**AQA Level 2 Certificate in FURTHER MATHEMATICS Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Exam details**

Paper 1 (non-calculator): 12th Jun 2024 - 1 hr 45 min Paper 2 (calculator): 19th Jun 2024 - 1 hr 45 min

**Useful websites**

* Past papers & mark schemes for the current specification:
<https://www.aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8365/assessment-resources?start_rank=1>
* Past papers and solutions from the **old** specification, but many of the papers from June 2016 onwards are still very useful:
<http://mrbartonmaths.com/students/aqa-level-2-certificate-in-further-mathematics/past-papers-and-solutions.html>
* Great video tutorials, worksheets & worked solutions for each topic:
<https://www.1stclassmaths.com/l2-further-maths>
<https://corbettmaths.com/more/further-maths/>

**Learning resources**

Lesson slides, homework sheets, past papers & other documents will regularly be placed on the shared student drive U: Mathematics/Further Mathematics

 **Grade boundaries**

Be aware that this is a very new course, so grade boundaries may be different from previous years (particularly during Covid years).

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|  | 9 | 8 | 7 | 6 | 5 | 4 |
| 2021 | 73% | 58% | 44% | 33% | 23% | 19% |
| 2022 | 81% | 69% | 58% | 46% | 34% | 28% |
| 2023 | 86% | 75% | 64% | 53% | 43% | 37% |

**Course content**

The “NEW” content will be covered in Further Maths lessons, but **you** are responsible for ensuring you understand the “GCSE” content. Be aware that these topics generally ask you to apply your GCSE knowledge (e.g. surds) in more difficult ways than a standard GCSE paper, so will still need lots of practice.

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| 1 NUMBER |  |
| 1.1 |  | Knowledge and use of numbers and the number system including fractions, decimals, percentages, ratio, proportion and order of operations are expected | GCSE |  |
| 1.2 | The product rule for counting | Work out how many 5-digit odd numbers can be formed using the digits 1 3 4 6 8 with no repetition of any digit | GCSE |  |
| 1.3 | Manipulation of surds, includingrationalising the denominator | Manipulation of surds, includingrationalising the denominatorThe use of surds in exact calculationsWrite $\sqrt{200}-\sqrt{72}+3\sqrt{162}$ in the form of $a\sqrt{2}$Rationalise and simplify $\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$Write your answer in the form $a+b\sqrt{3}$ where *a* and *b* are integers | GCSE |  |
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| 2 ALGEBRA |  |
| 2.1 | The basic processes of algebra | Knowledge and use of basic skills in manipulativealgebra including use of the associative, commutativeand distributive laws, are expected | GCSE |  |
| 2.2 | Definition of a function | Notation $f(x)$ will be used, eg. $f\left(x\right)=x^{2}-9$ | GCSE |  |
| 2.3 | Domain and range of a function | Domain may be expressed as, for example, $x>2$,or “for all $x$, except $x=0$” and range may be expressed as $f\left(x\right)>-1$ | NEW |  |
| 2.4 | Composite functions | The result of two or more functions, say $f$ and $g$,acting in succession. $fg\left(x\right)$is g followed by $f$ | GCSE |  |
| 2.5 | Inverse functions | The inverse function of $f$ is written $f^{-1}$Domains will be chosen for $f$ to make $f$ one-one | GCSE |  |
| 2.6 | Expanding brackets and collecting liketerms | Expand and simplify$$\left(y^{2}–2y+3\right)\left(2y–1\right) – 2(y^{3}– 3y^{2}+4y-2)$$ | GCSE |  |
| 2.7 | Expand $(a+b)^{n}$for positive integer $n$ | Expand and simplify $(5x+2)^{3}$ Use Pascal’s triangle to work out the coefficient of $x^{3}$ in the expansion of $(3+2x)^{5}$ | NEW |  |
| 2.8 | Factorising | Factorise fully $(2x+3)^{2}-(2x-5)^{2}$Factorise $15x^{2}\pm 34xy-16y^{2}$Factorise fully $x^{4}-25x^{2}$ | SOME NEW |  |
| 2.9 | Manipulation of rational expressions:Use of + – × ÷ for algebraic fractions withdenominators being numeric, linear or quadratic | Simplify $\frac{5}{x+2}-\frac{3}{2x-1}$Simplify $\frac{x^{3}+2x^{2}+x}{x^{2}+x}$Simplify $\frac{5x^{2}-14x-3}{4x^{2}-25}÷\frac{x-3}{4x^{2}+10x}$ | GCSE |  |
| 2.10 | Use and manipulation of formulae andexpressions | Rearrange $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ to make $v$the subject | GCSE |  |
| 2.11 | Use of the factor theorem for rationalvalues of the variable for polynomials | Factorise $x^{3}+x^{2}-5x+6$Show that 2*x* – 3 is a factor of 2*x*3 – *x*2 – 7*x* + 6Solve $x^{3}+x^{2}-10x+8=0$ Show that $x–7$ is a factor of $x^{5}-7x^{4}-x+7$  | NEW |  |
| 2.12 | Completing the square | Work out the values of *a, b* and *c* such that$$2x^{2}+6x+7≡a(x+b)^{2}+c$$ | GCSE |  |
| 2.13 | Drawing and sketching of functionsInterpretation of graphs | Graphs could be linear, quadratic, exponential and restricted to no more than 3 domains.Exponential graphs will be of the form $y=ab^{x}$ and $y=ab^{-x}$where *a* and *b* are rational numbersSketch the graph of $y=x^{2}-5x+6$Label clearly any points of the intersection with the axesA function $f(x)$ is defined as $f\left(x\right)=x^{2}$ $0\leq x<1$ $=1$ $1\leq x<2$ $=3-x^{2}$ 2$\leq x<3$Draw the graph of $f(x)$on the grid below for values of *x* from 0 to 3Given a sketch of $y=ab^{-x}$*,* and two points, work out the values of *a* and *b* | NEW |  |
| 2.14 | Solution of linear and quadraticequations | Solutions of quadratics to include solution by factorisation, by graph, by completing the square or by formula.Problems will be set in a variety of contexts, which result in the solution of linear or quadratic equations. | GCSE |  |
| 2.15 | Algebraic and graphical solution ofsimultaneous equations in two unknowns,where the equations could both be linearor one linear and one second order | Solve 4*x* – 3*y* = 0 and 6*x* + 15*y* = 13Solve $y=x+2$ and $y^{2}=4x+5$Solve $y=x^{2}$ and $y–5x=6$Solve $xy=8$ and $x+y=6$ | GCSE |  |
| 2.16 | Algebraic solution of linear equations inthree unknowns | Solve 2*x* – 5*y* + 4*z* = 22 *x* + *y* + 2*z* = 4 *x* – *y* – 6*z* = –4 | NEW |  |
| 2.17 | Solution of linear and quadraticinequalities | Solve 5(*x* – 7) > 2(*x* + 1)Solve $x^{2}<9$Solve $2x^{2}+5x⩽3$ | GCSE |  |
| 2.18 | Index laws, including fractional andnegative indices and the solution ofequations | Express as a single power of $x$ $\sqrt{x^{\frac{1}{2}}×x^{\frac{7}{2}}}$Express as a single power of $x$$\sqrt{\frac{x^{\frac{3}{2}}×x^{\frac{7}{2}}}{x^{2}}}$Solve $x^{-\frac{1}{2}}=3$Solve $\sqrt{x}-\frac{10}{\sqrt{x}}=3$ $x>0$ | GCSE |  |
| 2.19 | Algebraic proof | Prove that $(n+5)^{2}-\left(n+3\right)^{2} $ is divisible by 4 for any integer value of *n* | GCSE |  |
| 2.20 | Using *n*th terms of sequencesLimiting value of a sequence as $n\rightarrow \infty $  | Work out the difference between the 16th and 6thterms of the sequence with *n*th term $\frac{2n}{n+4}$Write down the limiting value of $\frac{2n}{n+4}$ as $n\rightarrow \infty $ | NEW |  |
| 2.21 | *n*th terms of linear sequences | A linear sequence starts 180 176 172 …By using the *n*th term, work out which term hasvalue –1000Work out the *n*th term of the linear sequence *r* + *s r* + 3*s r* + 5*s* … | GCSE |  |
| 2.22 | *n*th terms of quadratic sequences | Work out the *n*th term of the quadratic sequence10 16 18 16 …Which term has the value $0$? | GCSE |  |
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| 3 COORDINATE GEOMETRY |  |
| STRAIGHT LINE |  |
| 3.1 | Know and use the definition of a gradient |  | GCSE |  |
| 3.2 | Know the relationship between the gradientsof parallel and perpendicular lines | Show that *A* (0, 2), *B* (4, 6) and *C* (10, 0) form a right-angled triangle | GCSE |  |
| 3.3 | Use Pythagoras’ theorem to calculate thedistance between two points |  | GCSE |  |
| 3.4 | Use ratio to find the coordinates of a point on a line given the coordinates of two otherpoints. | Including midpoint | GCSE |  |
| 3.5 | The equation of a straight line$y=mx+c$and $y–y1=m(x–x1)$and other forms | Including interpretation of the gradient and *y* interceptfrom the equation | SOME NEW |  |
| 3.6 | Draw a straight line from given information |  | GCSE |  |
| CIRCLE |  |
| 3.7 | Understand that $x^{2}+y^{2}=r^{2}$ is the equation of a circle with centre (0, 0) and radius *r* | Including writing down the equation of a circle given centre (0, 0) and radiusThe application of circle geometry facts where appropriate:* the angle in a semi-circle is 90°;
* the perpendicular from the centre to a chord bisects the chord;
* the angle between tangent and radius is 90°;
* tangents from an external point are equal in length.
 | GCSE |  |
| 3.8 | Understand that $(x-a)^{2}+(y-b)^{2}=r^{2}$ is the equation of a circle with centre (*a, b*) and radius *r* | Including writing down the equation of any circle given centre and radius | NEW |  |
| 3.9 | The equation of a tangent at a point on acircle |  | GCSE |  |
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| 4 CALCULUS |  |
| 4.1 | Know that the gradient function $\frac{dy}{dx}$ gives thegradient of the curve and measures the rate of change of *y* with respect to *x* |  | NEW |  |
| 4.2 | Know that the gradient of a function is thegradient of the tangent at that point. |  | NEW |  |
| 4.3 | Differentiation of $kx^{n}$ where *n* is an integer, and the sum of such functions | Including expressions which need to be simplifiedfirstGiven $y=(3x+2)(x–3)$ work out $\frac{dy}{dx}$Given $y=\frac{5}{x^{3}}$ work out $\frac{dy}{dx}$ | NEW |  |
| 4.4 | The equation of a tangent and normal at anypoint on a curve |  | NEW |  |
| 4.5 | Increasing and decreasing functions | When the gradient is positive/negative a function is described as an increasing/decreasing function | NEW |  |
| 4.6 | Understand and use the notation $\frac{d^{2}y}{dx^{2}}$ | Know that $\frac{d^{2}y}{dx^{2}}$ measures the rate of change of the gradient function | NEW |  |
| 4.7 | Use of differentiation to find maxima and minima points on a curve | Determine the nature either by using increasingand decreasing functions or $\frac{d^{2}y}{dx^{2}}$ | NEW |  |
| 4.8 | Using calculus to find maxima and minima insimple problems | $$V=49x+\frac{81}{x} x>0$$Use calculus to show that *V* has a minimum value and work out the minimum value of *V* | NEW |  |
| 4.9 | Sketch/ interpret a curve with knownmaximum and minimum points |  | NEW |  |
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| 5 MATRIX TRANSFORMATIONS |  |
| 5.1 | Multiplication of matrices(All calculations will be restricted to 2 × 2 or 2 × 1 matrices) | Multiplying a 2 × 2 matrix by a 2 × 2 matrix or by a2 × 1 matrixMultiplication by a scalar | NEW |  |
| 5.2 | The identity matrix **I** | 2 × 2 only | NEW |  |
| 5.3 | Transformations of the unit square in the *x* –*y* plane | Representation by a 2 × 2 matrixTransformations restricted to rotations of 90o,180o or 270o about the origin, reflections in thelines *x* = 0, *y* = 0, *y* = *x, y* = –*x* andenlargements centred on the origin | NEW |  |
| 5.4 | Combination of transformations | Using matrix multiplicationsUse of **i** and **j** notation is not required | NEW |  |
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| 6 GEOMETRY |  |
| 6.1 |  | Knowledge of perimeter and area of rectangles and circles; and of the area of triangles, parallelograms and trapezia; and of the surface area and volume of prisms, cylinders, spheres, cones and pyramids.Knowledge of angle properties of parallel and intersecting lines, triangles, all special types of quadrilaterals and polygons.Understand and use circle theorems:* angle at the centre is twice the angle at the circumference;
* angles in the same segment are equal;
* opposite angles in cyclic quadrilateral add up to $180°$;
* alternate segment theorem;
* the angle in a semi-circle is 9$0°$;
* the perpendicular from the centre to a chord bisects the chord;
* the angle between tangent and radius is 9$0°$;
* tangents from an external point are equal in length.
 | GCSE |  |
| GEOMETRIC PROOF |  |
| 6.2 | Understand and construct geometrical proofsusing formal arguments | The use of theorems listed in the notes of section 6.1 | GCSE |  |
| TRIGONOMETRY IN TRIANGLES |  |
| 6.3 | Sine and cosine rules in scalene triangles;area of a triangle = $\frac{1}{2}ab\sin(c)$ | Knowledge and use of trigonometry to solveright-angled triangles are expected | GCSE |  |
| CIRCLE |  |
| 6.4 | Use of Pythagoras’ theorem in 2D and 3D | Recognise Pythagorean triples; 3, 4, 5; 5, 12, 13; 8, 15, 17; 7, 24, 25 and simple multiples of these | MOSTLYGCSE |  |
| 6.5 | Be able to apply trigonometry and Pythagoras’ theorem to 2 and 3 dimensionalproblems | Including the angle between a line and a plane and the angle between two planes; including triangles that do not have right angles | GCSE |  |
| RATIOS OF ANGLES AND THEIR GRAPHS |  |
| 6.6 | Sketch and use graphs of $y=\sin(x)$ , $y=\cos(x)$, and $y=\tan(x)$ for angles of any size |  | GCSE |  |
| 6.7 | Be able to use the definitions $\sin(θ)$, $\cos(θ)$, and $\tan(θ)$ for any positive angle up to $360°$ (measured in degrees only) | Angles measured anticlockwise will be taken as positive | GCSE |  |
| 6.8 | Knowledge and use of 30°, 60°, 90° trianglesand 45°, 45°, 90° triangles | The use of the ratios $1 : \sqrt{3 } : 2$ and $1 : 1 : \sqrt{2 }$  | GCSE |  |
| 6.9 | Know and use $\tan(θ)=\frac{\sin(θ)}{\cos(θ), }$and $sin^{2}θ+cos^{2}θ=1$ | Including expressions to be simplified, proofs of identities and equations solved | NEW |  |
| 6.10 | Solution of simple trigonometric equations ingiven intervals | Equations will be restricted to single angles:$$\sin(x)=0.5$$$\sqrt{2}\sin(x=\cos(x))$ for $0°\leq x\leq 360°$$sin^{2}x=\frac{1}{4} $ for $0°\leq x\leq 360°$ | NEW |  |
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