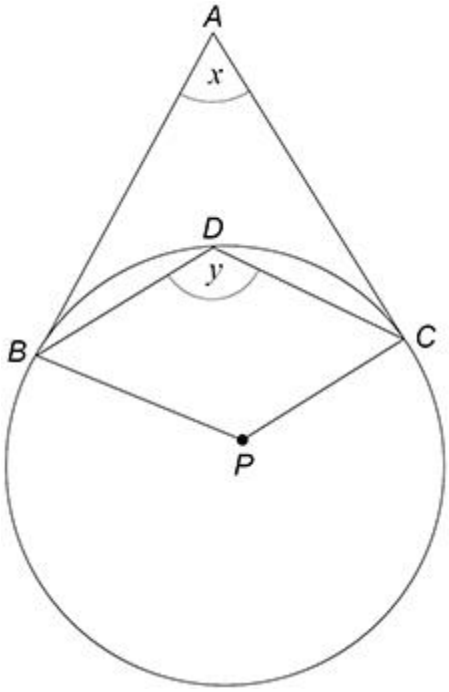
Angle *PSR* = 4(*x* + 15°) Angle *PQR* is 40° smaller than angle *PSR*.

Work out the value of *x*. **(3 marks)**

Q5. *B*, *C* and *D* are points on a circle, centre *P*.

AB and *AC* are tangents to the circle.



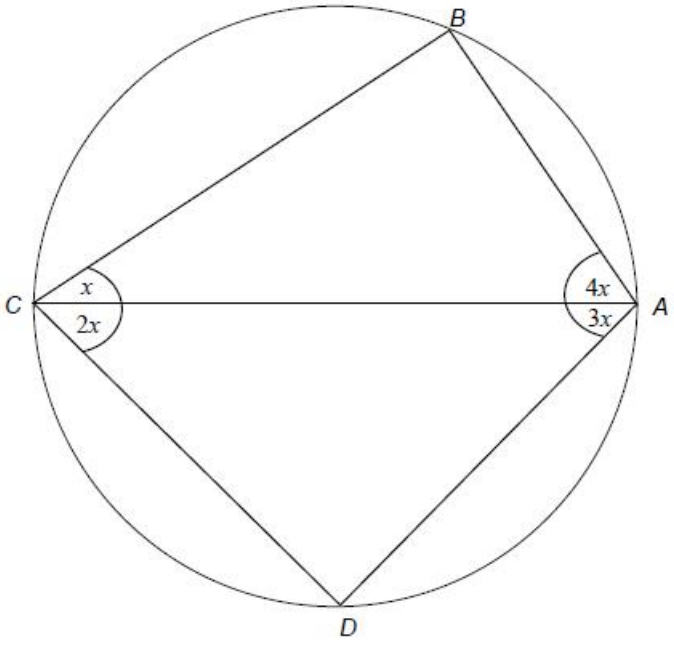
Not drawn accurately

Prove that $y = 90 + \frac{x}{2}$

(5 marks)

Q6. *A*, *B*, *C* and *D* are points on a circle.

$\angle BCA = x$ $\angle ACD = 2x$ $\angle CAD = x$ $\angle CAB = 4x$



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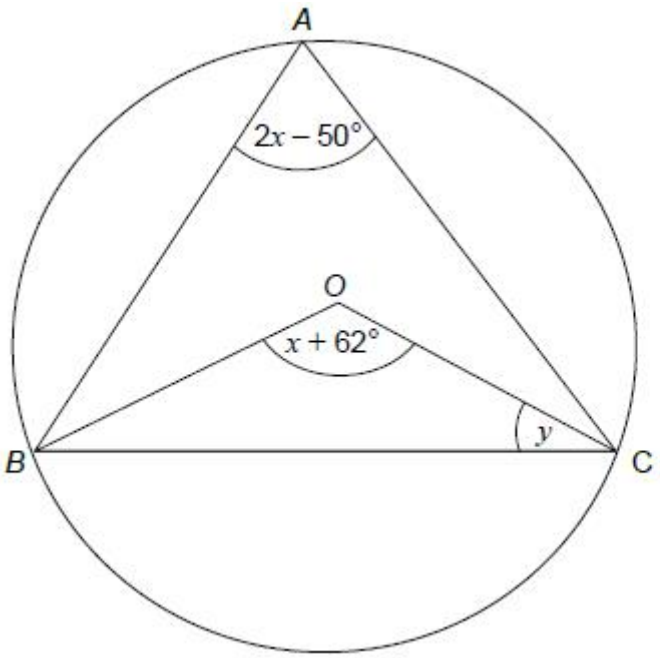
Prove that *AC* is a diameter. **(4 marks)**

Q7.

A, *B* and *C* are points on a circle, centre *O*.



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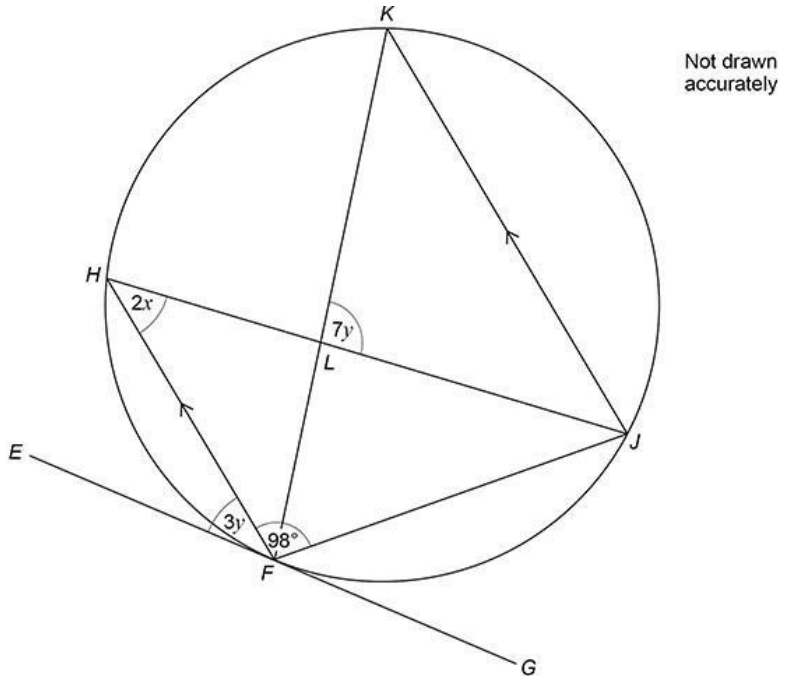
Work out the size of angle *y*.

(5 marks)

Q8. *F, H, K* and *J* are points on a circle.

Chords *HJ* and *KF* intersect at *L*. *EFG* is a tangent to the circle. *FH* and *JK* are parallel.



Angle *FHJ* = 2*x*

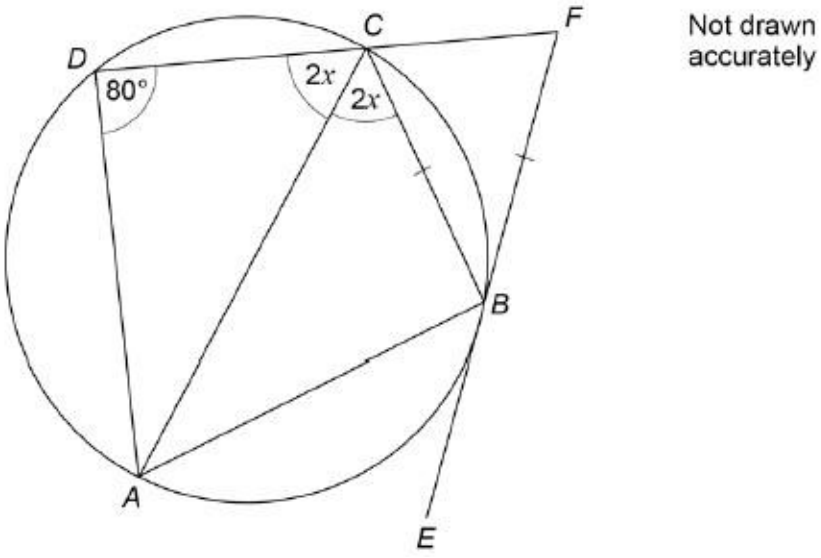
(a) Give reasons why angle *FKJ* and angle *HJK* are also equal to 2*x*. **(2 marks)**

(b) Work out the values of *x* and *y*. You **must** show your working. **(4 marks)**

Q9. *A*, *B*, *C* and *D* are points on a circle. *EBF* is a tangent. *DCF* is a straight line.

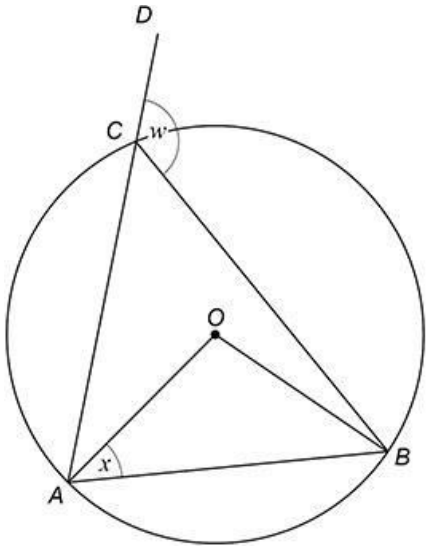
Angle *DCA* = angle *ACB* = 2*x*

BC = *BF*



Work out the value of *x*. **(5 marks)**

Q10. *A*, *B* and *C* are points on a circle, centre *O*. *ACD* is a straight line. Angle *BCD* = *w*



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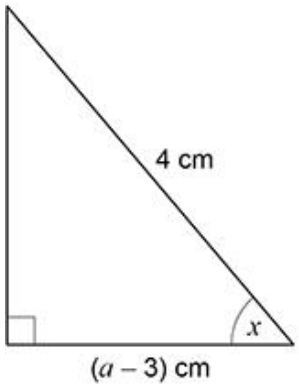
Prove that $w = x + 90°$ **(5 marks)**

Section 6.3 – 6.5

Q1. Here is a right-angled triangle.



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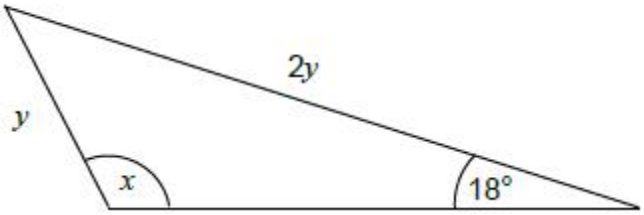
You are given that $a > 5$ Use trigonometry to work out the range of values of x. **(2 marks)**

Q2. Use the sine rule to work out the size of obtuse angle x.



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(3 marks)

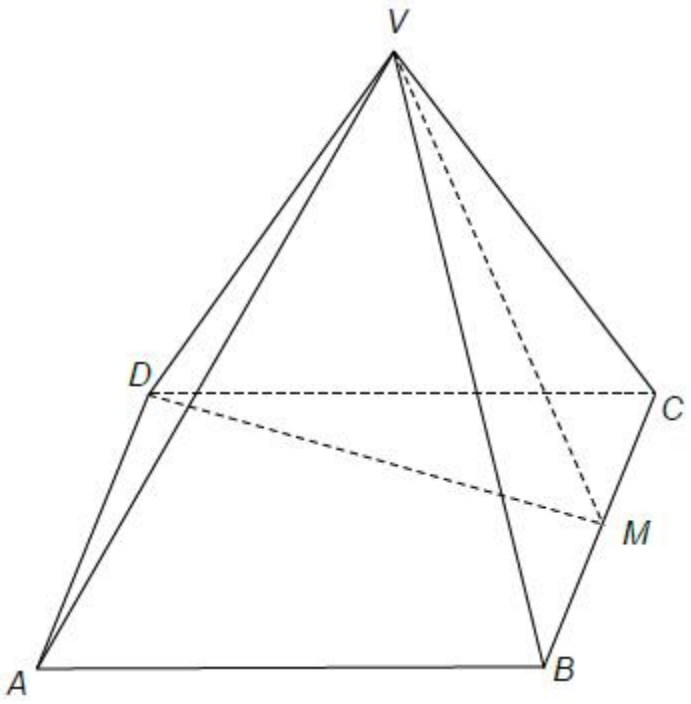
Q3. *VABCD* is a pyramid with a horizontal rectangular base *ABCD*.

V is directly above the centre of the base.

VA = *VB* = *VC* = *VD* = 10 cm

AB = 8 cm *BC* = 6 cm

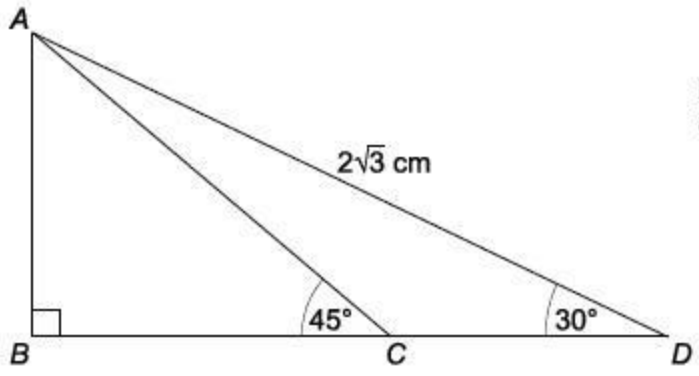
M is the midpoint of *BC*.



Work out the size of angle *VMD*. **(5 marks)**

Q4. In the diagram, *BCD* is a straight line.

$AD = 2\sqrt{3}$ cm



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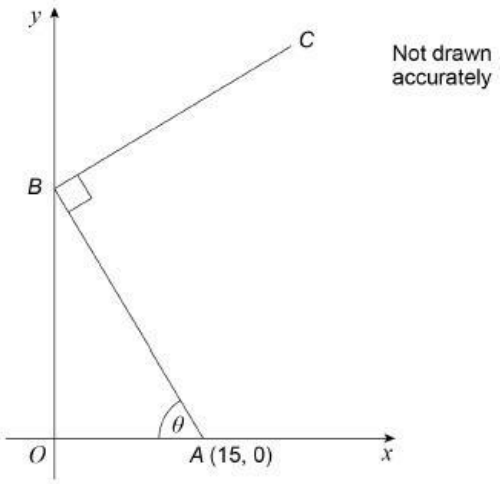
Work out the exact length of *CD*.

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers. **(4 marks)**

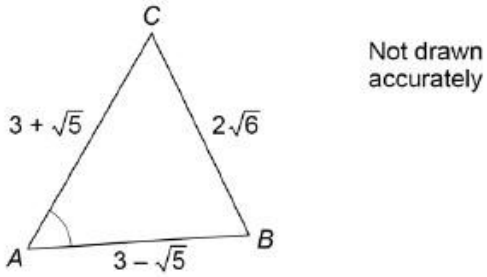
Q5. In the diagram, *A* is the point (15, 0) and *B* lies on the *y*-axis.

Angle *ABC* = 90° and tan $\theta = \frac{5}{3}$



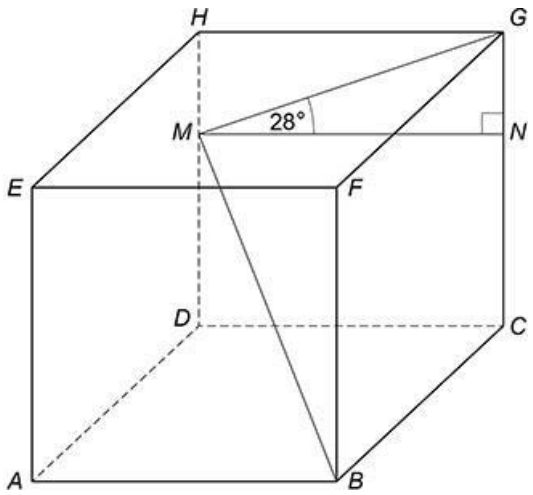
Work out the equation of the line *BC*. **(4 marks)**

Q6. *ABC* is a triangle. All lengths are in centimetres.



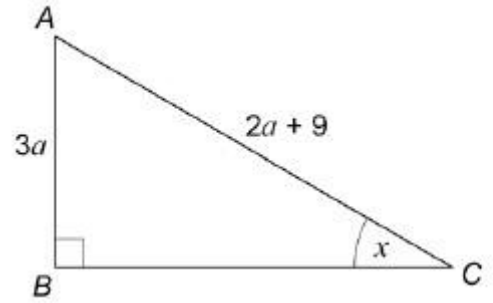
Show that angle *CAB* = 60° **(5 marks)**

Q7. *ABCDEFGH* is a cube with side length 32 cm *M* and *N* are points on *DH* and *CG* respectively.



Work out the size of the angle that the line *BM* makes with the plane *ABCD*. **(5 marks)**

Q8. *ABC* is a right-angled triangle. All lengths are in centimetres.



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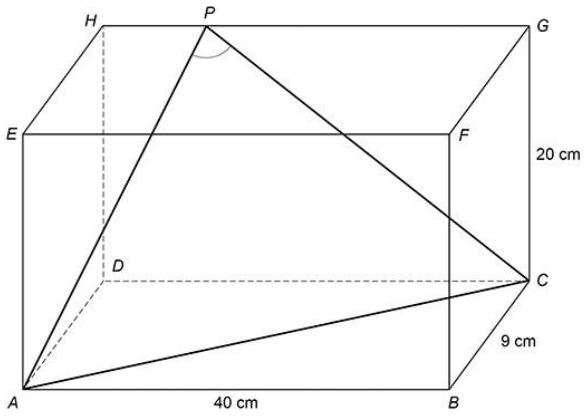
$\sin x = \frac{3}{5}$ Work out the length *BC*. **(5 marks)**

Q9. *ABCDEFGH* is a cuboid.

AB = 40 cm *BC* = 9 cm *CG* = 20 cm

P is a point on *HG* such that *HP* : *PG* = 3 : 7

AP = 25 cm

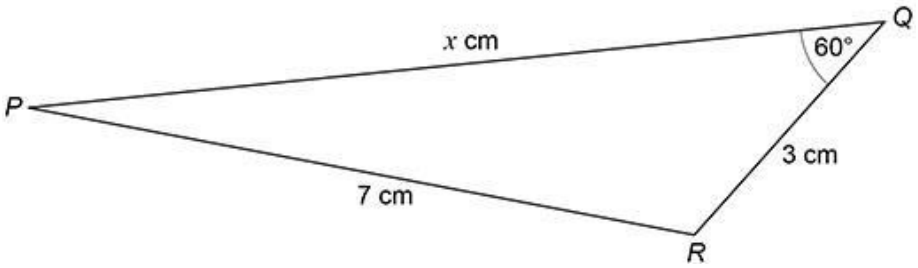


Work out the size of angle *APC*. **(5 marks)**

Q10.

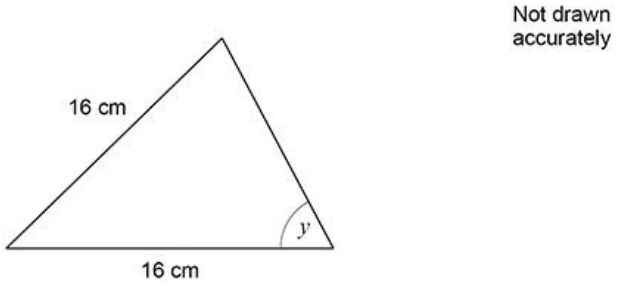
Here is a triangle.

Not drawn accurately



Use the cosine rule to work out the value of x. **(4 marks)**

Q11. Here is an isosceles triangle. All the angles are acute.



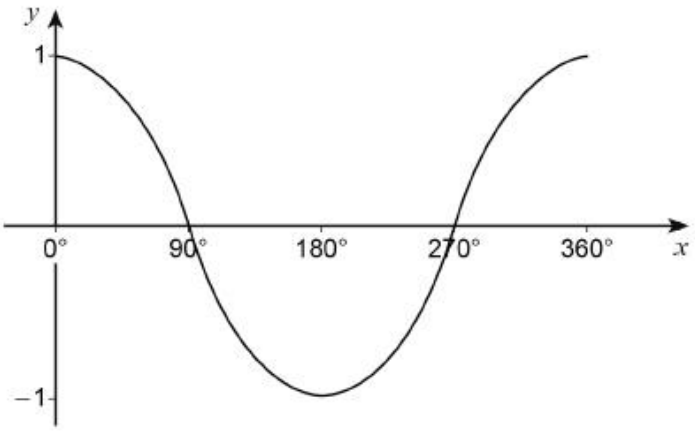
The area of the triangle is 120 cm^2 Work out the size of angle y. **(4 marks)**

Q12. ABC is a right-angled triangle with vertices A (–1, 5), B (–2, 5) and C $\left(-1, 5\frac{3}{4}\right)$

Work out the length of BC. **(3 marks)**

Section 6.6 – 6.7

Q1. Here is a sketch graph of $y = \cos x$ for $0° \leq x \leq 360°$



You are given that $\cos 36° = 0.8090$

Solve $\cos x = -0.8090$ for $0° \leq x \leq 360°$ **(2 marks)**

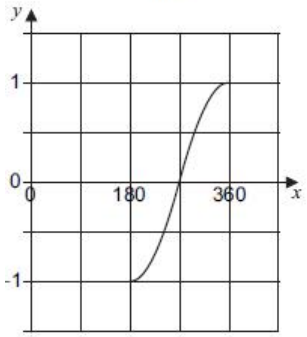
Q2. State the coordinates of each point where the graph

$y = \cos x$ for $0° \leq x \leq 360°$

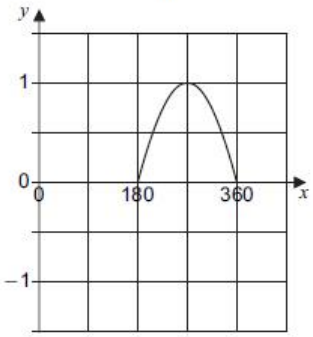
meets or intersects an axis. **(2 marks)**

Q3. Four graphs are shown for $180° \leq x \leq 360°$

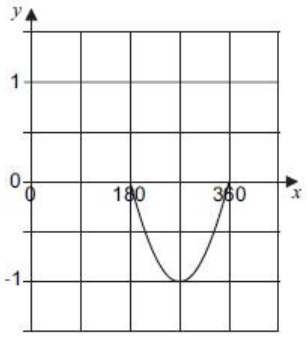
Graph A



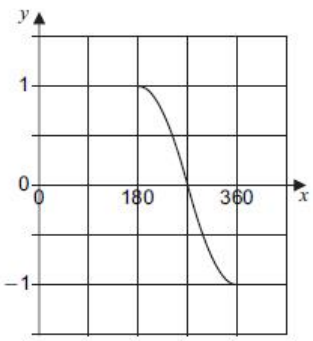
Graph B



Graph C



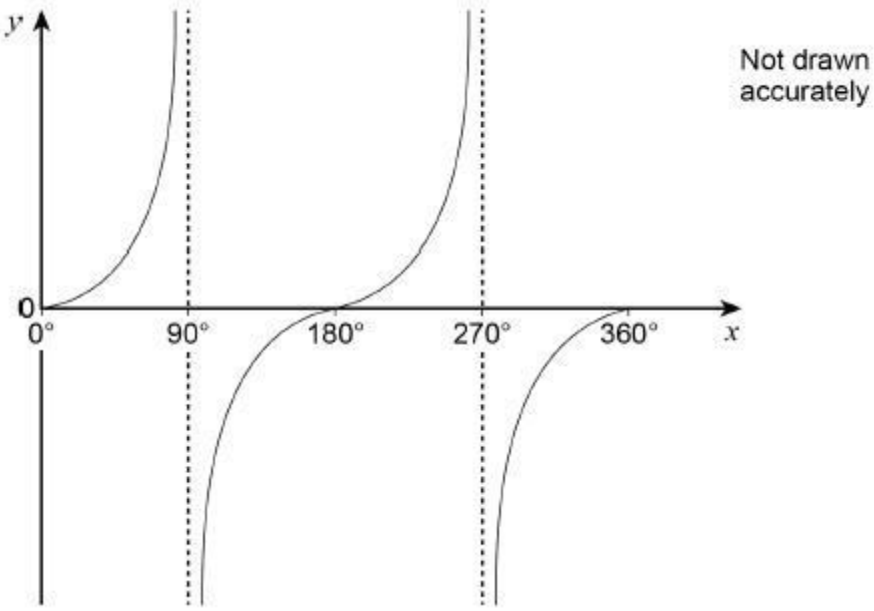
Graph D



(a) Which graph is $y = \sin x$? **(1 mark)**

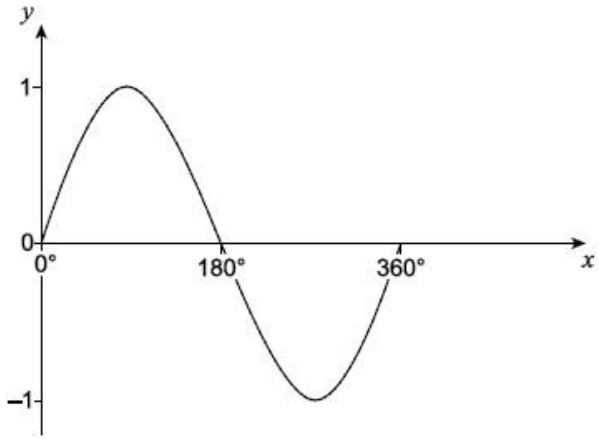
(b) Which graph is $y = \cos x$? **(1 mark)**

Q4. Here is a sketch of $y = \tan x$ for $0° \leq x \leq 360°$



How many solutions of $\tan x = k$ where $k > 0$ are between 90° and 360° ? **(1 mark)**

Q5. Here is a sketch of $y = \sin x$ for $0° \leq x \leq 360°$



α is an acute angle measured in degrees.

$\sin \alpha = k$ where k is a constant.

Write the answers to each of the following in terms of k, without involving trigonometric functions.

(a) $\sin(180° - \alpha)$ **(1 mark)**

(b) $\sin(360° - \alpha)$ **(1 mark)**

(c) $\cos \alpha$ **(2 marks)**

Section 6.9

Q1. Show that $\dfrac{4\cos^2 x + 3\sin^2 x - 4}{\cos^2 x} \equiv -\tan^2 x$ **(3 marks)**

Q2. (a) Show that $\dfrac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x}$ is equivalent to $\tan x$ **(3 marks)**

(b) Hence solve $\dfrac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = -1$ for $0° \leq x \leq 360°$ **(2 marks)**

Q3.

(a) Prove that $\sin^2 x - 3\cos^2 x \equiv 4\sin^2 x - 3$ **(2 marks)**

(b) Hence, or otherwise, work out the values of x between 0° and 360° for which

$\sin^2 x - 3\cos^2 x = 0$ **(4 marks)**

Q4. Prove that $\dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \tan\theta$ **(3 marks)**

Q5. Express $1 - \tan\theta \sin\theta \cos\theta$ in terms of $\cos\theta$. **(3 marks)**

Q6. Prove that $\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$ **(3 marks)**

Section 6.10

Q1. Work out the value of x where $0° \leqslant x \leqslant 90°$ for which $3\tan^2 x = 1$ **(2 marks)**

Q2. Solve $\sin x = 0.5$ for $0° \leq x \leq 360°$ **(2 marks)**

Q3. One solution of $\tan x = -\sqrt{3}$ is 120°

Circle another solution.

210° 240° 300° 330° **(1 mark)**

Q4. Solve $3\cos^2\theta - 1 = 0$ for $0° \leq \theta \leq 180°$ **(4 marks)**

Q5. Solve $\tan^2\theta + 3\tan\theta = 0$ for $0° < \theta < 360°$ **(5 marks)**

Q6. $0 < p < 1$

How many solutions of $\sin x = p - 1$ are between 0° and 180°?
You may use a sketch graph to help you. **(1 mark)**