AQA Level 2 Certificate in FURTHER MATHEMATICS Name: _____

Exam details

Paper 1 (non-calculator): 1 hr 45 min Paper 2 (calculator): 1 hr 45 min

Useful websites

- Past papers & mark schemes for the current specification:
  https://www.aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8365/assessment-resources?start_rank=1
- Past papers and solutions from the **old** specification, but many of the papers from June 2016 onwards are still very useful:
  http://mrbartonmaths.com/students/aqa-level-2-certificate-in-further-mathematics/past-papers-and-solutions.html
- Great video tutorials, worksheets & worked solutions for each topic:
  https://www.1stclassmaths.com/l2-further-maths
  https://corbettmaths.com/more/further-maths/

Learning resources

Lesson slides, homework sheets, past papers & other documents will regularly be placed on the shared student drive U: Mathematics/Further Mathematics

Grade boundaries

	9	8	7	6	5	4
2023	86%	75%	64%	53%	43%	37%
2024	85%	74%	64%	54%	43%	38%

Course content

The "NEW" content will be covered in Further Maths lessons, but **you** are responsible for ensuring you understand the "GCSE" content. Be aware that these topics generally ask you to apply your GCSE knowledge (e.g. surds) in more difficult ways than a standard GCSE paper, so will still need lots of practice.

1 NUMBER				
1.1		Knowledge and use of numbers and the number system including fractions, decimals, percentages, ratio, proportion and order of operations are expected	GCSE	
1.2	The product rule for counting	Work out how many 5-digit odd numbers can be formed using the digits 1 3 4 6 8 with no repetition of any digit	GCSE	
1.3	Manipulation of surds, including rationalising the denominator	Manipulation of surds, including rationalising the denominator The use of surds in exact calculations Write $\sqrt{200}-\sqrt{72}+3\sqrt{162}$ in the form of $a\sqrt{2}$ Rationalise and simplify $\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$ Write your answer in the form $a+b\sqrt{3}$ where *a* and *b* are integers	GCSE	
2 ALGEBRA				
2.1	The basic processes of algebra	Knowledge and use of basic skills in manipulative algebra including use of the associative, commutative and distributive laws, are expected	GCSE	
2.2	Definition of a function	Notation $f(x)$ will be used, eg. $f(x)=x^2-9$	GCSE	
2.3	Domain and range of a function	Domain may be expressed as, for example, $x>2$, or "for all x, except $x=0$" and range may be expressed as $f(x)>-1$	NEW	
2.4	Composite functions	The result of two or more functions, say f and g, acting in succession. $fg(x)$ is g followed by f	GCSE	
2.5	Inverse functions	The inverse function of f is written f^{-1} Domains will be chosen for f to make f one-one	GCSE	
2.6	Expanding brackets and collecting like terms	Expand and simplify $(y^2-2y+3)(2y-1)\ -\ 2(y^3-\ 3y^2+4y-2)$	GCSE	

2.7	Expand $(a+b)^n$ for positive integer n	Expand and simplify $(5x+2)^3$ Use Pascal's triangle to work out the coefficient of x^3 in the expansion of $(3+2x)^5$	NEW	
2.8	Factorising	Factorise fully $(2x+3)^2-(2x-5)^2$ Factorise $15x^2 \pm 34xy - 16y^2$ Factorise fully x^4-25x^2	SOME NEW	
2.9	Manipulation of rational expressions: Use of $+ - \times \div$ for algebraic fractions with denominators being numeric, linear or quadratic	Simplify $\frac{5}{x+2}-\frac{3}{2x-1}$ Simplify $\frac{x^3+2x^2+x}{x^2+x}$ Simplify $\frac{5x^2-14x-3}{4x^2-25} \div \frac{x-3}{4x^2+10x}$	GCSE	
2.10	Use and manipulation of formulae and expressions	Rearrange $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ to make v the subject	GCSE	
2.11	Use of the factor theorem for rational values of the variable for polynomials	Factorise x^3+x^2-5x+6 Show that $2x-3$ is a factor of $2x^3-x^2-7x+6$ Solve $x^3+x^2-10x+8=0$ Show that $x-7$ is a factor of x^5-7x^4-x+7	NEW	
2.12	Completing the square	Work out the values of a, b and c such that $2x^2+6x+7 \equiv a(x+b)^2+c$	GCSE	

2.13	Drawing and sketching of functions Interpretation of graphs	Graphs could be linear, quadratic, exponential and restricted to no more than 3 domains. Exponential graphs will be of the form $y = ab^x$ and $y = ab^{-x}$ where a and b are rational numbers Sketch the graph of $y = x^2 - 5x + 6$ Label clearly any points of the intersection with the axes A function $f(x)$ is defined as $f(x) = x^2 \quad 0 \leq x < 1$ $= 1 \quad 1 \leq x < 2$ $= 3 - x^2 \quad 2 \leq x < 3$ Draw the graph of $f(x)$ on the grid below for values of x from 0 to 3 Given a sketch of $y = ab^{-x}$, and two points, work out the values of a and b	NEW	
2.14	Solution of linear and quadratic equations	Solutions of quadratics to include solution by factorisation, by graph, by completing the square or by formula. Problems will be set in a variety of contexts, which result in the solution of linear or quadratic equations.	GCSE	
2.15	Algebraic and graphical solution of simultaneous equations in two unknowns, where the equations could both be linear or one linear and one second order	Solve $4x - 3y = 0$ and $6x + 15y = 13$ Solve $y = x + 2$ and $y^2 = 4x + 5$ Solve $y = x^2$ and $y - 5x = 6$ Solve $xy = 8$ and $x + y = 6$	GCSE	
2.16	Algebraic solution of linear equations in three unknowns	Solve $2x - 5y + 4z = 22$ $x + y + 2z = 4$ $x - y - 6z = -4$	NEW	
2.17	Solution of linear and quadratic inequalities	Solve $5(x - 7) > 2(x + 1)$ Solve $x^2 < 9$ Solve $2x^2 + 5x \leqslant 3$	GCSE	

2.18	Index laws, including fractional and negative indices and the solution of equations	Express as a single power of x $\sqrt{x^{\frac{1}{2}} \times x^{\frac{7}{2}}}$ Express as a single power of x $\sqrt{\frac{x^{\frac{3}{2}} \times x^{\frac{7}{2}}}{x^2}}$ Solve $x^{-\frac{1}{2}} = 3$ Solve $\sqrt{x} - \frac{10}{\sqrt{x}} = 3 \qquad x > 0$	GCSE	
2.19	Algebraic proof	Prove that $(n+5)^2 - (n+3)^2$ is divisible by 4 for any integer value of n	GCSE	
2.20	Using *n*th terms of sequences Limiting value of a sequence as $n \to \infty$	Work out the difference between the 16th and 6th terms of the sequence with *n*th term $\frac{2n}{n+4}$ Write down the limiting value of $\frac{2n}{n+4}$ as $n \to \infty$	NEW	
2.21	*n*th terms of linear sequences	A linear sequence starts 180 176 172 … By using the *n*th term, work out which term has value –1000 Work out the *n*th term of the linear sequence $r + s \qquad r + 3s \qquad r + 5s \ldots$	GCSE	
2.22	*n*th terms of quadratic sequences	Work out the *n*th term of the quadratic sequence 10 16 18 16 … Which term has the value 0?	GCSE	

3 COORDINATE GEOMETRY				
STRAIGHT LINE				
3.1	Know and use the definition of a gradient		GCSE	
3.2	Know the relationship between the gradients of parallel and perpendicular lines	Show that *A* (0, 2), *B* (4, 6) and *C* (10, 0) form a right-angled triangle	GCSE	
3.3	Use Pythagoras' theorem to calculate the distance between two points		GCSE	
3.4	Use ratio to find the coordinates of a point on a line given the coordinates of two other points.	Including midpoint	GCSE	
3.5	The equation of a straight line $y = mx + c$ and $y - y_1 = m(x - x_1)$ and other forms	Including interpretation of the gradient and y intercept from the equation	SOME NEW	
3.6	Draw a straight line from given information		GCSE	
CIRCLE				
3.7	Understand that $x^2 + y^2 = r^2$ is the equation of a circle with centre (0, 0) and radius r	Including writing down the equation of a circle given centre (0, 0) and radius The application of circle geometry facts where appropriate: ○ the angle in a semi-circle is 90°; ○ the perpendicular from the centre to a chord bisects the chord; ○ the angle between tangent and radius is 90°; ○ tangents from an external point are equal in length.	GCSE	
3.8	Understand that $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle with centre (a, b) and radius r	Including writing down the equation of any circle given centre and radius	NEW	
3.9	The equation of a tangent at a point on a circle		GCSE	

4 CALCULUS				
4.1	Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x		NEW	
4.2	Know that the gradient of a function is the gradient of the tangent at that point.		NEW	
4.3	Differentiation of kx^n where n is an integer, and the sum of such functions	Including expressions which need to be simplified first Given $y = (3x + 2)(x-3)$ work out $\frac{dy}{dx}$ Given $y = \frac{5}{x^3}$ work out $\frac{dy}{dx}$	NEW	
4.4	The equation of a tangent and normal at any point on a curve		NEW	
4.5	Increasing and decreasing functions	When the gradient is positive/negative a function is described as an increasing/decreasing function	NEW	
4.6	Understand and use the notation $\frac{d^2y}{dx^2}$	Know that $\frac{d^2y}{dx^2}$ measures the rate of change of the gradient function	NEW	
4.7	Use of differentiation to find maxima and minima points on a curve	Determine the nature either by using increasing and decreasing functions or $\frac{d^2y}{dx^2}$	NEW	
4.8	Using calculus to find maxima and minima in simple problems	$V = 49x + \frac{81}{x}$ $x > 0$ Use calculus to show that V has a minimum value and work out the minimum value of V	NEW	
4.9	Sketch/ interpret a curve with known maximum and minimum points		NEW	

5 MATRIX TRANSFORMATIONS				
5.1	Multiplication of matrices (All calculations will be restricted to 2 × 2 or 2 × 1 matrices)	Multiplying a 2 × 2 matrix by a 2 × 2 matrix or by a 2 × 1 matrix Multiplication by a scalar	NEW	
5.2	The identity matrix **I**	2 × 2 only	NEW	
5.3	Transformations of the unit square in the x – y plane	Representation by a 2 × 2 matrix Transformations restricted to rotations of 90^o, 180^o or 270^o about the origin, reflections in the lines $x = 0$, $y = 0$, $y = x$, $y = -x$ and enlargements centred on the origin	NEW	
5.4	Combination of transformations	Using matrix multiplications Use of **i** and **j** notation is not required	NEW	
6 GEOMETRY				
6.1		Knowledge of perimeter and area of rectangles and circles; and of the area of triangles, parallelograms and trapezia; and of the surface area and volume of prisms, cylinders, spheres, cones and pyramids. Knowledge of angle properties of parallel and intersecting lines, triangles, all special types of quadrilaterals and polygons. Understand and use circle theorems: ○ angle at the centre is twice the angle at the circumference; ○ angles in the same segment are equal; ○ opposite angles in cyclic quadrilateral add up to $180°$; ○ alternate segment theorem; ○ the angle in a semi-circle is $90°$; ○ the perpendicular from the centre to a chord bisects the chord; ○ the angle between tangent and radius is $90°$; ○ tangents from an external point are equal in length.	GCSE	
GEOMETRIC PROOF				
6.2	Understand and construct geometrical proofs using formal arguments	The use of theorems listed in the notes of section 6.1	GCSE	

TRIGONOMETRY IN TRIANGLES				
6.3	Sine and cosine rules in scalene triangles; area of a triangle = $\frac{1}{2}ab\sin c$	Knowledge and use of trigonometry to solve right-angled triangles are expected	GCSE	
CIRCLE				
6.4	Use of Pythagoras' theorem in 2D and 3D	Recognise Pythagorean triples; 3, 4, 5; 5, 12, 13; 8, 15, 17; 7, 24, 25 and simple multiples of these	MOSTLY GCSE	
6.5	Be able to apply trigonometry and Pythagoras' theorem to 2 and 3 dimensional problems	Including the angle between a line and a plane and the angle between two planes; including triangles that do not have right angles	GCSE	
RATIOS OF ANGLES AND THEIR GRAPHS				
6.6	Sketch and use graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$ for angles of any size		GCSE	
6.7	Be able to use the definitions $\sin\theta$, $\cos\theta$, and $\tan\theta$ for any positive angle up to $360°$ (measured in degrees only)	Angles measured anticlockwise will be taken as positive	GCSE	
6.8	Knowledge and use of 30°, 60°, 90° triangles and 45°, 45°, 90° triangles	The use of the ratios $1 : \sqrt{3} : 2$ and $1 : 1 : \sqrt{2}$	GCSE	
6.9	Know and use $\tan\theta = \frac{\sin\theta}{\cos\theta,}$ and $\sin^2\theta + \cos^2\theta = 1$	Including expressions to be simplified, proofs of identities and equations solved	NEW	
6.10	Solution of simple trigonometric equations in given intervals	Equations will be restricted to single angles: $\sin x = 0.5$ $\sqrt{2}\sin x = \cos x$ for $0° \le x \le 360°$ $\sin^2 x = \frac{1}{4}$ for $0° \le x \le 360°$	NEW	