6 GEOMETRY – Further Maths

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Section 6.1 (Area & Volume)

Mark schemes

Q1.

Answer	Mark	Comments
$\frac{4}{3}\pi x^3$ (=) $\frac{2}{3}\pi y^3$	M1	oe eg 1 $\frac{4}{3}\pi \times x^3$ (=) $\frac{1}{2} \times \frac{4}{3}\pi \times y^3$ eg 2 $y^3 = 2x^3$
($\frac{y^3}{x^3}$ =) $\frac{\frac{4}{3}\pi}{\frac{2}{3}\pi}$ **or** $y = \sqrt[3]{2}\,x$	M1Dep	oe eg $\frac{y^3}{x^3}$ = 2
$2^{\frac{1}{3}}$	A1	$\sqrt[3]{2}$ scores M2 A0

Q2.

Answer	Mark	Comments
Alternative method 1		
$\pi \times r \times 3r = 60\pi$	M1	oe
$r^2 = 20$ or $r = \sqrt{20}$ or $r = 2\sqrt{5}$	A1	oe
$(l =)\ 3\sqrt{20}$ or $(l =)\ 6\sqrt{5}$ or $(l =)\sqrt{180}$ or $l^2 = 180$	A1	oe

$(h^2 =)\ (3\sqrt{20})^2 - (\sqrt{20})^2$ or $(h^2 =)\ (6\sqrt{5})^2 - (2\sqrt{5})^2$ or $(h^2 =)\ (\sqrt{180})^2 - (\sqrt{20})^2$ or $(h^2 =)$ 160	M1	oe using their l and r (this is independent so l and r can be anything) condone missing brackets
$(h =)\ 4\sqrt{10}$	A1	

Alternative method 2		
$\pi \times \frac{l}{3} \times l = 60\pi$	M1	oe
$l^2 = 180$ or $l = \sqrt{180}$ or $l = 3\sqrt{20}$ or $l = 6\sqrt{5}$	A1	oe
$r^2 = 20$ or $(r =)\sqrt{20}$ or $(r =)\ 2\sqrt{5}$	A1	oe
$(h^2 =)\ (3\sqrt{20})^2 - (\sqrt{20})^2$ or $(h^2 =)\ (6\sqrt{5})^2 - (2\sqrt{5})^2$ or $(h^2 =)\ (\sqrt{180})^2 - (\sqrt{20})^2$ or $(h^2 =)$ 160	M1	oe using their l and r (this is independent so l and r can be anything) condone missing brackets
$(h =)\ 4\sqrt{10}$	A1	

Alternative method 3		
$\pi \times r \times 3r = 60\pi$ or $\pi \times \frac{l}{3} \times l = 60\pi$	M1	oe
$r^2 = 20$ or $r = \sqrt{20}$ or $r = 2\sqrt{5}$ or $l = 3\sqrt{20}$ or $l = 6\sqrt{5}$ or $l = \sqrt{180}$ or $l^2 = 180$	A1	oe
$r^2 + h^2 = (3r)^2$ or $(h^2 =)\ 9r^2 - r^2$ or $\left(\frac{l}{3}\right)^2 + h^2 = l^2$ or $(h^2 =)\ l^2 - \frac{l^2}{9}$	M1	oe to form an equation with only 2 variables using their l or r (this is independent so l and r can be anything)

$(h =)\ r\sqrt{8}$ or $(h^2 =)\ 160$	A1	oe
$(h =) 4\sqrt{10}$	A1	

Additional Guidance	
Second M mark is independent of first M mark	
Answer with no working will not gain any marks	
Minimum working for full marks would be a correct expression in the second M mark for alt method 1 and alt method 2. In this the candidate would show l and r so the first M mark would be implied. On alt method 3 they would need to show correct evidence in the first A mark and second M mark as a minimum expectation	M1, A1, A1, M1, A1

Q3.

Answer	Mark	Comments
$\frac{1}{3}$ (×) π (×) $(2p)^2$ (×) $5p$ (= $\frac{20\pi}{3}$ p^3)	B1	oe Missing brackets B0 unless recovered May be implied by working for M1
their $\frac{1}{3}$(×) π (×) $(2p)^2$ (×) $5p$ = 22 500π	M1	oe eg $\frac{20\pi}{3}$ p^3 = 22 500π π may already be cancelled or value for π may be substituted in Must be equating two volumes
Correctly rearranges to p^3 = eg p^3 = 22500π ÷ their $\frac{20\pi}{3}$	M1dep	oe eg p = $\sqrt[3]{3375}$
15	A1	SC3 [18.8, 18.9]

Q4.

Answer	Mark	Comments
$\frac{1}{2}$ × (8 + 4) × a (= 63)	M1	any letter oe eg 12a = 126

or $\frac{1}{2} \times 12 \times a$ (= 63) or $6a$ (= 63) or 63 ÷ 6		or $\frac{1}{2} \times 3 \times a + 4 \times a + \frac{1}{2} \times 1 \times a$ (= 63)
10.5 or $10\frac{1}{2}$ or $\frac{21}{2}$	A1	

Additional Guidance
M1 is for a full area calculation (= 63)

Q5.

	Answer	Mark	Comments
(a)	$2\pi r(r + 5)$ seen	M1	oe eg $2 \times \pi \times r(r + 5)$
	$\frac{9\pi r^2}{2}$	M1	oe eg $\pi \times r \times \frac{9r}{2}$
	$\pi r^2 + 2\pi r^2 + 10\pi r + \frac{9\pi r^2}{2}$ or $\frac{2\pi r^2 + 4\pi r^2 + 20\pi r + 9\pi r^2}{2}$ or $3\pi r^2 + 10\pi r + \frac{9\pi r^2}{2}$ or $\frac{6\pi r^2 + 20\pi r + 9\pi r^2}{2}$	A1	Correct unsimplified expression with brackets $2\pi r(r + 5)$ expanded May still contain multiplication signs
	$\frac{15\pi r^2}{2} + 10\pi r = \frac{5\pi r}{2}(3r + 4)$ or $\frac{15\pi r^2 + 20\pi r}{2} = \frac{5\pi r}{2}(3r + 4)$	A1	Must see M2 A1

(b)	$\frac{5\pi r}{2}(3r + 4) = 1200\pi$	M1	oe Allow $1200\pi \longrightarrow 1200$
	Correct equation or 3 term expression with no unexpanded brackets eg 1 $3r^2 + 4r - 480$ (= 0)	A1	oe

eg 2 $15r^2 + 20r = 2400$ eg 3 $\frac{15\pi}{2} r^2 + 10\pi r = 1200\pi$		
Attempt to factorise their 3 term quadratic eg for $3r^2 + 4r - 480$ $(3r + a)(r + b)$ where $ab = \pm480$ or $3b + a = \pm4$ or Attempt to substitute in the formula for their 3 term quadratic (allow one sign error) eg for $3r^2 + 4r - 480$ $\frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times -480}}{2 \times 3}$ or $\frac{4 \pm \sqrt{4^2 - 4 \times 3 \times -480}}{2 \times 3}$ (1 sign error)	M1dep	oe Attempt to complete the square for their 3 term quadratic eg for $3r^2 + 4r - 480$ $(3)\ [(r + \frac{2}{3})^2 ....]$
Correctly factorises their 3 term quadratic eg for $3r^2 + 4r - 480$ (= 0) $(3r + 40)(r - 12)$ (= 0) or Correct substitution in formula for their 3 term quadratic eg for $3r^2 + 4r - 480$ (= 0) $\frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times -480}}{2 \times 3}$	A1ft	ft M1 A0 M1dep oe Correct completion of square for their 3 term quadratic eg for $3r^2 + 4r - 480$ $(3)\ [(r + \frac{2}{3})^2 - (\frac{2}{3})^2 - 160]$ oe
12	A1	Do not award if negative solution also included

Q6.

Answer	Mark	Comments

(a)	$2\pi r^2 = \pi rl$ leading to $2r = l$ or $\frac{4\pi r^2}{2} = \pi rl$ leading to $2r = l$	B1	oe Allow verification

Additional Guidance	
$2\pi r^2 = \pi rl$ with appropriate cancelling shown	B1
Any incorrect working	B0
Verification example (Cone =) $\pi rl = \pi r \times 2r = 2\pi r^2$ Hemisphere is $2\pi r^2$ (Must link $2\pi r^2$ with the hemisphere)	B1

(b)	$(2r)^2 = r^2 + h^2$	M1	oe
	$h = r\sqrt{3}$ or $h = \sqrt{3r^2}$	A1	
	$\frac{2}{3}\pi r^3$ (+)$\frac{1}{3}\pi r^2 \times$ their $r\sqrt{3}$	M1	Must replace h with an expression in terms of r Allow $\frac{2}{3}\pi r^3$ to be $\frac{4}{3}\pi r^3$ or $\frac{8}{3}\pi r^3$
	$\frac{1}{3}\pi r^3(2 + \sqrt{3})$ with correct method seen	A1	

Additional Guidance	
$2r^2 = r^2 + h^2$ is M0 unless recovered	
$2r^2 = r^2 + h^2$ $h = r$ $\frac{8}{3}\pi r^3 + \frac{1}{3}\pi r^3$ $3\pi r^3$	M0 A0 M1 A0
Ignore units	

Q1.

Answer	Mark	Comments
Alternative method 1		
angle *BAC* = 2*y*	M1	
2*y* + *x* + 2*x* = 180 with M1 seen	M1dep	
$y = 90 - \frac{3}{2}x$ and angles in same segment (are equal) and angle sum of triangle (is 180°) with M2 seen	A1	

Answer	Mark	Comments
Alternative method 2		
angle *ACD* = *x* or angle *CED* = 2*x*	M1	
angle *ACD* = *x* and angle CED = 2*x* and 2*y* + *x* + 2*x* = 180 with M1 seen	M1dep	
$y = 90 - \frac{3}{2}x$ and angles in same segment (are equal) and vertically opposite angles (are equal)	A1	

and angle sum of triangle (is 180°) with M2 seen		

Alternative method 3		
angle *BAE* = 180 – 3*x*	M1	
2*y* = 180 – 3*x* with M1 seen	M1dep	
$y = 90 - \frac{3}{2}x$ and angle sum of triangle (is 180°) and angles in same segment (are equal) with M2 seen	A1	

Additional Guidance	
Statement must be made – do not accept if angles are only shown on the diagram	
Allow unambiguous indication of angles eg allow *A* for *BAC* but do not allow *E* for *CED*	

Q2.

Answer	Mark	Comments
Alternative method 1		
reflex angle *AOC* = 2 × 2*x* or 4*x*	M1	
their 4*x* + *x* + 75 = 360	M1dep	oe If they start with this equation, the first M1, for reflex angle *AOC* = 4*x*, is implied
(*x* =) 57	A1	

Alternative method 2

reflex angle $AOC = 360 - (x + 75)$ or $285 - x$	M1	oe
$360 - (x + 75) = 2(2x)$ or their $285 - x = 2(2x)$	M1dep	oe
$(x =)$ 57	A1	

Alternative method 3		
angle at circumference $= 180 - 2x$	M1	creating a cyclic quadrilateral
$x + 75 = 2(180 - 2x)$ or $x + 75 = 360 - 2(2x)$	M1dep	oe
$(x =)$ 57	A1	

Alternative method 4		
angle at circumference $= \frac{x + 75}{2}$	M1	oe creating a cyclic quadrilateral
$\frac{x + 75}{2} + 2x = 180$	M1dep	oe $\frac{x}{2} + \frac{\text{their } 75}{2} + 2x = 180$ scores this mark
$(x =)$ 57	A1	

Additional Guidance
$4x = x + 75$ (ans $x = 25$) and $x + 75 + 2x = 180$ (ans $x = 35$) both score 0 marks

Q3.

	Answer	Mark	Comments
(a)	Valid reason eg 1 Triangle OTS is isosceles eg 2 $OT = OS$ eg 3 OT and OS are radii	B1	

(b)			
	Correct equation eg 1 $5x = 2(x + 30)$ eg 2 $2.5x = x + 30$ eg 3 $(180 - 2x) + 120 + 5x = 360$ eg 4 $x + 30 + x + 30 + 360 - 5x = 360$	M1	oe Brackets not needed in eg 3
	Collects terms for their initial equation eg 1 $5x - 2x = 60$ eg 2 $2.5x - x = 30$ eg 3 $-2x + 5x = 360 - 180 - 120$	M1	oe their initial equation must have ≥ 2 terms in x Any brackets must be expanded correctly
	20	A1	

Q4.

Answer	Mark	Comments
$4(x + 15) + 4(x + 15) - 40 = 180$ or $8(x + 15) - 40 = 180$ or $4(x + 15) = \frac{180+40}{2}$ or $4(x + 15) - 40 = \frac{180-40}{2}$ or $y + 4(x + 15) = 180$ **and** $y = 4(x + 15) - 40$	M1	oe equation in x or pair of equations in x and y y may be any letter other than x eg $180 - (4x + 60) + 40 = 4x + 60$ or $4(x + 15) = 110$ or $4(x + 15) - 40 = 70$ or $y + 4x = 120$ **and** $y = 4x + 20$ implied by $y = 70$
$4x + 60 + 4x + 60 - 40 = 180$ or $8x + 120 - 40 = 180$ or $8x = 100$ or $100 \div 8$ or $4x = 50$ or $50 \div 4$	M1dep	oe equation or calculation equation with brackets expanded and fractions eliminated eg $120 - 4x + 40 = 4x + 60$ or $8x + 80 = 180$ or $4x + 60 = 110$

		or $4x + 20 = 70$
12.5 or $\frac{25}{2}$ or $12\frac{1}{2}$	A1	oe eg $\frac{100}{8}$ or $\frac{50}{4}$ SC2 2.5 oe

Additional Guidance	
Ignore simplification or conversion if correct answer seen	
2nd M1 Allow unnecessary brackets eg $(4x + 60) + (4x + 60) – 40 = 180$	M1M1
1st M1 may be implied if expansion error seen eg $4(x + 15) = 4x + 15$ (may be seen on diagram) $4x + 15 + 4x + 15 – 40 = 180$	M1M0
Only $4x + 15 + 4x + 15 – 40 = 180$	M0
SC2 is when they have angle *PQR* 40° larger than angle *PSR*	

Q5.

Answer	Mark	Comments
States that ∠*ABP* or ∠*ACP* is 90	B1	can be seen on diagram (either 90 or a square angle)
Any one further angle correct (not ∠*ABP* or ∠*ACP*)	B1	minor ∠*BPC* = $180 – x$ or $360 – 2y$ or major ∠*BPC* = $2y$ or $180 + x$ or ∠*BQC* = $180 – y$ or $90 – \frac{x}{2}$ (where Q is a point on the major arc)
Another further angle correct (not ∠*ABP* or ∠*ACP*)	B1	any two of minor ∠*BPC* = $180 – x$ or $360 – 2y$ or major ∠*BPC* = $2y$ or $180 + x$ or ∠*BQC* = $180 – y$ or $90 – \frac{x}{2}$ (where Q is a point on the major arc)

		could be the same angle found in the previous B mark but an expression in y rather than x
A correct equation in terms of x and y and rearrange to $y = 90 + \frac{x}{2}$	B1dep	dependent on first three B marks awarded doesn't imply the first 3 B marks
3 reasons given for the theorems used correctly for the angles stated in the first three marks	B1dep	dependent on first three B marks awarded reason - angle formed from a tangent and a radius is a right angle (can only be used once) reason - angles in a quadrilateral add up to 360 reason - angle at the centre is twice the angle at the circumference reason - opposite angles in a cyclic quadrilateral add up to 180 reason - angles at a point (or in a circle) add up to 360 reason - alternate segment theorem

Additional Guidance	
Angles must be identified with either our terminology such as $\angle ABP$ or their own labelling such as m or θ or can be seen on the diagram Accept supplementary for angles adding to 180 Accept complementary for angles adding to 90 Use of obtuse and reflex or interior and exterior instead of minor and major is fine. If it's not clear then assume it's the minor arc they are referring to Check candidates are not assuming that *BDCP* is a kite and using symmetry of this shape Check candidates are not using *BDCP* as a cyclic quadrilateral No credit for numbers used instead of x and y Mark the first three B marks positively Note – *ABPC* is a cyclic quadrilateral but *D* is not the centre of that circle Note – *D* is not the middle of minor arc *BC*	

Q6.

Answer	Mark	Comments
$x + 2x + 3x + 4x = 180$ or $10x = 180$	M1	oe
$x = 18$ or $5x = 90$	M1dep	must see working for first M1
$\angle ABC = 90$ or $\angle ADC = 90$ and (converse of) angle in a semicircle and *AC* is a diameter	A1	must see working for M1M1
(sum of) opposite angles of a cyclic quad = 180 and angle sum of a triangle = 180	A1	must see working for M1M1

Additional Guidance
The final A1 is likely to be seen within the working for M1M1A1

Q7.

Answer	Mark	Comments
$x + 62 = 2(2x - 50)$	M1	oe
$62 + 100 = 4x - x$ or $3x = 162$	M1dep	oe correct expansion and collection of terms
$x = 54$	A1	
$\frac{180 - 62 - \text{their } 54}{2}$	M1dep	
32	A1ft	ft their x with first and third M1 gained

Q8.

	Answer	Mark	Comments
(a)	Angles in the same segment	B1	oe eg angles at the circumference are equal
	Alternate angles	B1	do not accept alternative or alternating

Additional Guidance	
Angles on the circumference from a chord	B1
Angles in the same sector, opposite angles, parallel lines, angles from a chord, similar triangles, isosceles triangle, corresponding angles, triangles on a chord, intersecting chords, allied angles, alternate segment theorem	B0

(b)	$\angle HJF = 3y$ or $\angle JFG = 2x$ or $\angle HFL = 2x$	M1	may be on the diagram implied by one correct equation in x and y
	$2x + 3y + 98 = 180$ and $4x + 7y = 180$	M1dep	two correct equations in x and y
	A correct attempt to eliminate one of the variables from the two equations	M1dep	eg $(4x + 7y) – 2(2x + 3y)$
	$x = 17$ and $y = 16$	A1	

Q9.

Answer	Mark	Comments
Any 3 of angle $ABC = 100$ or angle $ABE = 2x$ or angle $BCF = 180 – 4x$		oe eg angle $BCF = 180 – 2x – 2x$ or angle $CBF = 180 – 100 – 2x$ or angle $CBF = 180 – 2(180 – 4x)$

or angle *CBF* = 80 – 2x or angle *CBF* = 8x – 180 or angle *BCF* = 50 + x	B3	or angle *BCF* = $\frac{180-(80-2x)}{2}$ B2 any two angles correct B1 any one angle correct angles may be seen on the diagram
180 – 4x = 50 + x or 2x + 2x + 50 + x = 180 or 8x – 180 + 100 + 2x = 180	M1	oe eg 180 – 4x = $\frac{180-(80-2x)}{2}$ or 2x + 2x + $\frac{180-(80-2x)}{2}$ = 180
26	A1	

Additional Guidance	
M1 implies B3	

Q10.

Answer	Mark	Comments
Alternative method 1		
angle *ABO* = x	M1	may be seen on diagram implied by angle *AOB* = 180 – 2x
angle *ACB* = 180 – w	M1	oe eg angle *ACB* + w = 180 may be seen on diagram
angle *AOB* = 2 × (180 – w) or angle *AOB* = 360 – 2w	M1dep	may be seen on diagram dep on 2nd M1 angle *AOB* may be seen as 180 – 2x
x + x + 2 × (180 – w) = 180	M1dep	oe eg 2(180 – w) = 180 – 2x dep on M3
w = x + 90 with M4 and	A1	eg of reasons isosceles triangle

all reasons given		and angles on a straight line and angle at centre and angle sum of triangle

Alternative method 2		
angle $ABO = x$	M1	may be seen on diagram implied by angle $AOB = 180 - 2x$
angle $AOB = 180 - x - x$ or angle $AOB = 180 - 2x$	M1dep	oe eg $2x$ + angle $AOB = 180$ may be seen on diagram
angle $ACB = \frac{1}{2} \times (180 - x - x)$ or angle $ACB = 90 - x$	M1dep	oe eg angle $ACB = \frac{1}{2} \times (180 - 2x)$ may be seen on diagram angle ACB may be seen as $180 - w$
$\frac{1}{2} \times (180 - x - x) + w = 180$	M1dep	oe eg $w = 180 - (90 - x)$
$w = x + 90$ with M4 and all reasons given	A1	eg of reasons isosceles triangle and angle sum of triangle and angle at centre and angles on a straight line

Alternative method 3 Draws tangent (eg PQ) at a		
angle $QAB = 90 - x$	M1	oe eg x + angle $QAB = 90$ may be seen on diagram
angle $ACB = 180 - w$	M1	oe eg angle $ACB + w = 180$ may be seen on diagram
angle QAB = angle ACB	M1	may be seen on diagram eg both angles labelled y
$90 - x = 180 - w$	M1dep	oe eg $90 - x + w = 180$ dep on M3
$w = x + 90$ with M4	A1	eg of reasons

and all reasons given		radius perpendicular to tangent and angles on a straight line and alternate segment

Additional Guidance	
Allow angle *BCD* for *w* throughout	
3rd M1 and 4th M1 may be seen in one line of working eg1 Alt 1 angle *ABO* = x angle *ACB* = 180 – w 180 – 2x = 2 × (180 – w) eg2 Alt 2 angle *ABO* = x angle *AOB* = 180 – 2x 180 – w = $\frac{1}{2}$ × (180 – 2x)	 M1 M1 M1M1 M1 M1 M1M1
Condone slips in notation only if angles are marked in correct position on the diagram eg1 Do not allow angle c = 180 – w unless marked in correct position on the diagram eg2 Allow *ACB* for angle *ACB*	
For reasons, allow if the intention is clear eg1 Allow isos triangle for isosceles triangle eg2 Allow angles in a triangle for angle sum of a triangle eg3 Allow angles on a line for angles on a straight line	
For reasons do not allow incorrect statements eg do not allow angles in a triangle add to 360	

<u>Section 6.3 – 6.5</u>

Mark schemes

Q1.

Answer	Mark	Comments
(0 <) x < 60 or (0 ⩽) x < 60	B2	B1 cos $x > \frac{5-3}{4}$ or cos $x > \frac{2}{4}$ or cos $x > \frac{1}{2}$ or $x < \cos^{-1}\frac{1}{2}$ or $a < x < 60$ where a is a non-zero value less than 60 or $b \leqslant x < 60$ where b is a value less than 60 SC1 (0 <) $x \leqslant 60$ or (0 ⩽) $x \leqslant 60$

Additional Guidance	
Answer (0 <) x < 60 (can ignore working lines)	B2
60 > x > 0 is equivalent to 0 < x < 60 etc	
0 < x < 60 is equivalent to the two inequalities x > 0 x < 60 etc	B2
Allow decimals for B1 responses eg cos x > 0.5	B1
For B1 condone cos x = > $\frac{1}{2}$ for cos $x > \frac{1}{2}$	
cos $x > \frac{1}{2}$ followed by $x > \cos^{-1}\frac{1}{2}$	B1
Only $x > \cos^{-1}\frac{1}{2}$	B0
(0, 60)	B2
[0, 60)	B2
(0, 60]	SC1
[0, 60]	SC1

Q2.

Answer	Mark	Comments
$\frac{\sin x}{2y} = \frac{\sin 18}{y}$	M1	oe
sin x = 2 sin 18 or sin x = [0.61, 0.62]	M1dep	oe eliminates y

or $\sin^{-1}$ [0.61, 0.62] or 38.(17…) or 38.(2)		
141.8… or 142	A1	

Q3.

Answer	Mark	Comments
(VM^2=) $10^2 - 3^2$ or 100 – 9 or 91	M1	oe
(DM^2=) $8^2 + 3^2$ or 64 + 9 or 73	M1	oe
10^2 = their 91 + their 73 $- 2 \times \sqrt{\text{their } 91}$ $\times \sqrt{\text{their } 73} \times \cos VMD$	M1dep	oe dep on M2 may be implied
($\cos VMD$ =) $\dfrac{\text{their } 91 + \text{their } 73 - 10^2}{2 \times \sqrt{\text{their } 91} \times \sqrt{\text{their } 73}}$	M1dep	oe dep on M3
[66.8, 66.9] or 67	A1	

Q4.

Answer	Mark	Comments
$AB = \sqrt{3}$	B1	
Any one of these responses ... $\dfrac{BD}{2\sqrt{3}}$ = cos 30° $\dfrac{BD}{2\sqrt{3}}$ = sin 60° $\dfrac{\sqrt{3}}{BD}$ = tan 30° $\dfrac{BD}{\sqrt{3}}$ = tan 60° $BD^2 + (\sqrt{3})^2 = (2\sqrt{3})^2$ oe	M1	... or these ... $\dfrac{BD}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$ $\dfrac{BD}{\sqrt{3}} = \sqrt{3}$ $\dfrac{\sqrt{3}}{BD} = \dfrac{1}{\sqrt{3}}$ $\dfrac{BD}{\sin 60°} = \dfrac{\sqrt{3}}{\sin 30°}$ $\dfrac{BD}{\sqrt{3}/2} = \dfrac{\sqrt{3}}{1/2}$
$BD = 3$	A1	
$CD = 3 - \sqrt{3}$	A1	oe

Additional Guidance
SC1 for a **final** answer of $\frac{2\sqrt{3}\sin 15°}{\sin 135°}$, possibly with $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ for sin 135°

Q5.

Answer	Mark	Comments
$\frac{5}{3} \times 15$ or 25 seen as the length of *OB* or the coordinates of *B*	M1	
gradient *AB* = $\frac{0 - \text{their } 25}{15 - 0}$ or $-\frac{5}{3}$	M1	oe
gradient *BC* = −1 ÷ (their $-\frac{5}{3}$) or $\frac{3}{5}$	M1	oe
$y = \frac{3}{5}x + 25$	A1	oe eg $y = \frac{15}{25}x + 25$ or $5y = 3x + 125$

Additional Guidance
We must see *y* = ... for A1 (or any other correct equation) Look for this in their working if it isn't written on the answer line. A sign error in their gradient *AB*, after a correct expression, can be recovered. eg gradient *AB* = $\frac{0 - 25}{15 - 0} = \frac{25}{15} = \frac{5}{3}$ gradient *BC* = $\frac{3}{5}$ (positive gradient because they can see it from the diagram) equation *BC* is $y = \frac{3}{5}x + 25$ ... this scores 4 marks similarly, recovery can be from ...

gradient $AB = \frac{25}{15} = \frac{5}{3}$ ... without seeing $\frac{0-25}{15-0}$ ... and can still lead to 4 marks

Q6.

Answer	Mark	Comments
(cos *CAB* =) $\frac{(3+\sqrt{5})^2+(3-\sqrt{5})^2-(2\sqrt{6})^2}{2(3+\sqrt{5})(3-\sqrt{5})}$	M1	oe eg $(2\sqrt{6})^2 = (3+\sqrt{5})^2 + (3-\sqrt{5})^2$ $- 2(3+\sqrt{5})(3-\sqrt{5})\cos CAB$
$((3+\sqrt{5})^2=)\ 9+3\sqrt{5}+3\sqrt{5}+5$ or $((3-\sqrt{5})^2=)\ 9-3\sqrt{5}-3\sqrt{5}+5$ or $((2\sqrt{6})^2=)\ \ 4\times 6$ or $((3+\sqrt{5})(3-\sqrt{5})=)$ $9-3\sqrt{5}+3\sqrt{5}-5$	M1	oe eg $9 + 6\sqrt{5} + 5$ or $9 - 6\sqrt{5} + 5$ or 24 or 9 – 5 or 4
Any three of $((3+\sqrt{5})^2=)\ 9+3\sqrt{5}+3\sqrt{5}+5$ or $((3-\sqrt{5})^2=)\ 9-3\sqrt{5}-3\sqrt{5}+5$ or $((2\sqrt{6})^2=)\ \ 4\times 6$ or $((3+\sqrt{5})(3-\sqrt{5})=)$ $9-3\sqrt{5}+3\sqrt{5}-5$	M1dep	
cos *CAB* = $\frac{14+14-24}{8}$	A1	must have cos *CAB* =
cos *CAB* = $\frac{4}{8}$ and 60		

or cos *CAB* = $\frac{1}{2}$	A1	

Additional Guidance	
2nd M1 is not dependent on the 1st M1	
Allow cos A or cos x etc	

Q7.

Answer	Mark	Comments
Alternative method 1 Works out *MD* and *BD* and uses tan *MBD*		
tan 28 = $\frac{GN}{32}$ or 32 tan 28 or [17, 17.015]	M1	oe eg $\frac{32}{\tan(90-28)}$ working out *GN* or *HM*
32 – 32 tan 28 or [14.985, 15]	M1dep	oe working out *NC* or *MD*
$\sqrt{32^2+32^2}$ or $\sqrt{2048}$ or [45.2, 45.3]	M1	oe eg $32\sqrt{2}$ working out *BD*
tan *MBD* = $\frac{\text{their }[14.985, 15]}{\text{their }[45.2, 45.3]}$	M1dep	oe eg $\tan^{-1}\frac{\text{their }[14.985, 15]}{\text{their }[45.2, 45.3]}$ dep on M3
[18.3, 18,4]	A1	

Alternative method 2 Works out *BD* and *MB* and uses cos *MBD*		
tan 28 = $\frac{GN}{32}$ or 32 tan 28 or [17, 17.015]	M1	oe eg $\frac{32}{\tan(90-28)}$ working out *GN* or *HM*
32 – 32 tan 28 or [14.985, 15]	M1dep	oe working out *NC* or *MD*
$\sqrt{32^2+32^2}$ or $\sqrt{2048}$ or [45.2, 45.3] or $\sqrt{32^2+32^2+\text{their }[14.985,15]^2}$	M1	oe eg $32\sqrt{2}$ working out *BD* or *MB* if awarding this mark for working out *MB* it is dependent on M2

or [47.67, 47.7]		
cos *MBD* = $\dfrac{\sqrt{32^2+32^2}}{\sqrt{32^2+32^2+\text{their }[14.985,\ 15]^2}}$	M1dep	oe eg $\cos^{-1}\dfrac{\sqrt{32^2+32^2}}{\sqrt{32^2+32^2+\text{their }[14.985,\ 15]^2}}$ dep on M3
[18.3, 18,4]	A1	

Alternative method 3 Works out *MD* and *MB* and uses sin *MBD*		
tan 28 = $\dfrac{GN}{32}$ or 32 tan 28 or [17, 17.015]	M1	oe eg $\dfrac{32}{\tan(90-28)}$ working out *GN* or *HM*
32 – 32 tan 28 or [14.985, 15]	M1dep	oe working out *NC* or *MD*
$\sqrt{32^2+32^2+\text{their }[14.985,15]^2}$ or [47.67, 47.7]	M1dep	oe working out *MB*
sin *MBD* = $\dfrac{\text{their }[14.985,15]}{\sqrt{32^2+32^2+\text{their }[14.985,\ 15]^2}}$	M1dep	oe eg $\sin^{-1}\dfrac{\text{their }[14.985,15]}{\sqrt{32^2+32^2+\text{their }[14.985,\ 15]^2}}$
[18.3, 18,4]	A1	

Additional Guidance	
1st M1 *GN* may be seen as a letter, eg x, but do not award if subsequently used as the length of an incorrect side (eg *MN*)	
4th M1 *MBD* may be seen as a letter, eg y, but do not award if subsequently used as the size of an incorrect angle (eg *DMB*)	
Alt 1 or Alt 2 $32\sqrt{1^2+1^2}$	3rd M1
Alt 1 tan *MBD* = $\dfrac{32(1-\tan 28)}{32\sqrt{2}}$ or tan *MBD* = $\dfrac{(1-\tan 28)}{\sqrt{2}}$	M4

Q8.

Answer	Mark	Comments

$\frac{3a}{2a+9} = \frac{3}{5}$	M1	
$15a = 6a + 27$	M1dep	oe eg $9a = 27$
$a = 3$	A1	
$15^2 - 9^2$ or $225 - 81$ or 144	M1	ft their 3 if less than 9
12	A1ft	ft their 3 if less than 9

Additional Guidance	
ft answer must be exact or to 1 dp or better	

Q9.

Answer	Mark	Comments
$\frac{40}{3+7} \times 7$ or 28	M1	oe eg $40 - \frac{40}{3+7} \times 3$ or $40 - 12$ may be seen on diagram may be implied
$20^2 + \text{their } 28^2$ or $400 + 784$ or 1184 or $4\sqrt{74}$ or [34.4, 34.41]	M1	oe eg $\sqrt{20^2 + \text{their}28^2}$ or $\sqrt{1184}$ their 28 must be < 40 may be seen on diagram
$40^2 + 9^2$ or $1600 + 81$ or 1681 or 41	M1	oe eg $\sqrt{40^2 + 9^2}$ or $\sqrt{1681}$ may be seen on diagram
their $1681 = 25^2 + \text{their } 1184$ $- 2 \times 25 \times \sqrt{\text{their}1184} \times \cos x$	M1dep	oe eg $\cos^{-1} \frac{25^2 + \text{their}1184 - \text{their}1681}{2 \times 25 \times \sqrt{\text{their}1184}}$ or $\cos^{-1}$ [0.07, 0.07442] dep on 2nd and 3rd M1 x may be *APC* or *A* etc
[85.7, 86]	A1	

Additional Guidance	
Up to M4 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	

If their *PG* is 28 do not allow use of a value other than 28 in subsequent working	
$\frac{40}{3+7} \times 3 = 12$ $20^2 + 12^2 = 544$ $40^2 + 9^2 = 1681$ $\cos^{-1} \frac{25^2 + 544 - 1681}{2 \times 25 \times \sqrt{544}}$	M0 M1 M1 M1A0
4th M1 Condone $\cos^{-1} = 0.07$ for $\cos^{-1} 0.07$ etc	
4th M1 oes must be a fully correct method eg Uses cosine rule to work out angle *PCA* then uses sine rule to work out angle *APC* Must get to correct sine rule equation with no errors in method	
Missing brackets must be recovered eg 4th M1 Do not allow $4\sqrt{74}^2$ unless recovered in subsequent working	
When *AP* is used it must be 25	

Q10.

Answer	Mark	Comments
$7^2 = x^2 + 3^2 - 2 \times 3 \times x \cos 60°$	M1	oe
$x^2 - 3x - 40$ (= 0)	A1	
$(x - 8)(x + 5)$ (= 0) or $\frac{--3 \pm \sqrt{(-3)^2 - 4 \times 1 \times -40}}{2 \times 1}$	M1	oe follow through their three term quadratic
8	A1	

Additional Guidance	
If –5 is also given as an answer then do not award final A mark	

Q11.

Answer	Mark	Comments
Alternative method 1 Uses $\frac{1}{2}ab\sin C$		
$\frac{1}{2} \times 16 \times 16 \times \sin x$ or $128 \sin x$	M1	oe eg $\frac{1}{2} \times 16 \times 16 \times \sin(180 - 2y)$ x can be any letter or expression may be implied
$\sin x = 120 \div \left(\frac{1}{2} \times 16 \times 16\right)$ or $\sin x = \frac{15}{16}$ or $\sin^{-1} \frac{15}{16}$ or $\sin^{-1}$ [0.93, 0.94] or [68.4, 70.12313]	M1dep	oe eg $\sin x = \frac{240}{256}$ or $\sin x$ = [0.93, 0.94] equation must have $\sin x$ = x can be any letter or expression
$\frac{180 - \text{their}[68.4, 70.12313]}{2}$	M1dep	oe
[54.93, 55.8]	A1	SC2 [75.82, 76.4]

Alternative method 2 Works out perpendicular height		
$120 \div \left(\frac{1}{2} \times 16\right)$ or $120 \div 8$ or 15	M1	
$\cos x = \frac{\sqrt{16^2 - (\text{their}15)^2}}{16}$ or $\cos^{-1} \frac{\sqrt{31}}{16}$ or $\tan x = \frac{15}{\sqrt{16^2 - (\text{their}15)^2}}$ or $\tan^{-1} \frac{15}{\sqrt{31}}$ or [68.4, 70.12313]	M1dep	oe eg $\sin x = \frac{15}{16}$ or $\sin x$ = [0.93, 0.94] or $\cos x$ = [0.34, 0.35] or $\tan x$ = [2.69, 2.7] x can be any letter or expression
$\frac{180 - \text{their}[68.4, 70.12313]}{2}$	M1dep	oe
[54.93, 55.8]	A1	SC2 [75.82, 76.4]

Alternative method 3 Works out perpendicular height		
$120 \div \left(\frac{1}{2} \times 16\right)$ or 120 ÷ 8 or 15	M1	oe
$16 - \sqrt{16^2 - (\text{their}15)^2}$ or 16 − √31 or [10.4, 10.44]	M1dep	oe eg tan $y = \frac{15}{16 - \sqrt{16^2 - (\text{their}15)^2}}$ y can be any letter or expression
$\tan^{-1} \frac{15}{\text{their}[10.4, 10.44]}$	M1dep	oe eg tan $^{-1}$ [1.43, 1.44231]
[54.93, 55.8]	A1	SC2 [75.82, 76.4]

Additional Guidance	
Alt 1 y = [68.4, 70.12313]	M1M1
Condone sin = for sin x = etc Condone sin $^{-1}$ = 0.9375 for sin $^{-1}$ 0.9375 etc	
SC2 is for omitting the 0.5 from the area of triangle formula	
After scoring M1M1, the 3rd M1 is for any full method eg Alt 1 68.6 Cosine rule used to work out the third side of the triangle followed by sine rule to work out y (up to sin $^{-1}$…) If there are no errors seen in the method the 3rd M1 is awarded and possibly the A1 as well	M1M1

Q12.

Answer	Mark	Comments
(AB) = 1 and (AC) = 0.75	M1	oe could be seen on diagram allow AB = –1 and/or AC = –0.75
(BC^2 =) $1^2 + \left(\frac{3}{4}\right)^2$	M1dep	oe eg $(-2 - -1)^2 + \left(5\frac{3}{4} - 5\right)^2$ $\sqrt{1.5625}$ or $\sqrt{\frac{25}{16}}$ or $\sqrt{1\frac{9}{16}}$ would imply this mark
(BC =) $\frac{5}{4}$ or $1\frac{1}{4}$ or 1.25	A1	

Additional Guidance	
Candidates may spot it's a $\frac{3}{4}, 1, \frac{5}{4}$ Pythagorean triple which gains the M marks and will probably go on to score all marks	
Ignore further rounding or truncating after correct answer seen eg $\frac{5}{4}$ followed by = 1.2 would score the A mark	M2A1
Condone $\frac{3}{4}^2$ without the brackets. Condone -1^2 without the brackets $\frac{5}{4}$ followed by = 0.8 is incorrect further working	M2A0

Section 6.6 – 6.7

Mark schemes

Q1.

Answer	Mark	Comments
144°	B1	answers should be on answer line but can be accepted if they are the only angles written on the diagram (other than 36° which is the question so fine) condone missing degree sign
216°	B1	

Additional Guidance	
Don't accept cos144°, cos216°, cos x = 144°, cos x = 216°	B0
Accept cos144° = −0.8090 and cos216° = −0.8090	B1, B1
If more than 2 angles offered this is choice	
4 or more angles	B0
2 wrong 1 right	B0
1 wrong 2 right	B1
1 wrong 1 right	B1

Q2.

Answer	Mark	Comments
(0, 1) (90, 0) (270, 0) with no other points	B2	B1 two answers, both correct or three answers, two correct or four answers, three correct

Additional Guidance	
Condone 0, 1 for (0, 1) etc	
0, 90, 270	B0
(1, 0) (0, 90) (0, 270)	B0

Q3.

	Answer	Mark	Comments
(a)	*C*	B1	Do not allow if more than one answer selected
(b)	*A*	B1	Do not allow if more than one answer selected

Q4.

Answer	Mark	Comments
1	B1	allow in words

Q5.

	Answer	Mark	Comments
(a)	k	B1	

Additional Guidance	
$k = 0$ or $k = 1$ etc	B0

	Answer	Mark	Comments
(b)	$-k$	B1	

Additional Guidance	
$-k = 0$ or $-k = 1$ etc	B0

(c)	$k^2 + \cos^2 \alpha = 1$ or $1 - k^2$	M1	oe eg $(1 + k)(1 - k)$
	$\sqrt{1-k^2}$ or $\sqrt{(1+k)(1-k)}$	A1	

Additional Guidance	
Answer $-\sqrt{1-k^2}$ or $\pm\sqrt{1-k^2}$	M1A0
Correct answer followed by incorrect further work	M1A0
Answer $1 - k^2$	M1A0
Allow $\cos^2 x$ or $\cos^2 \theta$ etc or $\cos^2$ or c^2 or $(\cos \alpha)^2$ for $\cos^2 \alpha$	
Condone $\cos \alpha^2$ for $\cos^2 \alpha$	
$\cos(\sin^{-1}k)$	M0A0

Section 6.9

Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		
LHS Use of: $\cos^2 x \equiv 1 - \sin^2 x$ or $\sin^2 x \equiv 1 - \cos^2 x$ or $3\sin^2 x + 3\cos^2 x \equiv 3$ in numerator to get: $4(1 - \sin^2 x) + 3\sin^2 x - 4$	M1	oe must be used as part of a solution (nothing for just stating it)

or $4\cos^2 x + 3(1 - \cos^2 x) - 4$ or $3 + \cos^2 x - 4$		
LHS $\dfrac{4 - 4\sin^2 x + 3\sin^2 x - 4}{\cos^2 x}$ or simplification of the other forms leading to $\dfrac{\cos^2 x - 1}{\cos^2 x}$	M1dep	one step away from the A mark this could imply the first M1 provided they have stated the identity used from the list in the first M mark
$-\dfrac{\sin^2 x}{\cos^2 x} \equiv -\tan^2 x$	A1	oe

Alternative method 2		
LHS $\dfrac{\left[4\cos^2 x + 3\sin^2 x - 4\left(\cos^2 x + \sin^2 x\right)\right]}{\cos^2 x}$	M1	
$\dfrac{\left[4\cos^2 x + 3\sin^2 x - 4\cos^2 x - 4\sin^2 x\right]}{\cos^2 x}$	M1	
$-\dfrac{\sin^2 x}{\cos^2 x} \equiv -\tan^2 x$	A1	oe

Alternative method 3		
RHS $-\tan^2 x \equiv -\dfrac{\sin^2 x}{\cos^2 x}$	M1	
$\dfrac{\left[4\left(\sin^2 x + \cos^2 x\right) - 4 - \sin^2 x\right]}{\cos^2 x}$	M1	
$\dfrac{\left[4\cos^2 x + 3\sin^2 x - 4\right]}{\cos^2 x}$	A1	

Additional Guidance	
Either starts with the left and finishes with the right or vice versa. Max M2 for any working that meets in the middle by trying to solve an equation Only mark using one of the alts – once the candidate starts to treat the solution as an equation by moving terms around from	

one side of the ≡ to the other then stop awarding marks The exception to this would be if a candidate uses identities to manipulate the LHS to an expression correctly and also then manipulates the RHS correctly to the same expression. They would then need to state that these two manipulations show the LHS ≡ RHS	

Q2.

	Answer		Mark	Comments
(a)	**Alternative method 1**			
	$2\sin^2 x - 1 + 1 - \sin^2 x$ or $2\sin^2 x - (\sin^2 x + \cos^2 x) + \cos^2 x$ or $2\sin^2 x - \sin^2 x - \cos^2 x + \cos^2 x$ or $2\sin^2 x - \sin^2 x$ or $\sin^2 x - \cos^2 x + \cos^2 x$ or $1 + \sin^2 x - 1$		M1	use of $\sin^2 x + \cos^2 x = 1$ in numerator ignore any denominator
	$\dfrac{\sin^2 x}{\sin x \cos x}$ with M1 seen	$\dfrac{\sin^2 x}{\tan x \cos^2 x}$ with M1 seen	M1dep	simplification to one step from $\dfrac{\sin x}{\cos x}$ or simplification to one step from $\dfrac{\tan^2 x}{\tan x}$
	$\dfrac{\sin x}{\cos x}$ and tax x with M2 seen	$\dfrac{\tan^2 x}{\tan x}$ and tax x with M2 seen	A1	SC3 equates given expression to tan x and cross multiplies to show equivalence with full working shown

Alternative method 2		
$2(1 - \cos^2 x) - 1 + \cos^2 x$	M1	use of $\sin^2 x + \cos^2 x = 1$ in

or $2 - 2\cos^2 x - 1 + \cos^2 x$			numerator ignore any denominator
$\frac{1-\cos^2 x}{\sin x \cos x}$ and $\frac{\sin^2 x}{\sin x \cos x}$ with M1 seen	$\frac{1-\cos^2 x}{\sin x \cos x}$ and $\frac{\sin^2 x}{\tan x \cos^2 x}$ with M1 seen	M1dep	simplification to one step from $\frac{\sin x}{\cos x}$ or simplification to one step from $\frac{\tan^2 x}{\tan x}$
$\frac{\sin x}{\cos x}$ and $\tan x$ with M2 seen	$\frac{\tan^2 x}{\tan x}$ and $\tan x$ with M2 seen	A1	SC3 equates given expression to $\tan x$ and cross multiplies to show equivalence with full working shown

Alternative method 3		
$\frac{2\sin x}{\cos x} - \frac{\sin^2 x}{\sin x \cos x}$	M1	from $\frac{2\sin^2 x}{\sin x \cos x} - \frac{1 - \cos^2 x}{\sin x \cos x}$
$2\tan x - \frac{\sin^2 x}{\sin x \cos x}$ or $\frac{2\sin x}{\cos x} - \frac{\sin x}{\cos x}$ with M1 seen	M1dep	simplification to one step from $2\tan x - \tan x$
$2\tan x - \tan x$ and $\tan x$ with M2 seen	A1	SC3 equates given expression to $\tan x$ and cross multiplies to show equivalence with full working shown

Additional Guidance	
Equating given expression to $\tan x$ and cross multiplying can score SC3 or M1M0A0 eg1 Alt 1 $\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = \tan x$ $2\sin^2 x - 1 + \cos^2 x = \tan x \sin x \cos x$ $2\sin^2 x - 1 + 1 - \sin^2 x = \tan x \sin x \cos x$ (scores M1 here for LHS)	M1, M0, A0

eg2 $\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = \tan x$ $2\sin^2 x - 1 + \cos^2 x = \tan x \sin x \cos x$ $2\sin^2 x - 1 + 1 - \sin^2 x = \tan x \sin x \cos x$ $\sin^2 x = \tan x \sin x \cos x$ $\sin^2 x = \frac{\sin x}{\cos x} \sin x \cos x$ $\sin^2 x = \sin^2 x$	SC3
Use of $\sin x = \frac{\text{opp}}{\text{hyp}}$ etc	M0, M0, A0
Allow sin or s for sin x etc	
Condone sin x^2 for $\sin^2 x$ etc	
Allow any letter for x	
Alts 1 and 2 For A1 $\frac{\sin x}{\cos x}$ is implied by $\frac{\sin^2 x}{\sin x \cos x}$ with cancelling shown	

(b)

135 and 315 with no other solutions [0, 360]	B2	B1 135 with no other solutions [0, 360] or 315 with no other solutions [0, 360] SC1 135 and 315 with one other solution [0, 360]

Additional Guidance	
Mark the answer line unless blank eg 135 and 315 in working with 135 on answer line	B1
−45 and 135 and 315	B2
−45 and 135	B1
Ignore incorrect solutions outside the range [0, 360] eg 135 and 315 and −90	B2
135 and 225 and 315	SC1

Both answers embedded ie tan 135 tan 315	B1
0 and 135 and 225 and 315	B0
45 and 135	B0
225 and 315	B0

Q3.

(a)

Answer	Mark	Comments
Alternative method 1 (LHS → RHS)		
$\sin^2 x - 3(1 - \sin^2 x)$	M1	Must see $(1 - \sin^2 x)$
$\sin^2 x - 3 + 3\sin^2 x = 4\sin^2 x - 3$	A1	Must see correct expansion SC1 Correct rearrangement of given identity to $3 \sin^2 x + 3 \cos^2 x = 3$ and $3 (\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

Alternative method 2 (LHS → RHS)		
$1 - \cos^2 x - 3 \cos^2 x = 1 - 4 \cos^2 x$ $= 1 - 4(1 - \sin^2 x)$	M1	Must see $(1 - \cos^2 x)$ and $(1 - \sin^2 x)$
$1 - 4 + 4 \sin^2 x = 4 \sin^2 x - 3$	A1	Must see correct expansion SC1 Correct rearrangement of given identity to $3 \sin^2 x + 3 \cos^2 x = 3$ and $3 (\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

Alternative method 3 (RHS → LHS)		
$4 \sin^2 x - 3(\sin^2 x + \cos^2 x)$	M1	Must see $(\sin^2 x + \cos^2 x)$
$4 \sin^2 x - 3 \sin^2 x - 3 \cos^2 x)$ $= \sin^2 x - 3 \cos^2 x$	A1	Must see correct expansion SC1 Correct rearrangement of given identity to $3 \sin^2 x + 3 \cos^2 x = 3$ and $3 (\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

Alternative method 4 (RHS → LHS)		
$4(1 - \cos^2 x) - 3 = 4 - 4\cos^2 x - 3$ $= 1 - 4\cos^2 x$ $= \sin^2 x + \cos^2 x - 4\cos^2 x$	M1	Must see $(1 - \cos^2 x)$ and $\sin^2 x + \cos^2 x$ and correct expansion
$= \sin^2 x - 3\cos^2 x$	A1	SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

Alternative method 5 (LHS and RHS → common expression)		
$1 - \cos^2 x - 3\cos^2 x = 1 - 4\cos^2 x$ and $4(1 - \cos^2 x) - 3 = 4 - 4\cos^2 x - 3$ $= 1 - 4\cos^2 x$	B2	Must see $(1 - \cos^2 x)$ and correct expansion SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

Additional Guidance	
As shown in the mark scheme, allow = signs but they may be seen (correctly) as the identity symbol	
= signs may be implied (eg working down the page, line by line)	
To give M1 the working must not need any further identities applying	
The other side of the identity may be seen throughout working in Alts 1 to 4 However, full working on one side of the identity is needed for M1 A1 eg (Alt 2) $1 - \cos^2 x - 3\cos^2 x = 4\sin^2 x - 3$ $1 - 4\cos^2 x = 4\sin^2 x - 3$ $1 - 4(1 - \sin^2 x) = 4\sin^2 x - 3$ $1 - 4 + 4\sin^2 x = 4\sin^2 x - 3$ (with $4\sin^2 x - 3 = 4\sin^2 x - 3$ it would be M1 A1)	M1 A0

Other examples may be seen, escalate if necessary	
Allow any variable or mixed variables or no variables	
Allow $(\sin x)^2$ for $\sin^2 x$ and $(\cos x)^2$ for $\cos^2 x$ Allow s^2 for $\sin^2 x$ and c^2 for $\cos^2 x$	
Do not allow $\sin x^2$ for $\sin^2 x$ (but could still gain M1) eg1 Alt 1 $\sin^2 x - 3(1 - \sin^2 x)$ $= \sin^2 x - 3 + 3 \sin x^2 = 4 \sin x^2 - 3$ eg1 Alt 1 $\sin x^2 - 3(1 - \sin^2 x)$ $= \sin^2 x - 3 + 3 \sin x^2 = 4 \sin x^2 - 3$	M1 A0 M0 A0
Do not allow recovery of missing brackets as this is a proof	
SC1 Instead of factorisation, they can divide by 3	
Other examples of SC1 may be seen where the identity is assumed to be correct and correct working with use of $\sin^2 x + \cos^2 x = 1$ is seen	

(b)

Alternative method 1		
$\sin^2 x = \frac{3}{4}$ or $\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = \sqrt{\frac{3}{4}}$ or 60 or 120	M1	oe eg $(\sin x)^2 = \frac{3}{4}$ Allow 0.86... or 0.87 for $\frac{\sqrt{3}}{2}$ Must have $\sin^2 x =$ or $\sin x =$ or $\sin^{-1}$ Allow s for $\sin x$ Do not allow $\sin x^2$ for $\sin^2 x$ but may be recovered
$\sin x = -\frac{\sqrt{3}}{2}$ or $\sin x = -\sqrt{\frac{3}{4}}$ or 240 or 300 or –60	M1	oe Allow –0.86... or –0.87 for $-\frac{\sqrt{3}}{2}$
60 and 120 and 240 and 300 with no other angles in range	A2	A1 60 and 120 or 240 and 300

Alternative method 2

$\tan^2 x = 3$ or $\tan x = \sqrt{3}$ or 60 or 240	M1	oe eg $(\tan x)^2 = 3$ Allow 1.73... for 3 Must have $\tan^2 x =$ or $\tan x =$ or $\tan^{-1}$ Allow t for $\tan x$ Do not allow $\tan x^2$ for $\tan^2 x$ but may be recovered
$\tan x = -\sqrt{3}$ or 120 or 300 or –60	M1	Allow –1.73... for $-\sqrt{3}$
60 and 120 and 240 and 300 with no other angles in range	A2	A1 60 and 240 or 120 and 300

Alternative method 3		
$\cos^2 x = \frac{1}{4}$ or $\cos x = \frac{1}{2}$ or $\cos x = \sqrt{\frac{1}{4}}$ or 60 or 300	M1	oe eg $(\cos x)^2 = \frac{1}{4}$ Must have $\cos^2 x =$ or $\cos x =$ or $\cos^{-1}$ Allow c for $\cos x$ Do not allow $\cos x^2$ for $\cos^2 x$ but may be recovered
$\cos x = -\frac{1}{2}$ or $\cos x = -\sqrt{\frac{1}{4}}$ or 120 or 240	M1	oe
60 and 120 and 240 and 300 with no other angles in range	A2	A1 60 and 300 or 120 and 240

Additional Guidance	
Ignore any solutions outside of $0 < x < 360$ ie 0 and 360 are outside the range and can be ignored	
All four solutions with extra solutions in range, $0 < x < 360$,are penalised one accuracy mark eg 60 90 120 150 240 300 Only penalise extra solutions in range when all four correct solutions are given	M1 M1 A1

Answer line blank, award any marks gained from working lines	
If angles are found in working lines but only some are listed on answer line award any method marks gained from the working lines award any accuracy marks gained from the answer line eg1 Working lines sin x = ± $\frac{\sqrt{3}}{2}$ 60 and 120 and 240 and 300 Answer line 60 and 120 and 240 eg2 Working lines tan x = $\sqrt{3}$ 60 240 Answer line 60 eg3 Working lines sin x = $\frac{\sqrt{3}}{2}$ 60 120 sin x = – $\frac{\sqrt{3}}{2}$ 300 Answer line 300	 M1 M1 A1 M1 M0 A0 M1 M1 A0
Answers only can score up to 4 marks All 4 correct → 4 marks 3 correct → 3 marks 2 correct → 2 marks 1 correct → 1 mark	
M1 M0 A1 or M0 M1 A1 are possible eg1 sin x = $\frac{\sqrt{3}}{2}$ 60 120 eg1 sin x = – $\frac{\sqrt{3}}{2}$ 240 300	M1 M0 A1 M0 M1 A1
Embedded answers can score up to M1 M1 A0	
Working in rads or grads can score M marks if method seen	

Q4.

Answer	Mark	Comments
Alternative method 1		
$\frac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \frac{\sin\theta(1-\sin^2\theta)}{\cos^3\theta}$	M1	
$\frac{\sin\theta - \cos^2\theta}{\cos^3\theta}$	M1	oe eg sin θ ($\sin^2\theta + \cos^2\theta - \sin^2\theta$)

$\frac{\sin\theta\cos^2\theta}{\cos^3\theta} \equiv \frac{\sin\theta}{\cos\theta} \equiv \tan\theta$	A1	

Alternative method 2		
$\frac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \frac{\sin\theta(1-\sin^2\theta)}{\cos^3\theta}$	M1	
$\frac{\sin\theta(1-\sin^2\theta)}{\cos\theta(1-\sin^2\theta)}$	A1	
$\frac{\sin\theta(1-\sin^2\theta)}{\cos\theta(1-\sin^2\theta)} \equiv \frac{\sin\theta}{\cos\theta} \equiv \tan\theta$	A1	

Q5.

Answer	Mark	Comments
Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$	M1	eg $1 - \frac{\sin\theta}{\cos\theta}\sin\theta\cos\theta$
$1 - \sin^2\theta$	M1dep	oe eg $\sin^2\theta + \cos^2\theta - \sin\theta\sin\theta$
$\cos^2\theta$	A1	Condone $(\cos\theta)^2$ but do not allow $\cos\theta^2$

Q6.

Answer	Mark	Comments
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\frac{1}{\tan\theta} \equiv \frac{\cos\theta}{\sin\theta}$	M1	oe
Denominator = $\sin\theta\cos\theta$	M1Dep	oe
$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ ($\sin^2\theta + \cos^2\theta \equiv 1$) and $\frac{1}{\sin\theta\cos\theta}$	A1	All steps clearly shown

Section 6.10

Mark schemes

Q1.

Answer	Mark	Comments
$\tan x = (\pm)\frac{1}{\sqrt{3}}$ or $\tan x = (\pm)\frac{\sqrt{3}}{3}$	M1	
30 with no incorrect solutions within the given range	A1	ignore correct solutions outside the given range.

Q2.

Answer	Mark	Comments
30 and 150 with no other solutions [0, 360]	B2	B1 30 with no other solutions [0, 360] or 150 with no other solutions [0, 360] SC1 30 and 150 with one other solution [0, 360]

Q3.

Answer	Mark	Comments
300°	B1	

Q4.

Answer	Mark	Comments
$\cos^2\theta = \frac{1}{3}$	B1	May be implied in working $\sin^2\theta = \frac{2}{3}$ or $\tan^2\theta = 2$
$\cos\theta = (\pm)\sqrt{\frac{1}{3}}$	M1	oe eg $\cos\theta = (\pm)$ [0.57(7), 0.6] $\sin\theta = (\pm)\sqrt{\frac{2}{3}}$ oe **or** $\tan\theta = (\pm)\sqrt{2}$ oe

[54.7, 54.7602]	A1	
[125.2398, 125.3]	A1ft	ft 180 – their [54.7, 54.7602] if M1 gained Correct or ft A0 if an incorrect solution [0, 180] also seen

Q5.

Answer	Mark	Comments
tan θ(tan θ + 3) **or** tan θ = 0 **or** sin θ(sin θ + 3cos θ) **or** sin θ = 0	M1	oe eg $t(t + 3)$ Must be correct
180	A1	
tan θ = –3	A1	
[108, 108.44]	A1	
[288, 288.44]	B1ft	ft 180 + any angle (other than 0 and 90) if in range

Q6.

Answer	Mark	Comments
0	B1	allow in words eg none or zero