3 COORDINATE GEOMETRY – Further Maths

Jump to:

Section 3.1 – 3.6

Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		
$-4 = \frac{3}{2} \times -6 + c$ or $c = 5$ $y - -4 = \frac{3}{2}(x - -6)$	M1	oe
(0, 5)	A1	

Alternative method 2		
Correctly adding at least 1 multiple of 2 to the right and 3 up eg –6 + 2 = –4 and –4 + 3 = –1	M1	oe needs to be added to both vertical and horizontal. Could be seen in coordinates eg (–4, –1) could be 1 right and 1.5 up or y coordinate of –4 + 1.5 × 6
(0, 5)	A1	

Alternative method 3		
Sketch drawn with straight line passing through (–6, –4) and (0, 5) with steps shown.	M1	just a line passing through 5 seen on the axis is enough for M1 but won't gain A1 unless written as coordinates
(0, 5)	A1	answer could be embedded in diagram
Additional Guidance		

(0, 5) seen without working will be 2 marks	M1A1

Q2.

Answer	Mark	Comments
Alternative method 1		
(*x*-coordinate of *A* =) 10 **and** (*y*-coordinate of *B* =) 8	B1	May be implied on diagram eg 10 written next to *A* **and** 8 written next to *B*
(*x*-coordinate of *P* =) $\frac{2}{2+3}$ × their 10 or $\frac{2 \times \text{their } 10 + 3 \times 0}{2+3}$ or 4	M1	oe their 10 must be their *x*-coordinate of *A* May be seen on diagram
(area of triangle *OBP* =) $\frac{1}{2}$ × their 8 × their 4	M1dep	oe their 8 must be their *y*-coordinate of *B*
16	A1ft	ft B0M2

Alternative method 2		
(*x*-coordinate of *A* =) 10 **and** (*y*-coordinate of *B* =) 8	B1	May be implied on diagram eg 10 written next to *A* **and** 8 written next to *B*
(area of triangle *OAB* =) $\frac{1}{2}$ × their 10 × their 8 or 40	M1	oe
(area of triangle *OBP* =) $\frac{2}{2+3}$ × their 40	M1dep	oe eg their 40 – $\frac{2}{2+3}$ × their 40
16	A1ft	ft B0M2

Alternative method 3		
(*x*-coordinate of *A* =) 10 **and** (*y*-coordinate of *B* =) 8	B1	May be implied on diagram eg 10 written next to *A* **and** 8 written next to *B*
(area of triangle *OAB* =)	M1	oe

$\frac{1}{2}$ × their 10 × their 8 or 40		
(*y*-coordinate of *P* =) $\frac{3}{2+3}$ × their 8 or 4.8 **and** (area of triangle *OPA* =) $\frac{1}{2}$ × their 10 × their 4.8 or 24 **and** (area of triangle *OBP* =) their 40 – their 24	M1dep	oe their 8 must be their *y*-coordinate of *B* *y*-coordinate of *P* may be seen on diagram
16	A1ft	ft B0M2

Alternative method 4		
(*x*-coordinate of *A* =) 10 **and** (*y*-coordinate of *B* =) 8	B1	May be implied on diagram eg 10 written next to *A* **and** 8 written next to *B*
(*AB* =) $\sqrt{\text{their } 10^2 + \text{their } 8^2}$ or $\sqrt{100+64}$ or $\sqrt{164}$ or $2\sqrt{41}$ or 12.8(...) **and** (*BP* =) $\frac{2}{2+3}$ × their 12.8(...) or 5.12(...) **and** (angle *OBP* =) $\tan^{-1}\frac{\text{their } 10}{\text{their } 8}$ or 51.3(...)	M1	oe their 10 must be their *x*-coordinate of *A* their 8 must be their *y*-coordinate of *B*
(area of triangle *OBP* =) $\frac{1}{2}$ × their 8 × their 5.12 × sin their 51.3	M1dep	oe their 8 must be their *y*-coordinate of *B*
16	A1ft	ft B0M2

Additional Guidance	
A = 10 and *B* = 8	B1
A (8, 0) and *B* (0, 10) is B0 but can subsequently score up to M2A1ft (answer 16)	
A (0, 10) and *B* (8, 0) is B0 but can score up to M2A1ft if uses *x*-coordinate of *A* as 10 and *y*-coordinate of *B* as 8 (answer 16)	
A (0, 8) and *B* (10, 0) is B0 but can score up to M2A1ft if uses *x*-coordinate of *A* as 8 and *y*-coordinate of *B* as 10 (answer 16)	
Area triangle *OBP* may be seen as the sum of two right-angled triangles	
Area triangle *OBP* may be seen as area trapezium *OBPX* – area triangle *OPX* *X* is on the *x*-axis with *PX* perpendicular to the *x*-axis	
Allow marks for valid working seen even if not subsequently used	
15.9(…) → answer 16 Answer 15.9(…)	4 marks B1M2A0

Q3.

Answer	Mark	Comments
Alternative method 1		
A (6, 0) or $x = 6$ (for *A*)	B1	May be on diagram or be implied
$\frac{1}{2} \times$ their $6 \times y = 24$	M1	
$y = 8$	A1ft	Only ft B0 M1
their $8 = 12 - 2x$	M1	
$y = 2$	A1ft	ft their y SC2 Answer (8, 2) with no valid working SC1 *B* (0, 12) or $y = 12$ (for *B*)

Alternative method 2		
A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 × 12 or 36 and $\frac{1}{2}$ × 12 × x = their 36 − 24	M1	
x = 2	A1ft	Only ft B0 M1
y = 12 − 2 × their 2	M1	
y = 8	A1ft	ft their y SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or y = 12 (for B)
Alternative method 3		
A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
$\frac{1}{2}$ × their 6 × y = 24	M1	
y = 8	A1ft	Only ft B0 M1
B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 × 12 or 36 and $\frac{1}{2}$ × 12 × x = their 36 − 24	M1	
x = 2	A1ft	Only ft B0 with 2nd M1 gained SC2 Answer (8, 2) with no valid working

		SC1 B (0, 12) or y = 12 (for B)
Alternative method 4		
A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 × 12 or 36 and $\frac{1}{2}$ × 12 × x = their 36 − 24	M1	
x = 2	A1ft	Only ft B0 M1
$\frac{1}{2}$ × their 6 × y = 24	M1	
y = 8	A1ft	Only ft B0 with 2nd M1 gained SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or y = 12 (for B)

Alternative method 5		
A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
B (0, 12) or y = 12 (for B) and (area OAB =) × their 6 × 12 or 36 and $\frac{24}{\text{their } 36} \times 12$	M1	
y = 8	A1ft	Only ft B0 M1
B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 ×	M1	

12 or 36 and $\frac{\text{their } 36 - 24}{\text{their } 36} \times$ their 6		
$x = 2$	A1ft	Only ft B0 with 2nd M1 gained SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or y = 12 (for B)

Q4.

Answer	Mark	Comments
Any pair of integer values for a and b for which $b = 12a + 26$	B2	B1 Correct equation in any form eg $\frac{b - -10}{a - -3} = 12$ or $b + 10 = 12(a + 3)$ or $\frac{y - -10}{x - -3} = 12$ or $y + 10 = 12(x + 3)$ or $b = 12a + c$ and $c = 26$ or $y = 12x + c$ and $c = 26$ or −3 + k and −10 + 12k where k is a non-zero integer

Additional Guidance	
Examples of B2 responses $a = -4$ and $b = -22$ or $a = -2$ and $b = 2$ or $a = -1$ and $b = 14$ or $a = 0$ and $b = 26$ or $a = 1$ and $b = 38$ or $a = 2$ and $b = 50$	B2

or a = 3 and b = 62 or a = 4 and b = 74	
a = −3 and b = −10 is point *P* so will not score B2 (B1 possible)	
−3 + 1 and −10 + 12	B1
−3 + 2 and −10 + 24	B1

Q5.

	Answer	Mark	Comments
(a)	$(\frac{c}{a}, 0)$	B1	
(b)	$-\frac{a}{b}$	B1	

Q6.

Answer	Mark	Comments
Alternative method 1		
Intention to work out gradient or reciprocal of gradient **or** Intention to work out the equation of the straight line	M1	Condone one sign error in the calculation, eg $\frac{-5-7}{6--4}$ **or** $\frac{-5-t}{6-8}$ **or** $\frac{7-t}{-4-8}$ **or** −1.2 oe eg 7 = −4m + c **or** −5 = 6m + c eg y − 7 = m(x − −4)
A correct value for m or a correct expression for m **and** an expression to calculate the value of t or the value of c **or** m = −1.2 **and** c = 2.2	M1dep	eg. (m =) $\frac{7--5}{-4-6}$ **or** (m =) $\frac{-6}{5}$ oe **and** eg $\frac{t--5}{8-6}=\frac{-5-7}{6--4}$ **or** t = $\frac{-6}{5}$ (8) + (7 − $\frac{24}{5}$) 7 = $\frac{-6}{5}$ (-4) + c **or** -5 = $\frac{-6}{5}$ (6)

		+ c
(t =) −7.4 **or** $-7\frac{2}{5}$ **or** $\frac{-37}{5}$	A1	

Alternative method 2		
−4 to 6 is +10 **and** 7 to −5 is −12	M1	oe Condone a sign error
6 to 8 is +2 **and** −5 to t is $\frac{-12}{5}$	M1	oe
(t =) −7.4 **or** $-7\frac{2}{5}$ **or** $\frac{-37}{5}$	A1	

Alternative method 3		
$\sqrt{[(-4-6)^2 + (7 - -5)^2]}$ (=√244) **and** stating −4 to 6 is 10 **and** 6 to 8 is 2	M1	Correct use of Pythagoras and identifying the correct displacements
$\sqrt{[(6-8)^2 + (-5-t)^2]} = (\sqrt{244}) \div 5$	M1	ft their 244
(t =) −7.4 or $-7\frac{2}{5}$ or $\frac{-37}{5}$	A1	

Additional Guidance
−7.4 seen on answer line is 3 marks −7.4 seen in the working but sign error on answer line is 3 marks 'Algebraic method' means the question must not be done graphically ... although a diagram is fine when used to do the gradient calculations $\frac{t--5}{8-6} = \frac{t-7}{8--4}$ seen implies M1 M1 Look at any diagram they may have drawn for evidence of the alt 2 method $\frac{7--5}{-4-6}$ (correct expression) = 1.2 (error) followed by 7 = (1.2)(−4) + c scores M1 M1 but will not lead to a correct final answer, so A0 m = − 1.2, but they use 1.2 instead ... 7 = 1.2(−4) + c giving c = 11.8 is M1 M1 A0 m = − 1.2, then t = −1.2 + 11.8 = 2.2 scores M1 M1 A0 because this is a

correct method for calculating c, and so scores the 2nd M1, even though they think they are calculating t

$m = \frac{-5-7}{6--4} = \frac{-12}{10} - \frac{-12}{10} \times 2 = \frac{-24}{20} = \frac{-6}{5}$ = −1.2 so t = −5 − 1.2 = − 6.2
M1 M1 A0

... because the only error is $\frac{-12}{10} \times 2 = \frac{-24}{20}$ ... if this had been −2.4 then t = −7.4

Q7.

Answer	Mark	Comments
$\left(\frac{4+6}{2}, \frac{1+9}{2}\right)$ or (5, 6)	M1	oe eg $\left(4+\frac{6-4}{2}, 1+\frac{11-1}{2}\right)$ may be on diagram
$\frac{1--3}{4-10}$ or $\frac{4}{-6}$ or $\frac{0-\text{their }6}{14-\text{their }5}$ or $\frac{-6}{9}$	M1	oe method for at least one gradient or at least one unsimplified gradient seen eg $\frac{-3-1}{10-4}$ or $\frac{-4}{6}$ or $\frac{\text{their }6-0}{\text{their }5-14}$ or $\frac{6}{-9}$ $\frac{6-0}{5-14}$ or $\frac{6}{-9}$ is M1M1
$\frac{1--3}{4-10}$ or $\frac{4}{-6}$ or $\frac{0-6}{14-5}$ or $\frac{-6}{9}$ and shows that the gradients are equal	A1	oe method for both gradients or two unsimplified gradients seen and gradients shown to be equal eg $\frac{4}{-6}$ and $\frac{-6}{9}$ and these are both $-\frac{2}{3}$ SC2 (5, 6) and at least one gradient given as $-\frac{2}{3}$ SC1 at least one gradient given as $-\frac{2}{3}$

Additional Guidance	
Mark intention for 1st M1 eg condone 5, 6	M1

$\frac{4}{-6} = -\frac{2}{3}$ and $\frac{-6}{9} = -\frac{2}{3}$	M2, A1
$\frac{1--3}{4-10} = -\frac{2}{3}$ and $\frac{0-6}{14-5} = -\frac{2}{3}$	M2, A1
$\frac{4}{-6} = \frac{-6}{9}$	M2, A1
$\frac{4}{-6}$ and $\frac{-6}{9}$ and parallel	M2, A0
$\frac{4}{6}$ is 2nd M0 unless recovered to $\frac{4}{-6}$	
$\frac{4}{6}$ recovered to $\frac{4}{-6}$ and $\frac{6}{9}$ recovered to $\frac{-6}{9}$ could go on to score full marks	
both gradients = $-\frac{2}{3}$ with no method or unsimplified gradients seen cannot score the A mark	
$\frac{4}{-6}x$ or $\frac{-6}{9}x$ do not score 2nd M1 unless recovered	
Equation of a line does not score 2nd M1 unless a method or unsimplified gradient seen	
Using the reciprocals of gradients can score a maximum of M1 M0 A0	
Allow −0.66… or −0.67 for $-\frac{2}{3}$ and $\frac{4}{-6}$ etc Ignore conversion attempt after a correct fraction is seen	
or method for $\frac{4}{-6}$ $1 = 4m + c$ and $-3 = 10m + c$ $4 = -6m$ $\frac{4}{-6} = m$ (similar method possible for $\frac{-6}{9}$)	(2nd) M1

Q8.

Answer	Mark	Comments
Alternative method 1		
$y + 4x = c$ or $y = -4x + c$	M1	oe

or gradient = –4		c can be any value other than 6 may be implied
$1 + 4 \times 2 = c$ or 1 = (their –4) × 2 + c or $c = 9$	M1	oe their –4 can only be 4 or $\frac{1}{4}$ implied by a correct equation of B eg $y - 1 = -4(x - 2)$ or $y + 4x = 9$ or $y = -4x + 9$
$2d + 4d$ = their 9 or $2d$ = (their –4)d + their 9 or $6d = 9$ or $9 \div 6$	M1dep	oe substitution of (d, $2d$) into their equation of B equation with no algebraic denominator dep on 2nd M1
$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	oe eg $\frac{9}{6}$

Alternative method 2		
$y + 4x = c$ or $y = -4x + c$ or gradient = –4	M1	oe c can be any value other than 6 may be implied
$\frac{2d-1}{d-2}$ = their –4	M1	oe their –4 can only be 4 or $\frac{1}{4}$ may be implied
$2d - 1$ = their –4($d - 2$) or $6d = 9$ or $9 \div 6$	M1dep	oe equation with no algebraic denominator dep on 2nd M1
$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	oe eg $\frac{9}{6}$

Additional Guidance	
Ignore simplification or conversion if correct answer seen	
Condone answer (1.5, 3) oe	

gradient = –4x must be recovered	
3rd M1 Allow (d, 2d) to be (x, 2x) etc	
3rd M1 Do not allow use of (2d, d) to be a misread	
A correct equation in d with no algebraic denominator implies M1M1M1 eg 2d – 1 = –4(d – 2) or 2d = –4d + 9 or 6d = 9	M1M1M1
Alt 1 gradient = 4 $y = 4x - 7$ $2d = 4d - 7$ $d = 3.5$	M0 M1 M1A0
Alt 1 gradient = $\frac{1}{4}$ $y = \frac{1}{4}x + \frac{1}{2}$ $2d = \frac{1}{4}d + \frac{1}{2}$ $d = \frac{2}{7}$	M0 M1 M1A0
gradient –4 followed by correct method using gradient 4 or $\frac{1}{4}$ for 2nd and 3rd marks can score a maximum of M2 eg Alt 1 gradient –4 $1 = 4 \times 2 + c$ $2d = 4d - 7$	M0M1M1
gradient –4 followed by correct method using gradient 4 or $\frac{1}{4}$ for 2nd mark (but not the 3rd mark) can score a maximum of M1 eg Alt 1 gradient –4 $y = \frac{1}{4}x + \frac{1}{2}$ (no further valid work)	M0M1M0

Q9.

Answer	Mark	Comments
(gradient =) 0.5 or $\frac{1}{2}$	M1	
0 = their 0.5 × 4 + c or c = −2 or y − 0 = their 0.5(x − 4)	M1	oe
$y = 0.5x - 2$ or $y = 0.5(x - 4)$	A1	oe simplified equation

Q10.

Answer	Mark	Comments
$p = 2.5$ or $\frac{5}{2}$ or $2\frac{1}{2}$	B1	
$r = -5$	B1	

Q11.

Answer	Mark	Comments
Alternative method 1		
$5 + \frac{2}{5} \times (5 - 3)$	M1	oe
$5.5 - \frac{2}{5} \times (7 - 5.5)$ or 4.9	M1	oe
5.8 or 4.9	A1	oe
(5.8, 4.9)	A1	oe

Alternative method 2		
$\frac{x-3}{x-5} = \frac{5+2}{2}$	M1	oe
$\frac{7-y}{5.5-y} = \frac{5+2}{2}$	M1	oe
5.8 or 4.9	A1	oe
(5.8, 4.9)	A1	oe

Alternative method 3		
$\frac{2 \times 3 + 5 \times x}{2+5} = 5$	M1	oe
$\frac{2 \times 7 + 5 \times y}{2+5} = 5.5$	M1	oe
5.8 or 4.9	A1	oe
(5.8, 4.9)	A1	oe

Q12.

Answer	Mark	Comments
(gradient =) $-\frac{3}{2}$	M1	
–1 ÷ their $-\frac{3}{2}$ or $\frac{2}{3}$	M1	
–1 = their $\frac{2}{3}$ × 3 + c or c = –3	M1dep	oe dep on 2nd M1
5 = their $\frac{2}{3}x$ + their –3	M1dep	dep on 2nd and 3rd M1
12	A1	

Q13.

Answer	Mark	Comments
10	B1	y-coordinate of c may be seen on the graph
$(-)\frac{\text{their } 10}{5}$ or $(-)2$	M1	± their gradient of L
$(y =) -\frac{\text{their } 10}{5}x + \text{their } 10$	M1dep	oe eg $y - 0 = -\frac{\text{their } 10}{5}(x-5)$ or $y - \text{their } 10 = -\frac{\text{their } 10}{5}(x-0)$ must use a negative gradient
$-\frac{\text{their } 10}{5}$ + their 10 = 3x + 2 or 5x = 8	M1dep	oe
1.6	A1ft	oe eg $\frac{8}{5}$ ft B0M3

Additional Guidance	
A1ft values must be exact or rounded to 1 decimal place or	

better	
Ignore any y-coordinate of b calculated after working out the x-coordinate	
Assuming the lines are perpendicular can score a maximum of B1	
y-coordinate of c = 8 gradient L = $-\frac{8}{5}$ $y = -\frac{8}{5}x + 8$ $-\frac{8}{5}x + 8 = 3x + 2$ 1.3 (Note that the exact value is $\frac{30}{23}$	B0 M1 M1 M1 A1ft

<u>Section 3.7 – 3.8</u>

Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		
$(x - 1)^2 + (y - 9)^2 = 25$	B3	B2 $(x - 1)^2 + (y - 9)^2 = 5^2$ or (1, 9) **and** radius = 5 or (1, 9) **and** radius^2 = 5^2 or (1, 9) **and** radius^2 = 25 B1 $(x - 1)^2 + (y - 9)^2 = k$ or $(x - \ldots)^2 + (y - \ldots)^2 = 25$ or $(x - \ldots)^2 + (y - \ldots)^2 = 5^2$ or (1, 9) or $\frac{-2+4}{2}$ oe **and** $\frac{5+13}{2}$ oe or radius = 5 or radius^2 = 5^2

		or radius^2 = 25

Alternative method 2 Uses perpendicular lines where (x, y) is a point on the circle		
$\frac{y-5}{x--2} \times \frac{y-13}{x-4} = -1$	M1	oe eg $(y-5)(y-13) = -1(x+2)(x-4)$
$y^2 - 18y + 65 + x^2 - 2x - 8 = 0$	M1dep	oe equation of circle with brackets expanded and fractions eliminated eg $y^2 - 18y + 65 = -x^2 + 2x + 8$
$(x-1)^2 + (y-9)^2 = 25$	A1	

Additional Guidance	
a = 1 b = 9 c = 25 implies $(x-1)^2 + (y-9)^2 = 25$	
Alt 1 (1, 9) may be implied eg x = 1 y = 9 or 1, 9	B1
Alt 1 $(x+3)^2 + (y+4)^2 = 5^2$	B1
Alt 1 $(x-1)^2 + (y-9)^2 = 5$ (with no indication that radius = 5)	B1
Alt 1 r = 5	B1
Alt 1 diameter = 10	B0
$(x-1)^2 + (y-9)^2 = 25$ in working lines with brackets expanded on answer line	B2

Q2.

Answer	Mark	Comments
$\pi \times 100$ (÷ 4) or 100π (÷ 4) or 25π or 25pi or $\pi \times 36$ (÷ 4) or 36π (÷ 4) or 9π	M1	oe
$\pi \times 100 \div 4 - \pi \times 36 \div 4$	M1dep	oe eg $\frac{100\pi - 36\pi}{4}$ or $\frac{64\pi}{4}$
16π	A1	SC2 2176π

Additional Guidance	
Use of circumference instead of area throughout	M0M0A0
Allow substitution of π = [3.14, 3.142] for M marks	
16π in working with eg 50.3 on answer line	M2A0
SC2 is for using radii of 100 and 36	
Omission of π in working must be recovered	

Q3.

Answer	Mark	Comments
$r = 5$ **or** $r^2 = 25$ **or** $r = \sqrt{25}$ **or** $d = 10$	B1	May be seen on diagram
$(2 \times \text{their } r)^2 - \pi \times \text{their } r^2$	M1	
[21.45, 21.5] **or** $100 - 25\pi$	A1ft	ft from B0 M1 Allow 21 with working (uses $25\pi = 79$) Ignore any units seen

Q4.

Answer	Mark	Comments
$(x - 1)^2$ (– 1) **or** $(y - 3)^2$ (– 3^2)	M1	
$(x - 1)^2$ (– 1) **and** $(y - 3)^2$ (– 3^2)	M1	
$(x - 1)^2 + (y - 3)^2 = 10$	A1	
Centre = (1, 3)	A1ft	ft from their equation if at least M1 earned
Radius = $\sqrt{10}$	A1ft	ft from their equation

Q5.

Answer	Mark	Comments
$x^2 + y^2 = 100$ or $x^2 + y^2 =$	B2	B1 radius = 10

10^2		

Q6.

	Answer	Mark	Comments
(a)	$\left[\frac{-4+2}{2}, \frac{3+11}{2}\right]$	M1	oe
	(−1, 7)	A1	SC1 for one coordinate correct
(b)	$(r^2 =)\ 3^2 + 4^2$ or $(r^2 =)\ 25$ or $(d^2 =)\ 6^2 + 8^2$ or $(d^2 =)$ 100	M1	oe ft their centre
	(r = 5)	A1ft	SC1 for 10
(c)	$(x + 1)^2 + (y - 7)^2 = 25$	B1ft	oe ft their centre and radius
(d)	$-\frac{1}{2}$ or − 0.5	B1	Accept $\frac{-1}{2}, \frac{1}{-2}$ or − .5

Q7.

Answer	Mark	Comments
(radius =) $\sqrt{289}$ or 17 or (radius =) $\sqrt{121}$ or 11	B1	
$(\frac{1}{4}\times)\ 2 \times \pi \times$ their 17 or 34π or $\frac{17\pi}{2}$ or [106.76, 107] or [26.69, 26.71] or $(\frac{1}{4}\times)\ 2 \times \pi \times$ their 11 or 22π or $\frac{11\pi}{2}$ or [69.08, 69.124] or [17.27,	M1	oe their 17 can be 289 their 11 can be 121

17.3]		
their 17 − their 11 or 6	M1	their 17 can be 289 their 11 can be 121 May be implied by 12 seen in next method mark
$\frac{1}{4} \times 2 \times \pi \times$ their 17 + $\frac{1}{4} \times 2 \times \pi \times$ their 11 + 2 × their 6	M1	their 17 can be 289 their 11 can be 121
$14\pi + 12$ or [55.96, 56(.0)]	A1	SC2 42π or [131.88, 132]

Q8.

Answer	Mark	Comments
$2^2 + 3^2$ or 4 + 9 or 13	M1	oe eg $\sqrt{2^2 + 3^2}$
$x^2 + y^2 = 13$	A1	
$(x - 2)^2 + (y - 3)^2 = 13$	A1	

Q9.

Answer	Mark	Comments
(x-coordinate of C =) $\frac{5+1}{2}$ or 3 or (radius =) $\frac{5+1}{2}$ or 3	M1	may be implied
(y-coordinate of C =) 2	M1	may be implied
$(x - 3)^2 + (y - 2)^2 = 9$	A1	allow $(x - 3)^2 + (y - 2)^2 = 3^2$

Section 3.9
Mark schemes

Q1.

	Answer	Mark	Comments
(a)	$(x-4)^2+(y+2)^2=20$	B2	B1 $(x-4)^2+(y+2)^2$ or 20

Additional Guidance	
$(x+4)^2+(y-2)^2=20$	B1
$(x-4)^2+(y+2)^2=4^2+(-2)^2$	B1
$(x-4)^2+(y+2)^2=\sqrt{20}$	B1
$(x-4)^2-(y+2)^2=20$	B1
$(x-4)^2+(y--2)^2=20$	B2
$(x-4)^2+(y--2)^2=(\sqrt{20})^2$	B2
ignore further working	

	Answer	Mark	Comments
(b)	(Gradient *AC* =) $\frac{0--2}{8-4}$ or $\frac{2}{4}$	M1	oe
	(Gradient of tangent =) negative reciprocal of their $\frac{2}{4}$ or –2	M1	oe ft their gradient *AC* only gradient –2 seen is M2
	$y=-2x+16$	A1	oe

Additional Guidance	
It is possible to find an incorrect gradient of *AC* and then get the second M mark for finding the negative reciprocal of this	M0M1A0

Q2.

	Answer	Mark	Comments
(a)	(1, −3)	B1	

Additional guidance	
Mark intention eg condone 1, −3	B1

(b)

Alternative method 1		
$-3 + \sqrt{25}$ (= 2) or −3 + 5 (= 2)	B1	oe eg 5 − 3 (= 2) or 2 + 3 = 5

Alternative method 2		
$(y + 3)^2 = 25$ and $y = 2$ or $y + 3 = 5$ and $y = 2$ or $(2 + 3)^2 = 25$	B1	oe eg $(1 - 1)^2 + (y + 3)^2 = 25$ and $y = 2$

Additional Guidance	
(1, −3) + (0, 5) = (1, 2) so y = 2	B0
Allow −3 + radius of 5	B1
$2 = 0x + c$ $c = 2$ so $y = 2$	B0

(c)

Alternative method 1 Using equation *PR*		
$\frac{-7 - \text{their } -3}{4 - \text{their } 1}$ or $-\frac{4}{3}$	M1	oe grad *PC* their −3 and their 1 from (a)
$-1 \div \text{their } -\frac{4}{3}$ or $\frac{3}{4}$	M1	oe grad *PR* their $-\frac{4}{3}$ must be a value (gradient *PR* =) $\frac{4}{3}$ is M2
$2 - -7 = \text{their } \frac{3}{4}(x - 4)$	M1dep	oe equation *PR* with y = 2 substituted eg $2 = \frac{3}{4}x - 10$ dep on 2nd M1
16	A1ft	only ft their −3 and their 1 from (a)

Alternative method 2 Using $RC^2 = CP^2 + PR^2$ *or* $PR^2 = QR^2$ *with R (x, 2)*		
$(x - \text{their } 1)^2 + (2 - \text{their } -3)^2$ $= (2 - \text{their } -3)^2 + (x - 4)^2 + (2 - -7)^2$	M1	oe eg $(x - 1)^2 = (x - 4)^2 + (2 - -7)^2$ their −3 and their 1 from (a)
$x^2 - 2x + 1 + 25$ $= 25 + x^2 - 8x + 16 + 81$	M1dep	oe brackets expanded
$96 = 6x$ or $96 \div 6$	M1dep	oe linear equation or calculation dep on M2
16	A1ft	only ft their −3 and their 1 from (a)

Alternative method 3 Using equation *CR*		
$\dfrac{-7-2}{4-\text{their } 1}$ or -3	M1	oe grad *PQ* their 1 from (a)
$-1 \div \text{their } -3$ or $\dfrac{1}{3}$	M1	oe grad *CR* their −3 must be a value (gradient CR =) $\dfrac{1}{3}$ is M2
$2 - \text{their } -3 = \text{their } \dfrac{1}{3}(x - \text{their } 1)$	M1dep	oe equation *CR* with $y = 2$ substituted eg $2 = \dfrac{1}{3}x - \dfrac{10}{3}$ dep on 2nd M1
16	A1ft	only ft their −3 and their 1 from (a)

Alternative method 4 Using equation *MR where M is the midpoint of PQ*		
$\dfrac{-7-2}{4-\text{their } 1}$ or -3	M1	oe grad *PQ* their 1 from (a)
$-1 \div \text{their } -3$ or $\dfrac{1}{3}$	M1	oe grad *MR* their −3 must be a value (gradient MR =) $\dfrac{1}{3}$ is M2

$\left(\frac{4 + \text{their } 1}{2}, \frac{-7+2}{2}\right)$ or (2.5, −2.5) and 2 − their −2.5 = their $\frac{1}{3}$ (*x* − their 2.5)	M1dep	oe midpoint of *PQ* and equation MR with *y* = 2 substituted eg $2 = \frac{1}{3}x - \frac{10}{3}$ dep on 2nd M1
16	A1ft	only ft their −3 and their 1 from (a)

Alternative method 5 Using equation *MC where M is the midpoint of PQ*		
$\left(\frac{4 + \text{their } 1}{2}, \frac{-7+2}{2}\right)$ or (2.5, −2.5)	M1	oe midpoint of *PQ* their 1 from (a)
$\frac{\text{their } -3 - \text{their } -2.5}{\text{their } 1 - \text{their } 2.5}$ or $\frac{1}{3}$	M1dep	oe grad *MC*
2 − their −3 = their $\frac{1}{3}$ (*x* − their 1) or 2 − their −2.5 = their $\frac{1}{3}$ (*x* − their 2.5)	M1dep	oe equation *MC* with *y* = 2 substituted eg $2 = \frac{1}{3}x - \frac{10}{3}$ dep on M2
16	A1ft	only ft their −3 and their 1 from (a)

Alternative method 6 Using *trigonometry where M is the midpoint of PQ*		
$(QM =)\ \frac{1}{2}\sqrt{(4 - \text{their } 1)^2 + (-7-2)^2}$ or $\frac{1}{2}\sqrt{90}$ or 4.74...	M1	
$\sin^{-1}\left(\frac{\text{their } 4.74...}{5}\right)$ or (angle *QCM* =) 71.5… or 71.6	M1dep	oe angle *QCM*
$\tan\ (\text{their } 71.5...) = \frac{x - \text{their } 1}{5}$	M1dep	using triangle *QCR*
16	A1ft	only ft their 1 from (a)

Additional Guidance	
Allow (16, …) to imply answer 16	
Alt 1 $-\frac{4}{3}x$ is M0 unless recovered	
(a) (1, −2) grad *PC* = $-\frac{5}{3}$ grad *PR* = $\frac{3}{5}$ Answer 19 (3rd M1 can be implied by A1ft answer)	M1, M1 M1, A1ft

Q3.

	Answer	Mark	Comments
(a)	$(-5)^2 + 2^2 = 29$	B1	oe involving use of −5 and 2 eg $(-5 - 0)^2 + (2 - 0)^2 = 29$ or $(0 - -5)^2 + (0 - 2)^2 = 29$ or $\sqrt{(-5)^2 + 2^2} = \sqrt{29}$ or $29 - (-5)^2 = 2^2$ or $29 - 2^2 = (-5)^2$ or $\sqrt{29 - (-5)^2} = 2$ or $\sqrt{29 - 2^2} = -5$

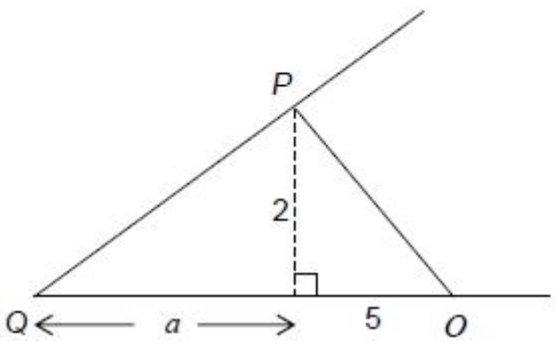
Additional Guidance	
25 + 4 = 29	B0
$-5^2 + 2^2 = 29$	B0
Allow 29 to be written as $\sqrt{29}^2$	

(b)	**Alternative method 1** Using gradients		
	(gradient *OP* =) $\frac{2-0}{-5-0}$ or $-\frac{2}{5}$ or −0.4	M1	oe may be implied eg $y = -\frac{2}{5}x$ or

		gradient of tangent = $\frac{5}{2}$ (with gradient *OP* not seen)
(gradient tangent =) $\frac{-1}{\text{their } -\frac{2}{5}}$ or $\frac{5}{2}$ or 2.5	M1	oe correct or ft their $-\frac{2}{5}$
$y - 2 = \text{their } \frac{5}{2}(x - -5)$ or $0 - 2 = \text{their } \frac{5}{2}(x - -5)$ or $2 = \text{their } \frac{5}{2} \times -5 + c$	M1dep	oe dep on 2nd M1 equation of their tangent with or without substitution of $y = 0$ implied by $y = \frac{5}{2}x + \frac{29}{2}$ oe or $0 = \frac{5}{2}x + \frac{29}{2}$ oe
$-\frac{29}{5}$ or −5.8	A1	oe allow $\left(-\frac{29}{5}, 0\right)$ SC2 answer −10 (grad tangent = $\frac{2}{5}$ SC2 answer $-\frac{21}{5}$ or −4.2 oe (grad tangent = $-\frac{5}{2}$)

Alternative method 2 Using similar triangles (see diagram in Additional Guidance)		
$\frac{a}{2} = \frac{2}{5}$	M1	oe equation any letter
$a = \frac{2}{5} \times 2$ or $a = \frac{4}{5}$	M1dep	
$-5 - \text{their } \frac{4}{5}$	M1dep	dep on M2
$-\frac{29}{5}$ or −5.8	A1	oe

		allow $\left(-\frac{29}{5}, 0\right)$ SC2 answer −10 (grad tangent = $\frac{2}{5}$) SC2 answer $-\frac{21}{5}$) or −4.2 oe (grad tangent = $-\frac{5}{2}$)

Additional Guidance	
Alt 1 2nd M mark is not dependent but there must be a numerical value for grad *OP* to ft	
grad *OP* = −0.4 and grad tangent = −0.4	M1M0 M0A0
$\left(0, -\frac{29}{5}\right)$	M3A0
Ignore any incorrect conversion between fraction and decimal after correct answer seen	
Alt 2 diagram 	

Q4.

	Answer	Mark	Comments
(a)	Radius of circle = 4	M1	4 could be seen in the solution or diagram without the word radius stated
	Use of 4cos 60 and 4sin 60 and	A1	= (2, 2√3) candidates could use the sine rule but it should look like this anyway

$4 \times \frac{1}{2}$ and $4 \times \frac{\sqrt{3}}{2}$		

Alternative method 2		
1 : √3 : 2 triangle seen or stated	M1	Pythagorean triple
2 : 2√3 : 4	A1	

Alternative method 3		
$\tan 60 = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$ or $\frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3} = \tan 60$	B1	shows that the point is on the line *OP*
$(2\sqrt{3})^2 + 2^2 = 12 + 4 = 16$	B1	shows that the point lies on the circle

Additional Guidance	
Candidates could find one coordinate and then substitute into the circle equation to show the second coordinate Candidates may try to use multiple alt methods – mark according to the method that gives them the best mark It is possible to show that the x coordinate is 2 by connecting P and (4,0) hence creating an equilateral triangle (this would need to be stated). Then drop a perpendicular from P which bisects the base line showing that the x coordinate is 2	M1A1

(b)

(Gradient of *OP* =) $\frac{2\sqrt{3}}{2}$ or = √3	M1	√3 either from part (a) or knowing that an angle of 60° gives it
(Gradient of tangent =) $\frac{-1}{\text{their}\sqrt{3}}$	M1	oe $\frac{-1}{\sqrt{3}}$ would imply the first M mark
$y - 2\sqrt{3} = \frac{-1}{\sqrt{3}}(x-2)$ or $2\sqrt{3} = \frac{-1}{\sqrt{3}}(2) + c$	M1dep	oe dependent on M2 already being awarded $c = \frac{8\sqrt{3}}{3}$
$x + \sqrt{3}\,y = 8$	A1	