4 CALCULUS – Further Maths

Section 4.1 – 4.3

Q1. $y = \dfrac{2x^2(3x^3 - 7x)}{x}$ Work out $\dfrac{dy}{dx}$ **(4 marks)**

Q2. $y = x^5 + x$

Work out the positive value of x at which the rate of change of y with respect to x is 81 **(3 marks)**

Q3. $y = \dfrac{x^6}{2} + \dfrac{x^4}{4}$ Work out $\dfrac{dy}{dx}$ Simplify your answer. **(2 marks)**

Q4. $y = 2x^{10} - \dfrac{3}{x^2}$ Work out $\dfrac{dy}{dx}$ **(3 marks)**

Q5. $y = x(2x^4 - 7x^3)$

Work out an expression for the rate of change of y with respect to x. **(3 marks)**

Q6. $y = (5x - 3)^2$ Work out $\dfrac{dy}{dx}$

Give your answer in the form $a(bx - c)$ **where** a, b and c are integers > 1 **(4 marks)**

Q7. Work out the gradient of the curve $y = \dfrac{12x^3 - 8x + 3}{4x^2}$ at the point where $x = -1$

You **must** show your working. **(5 marks)**

Q8. A curve has equation $y = ax^2 + 3x$ where a is a constant.

When $x = -1$, the gradient of the curve is -5 Work out the value of a. **(3 marks)**

Q9. $y = 2x^4\left(x^3 + 2 - \dfrac{3}{x}\right)$ Work out $\dfrac{dy}{dx}$ **(3 marks)**

Q10. $y = 2x^3 + ax$, where a is a constant.

The value of $\dfrac{dy}{dx}$ when $x = 2$ is twice the value of $\dfrac{dy}{dx}$ when $x = -1$

Work out the value of a. **(5 marks)**

Q11. For the curve $y = f(x)$, $\frac{dy}{dx} = \frac{3}{2}x - kx^4 + k$ where k is a constant.

When $x = -2$ the gradient of the curve is 12 Work out the value of k. **(3 marks)**

Q12. $y = \frac{2x^7 + 15x^2}{3x}$ Work out the value of x when $\frac{dy}{dx} = 133$ **(4 marks)**

Q13. A curve has the equation $y = x^3 + ax^2 - 7$ where a is a constant.
The gradient of the curve when $x = 4$ is **twice** the gradient of the curve when $x = -1$
Work out the value of a. **(5 marks)**

Q14. A curve has equation $y = x^3 - 5x^2$

At two points on the curve, the rate of change of y with respect to x is 4

(a) Work out an equation, in terms of x, to represent this information.
Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers. **(2 marks)**

(b) Hence, work out the two possible values of x.
Give your answers to 3 significant figures. **(2 marks)**

Section 4.4

Q1. A curve has equation $y = x^4 - 5x^2 + 9$

(a) Work out $\frac{dy}{dx}$ **(2 marks)**

(b) Work out the equation of the tangent to the curve at the point where $x = 2$
Give your answer in the form $y = mx + c$ **(4 marks)**

Q2. A curve has equation $y = 14x + \frac{3}{2x^2}$

Work out the equation of the normal to the curve at the point $\left(\frac{1}{2}, 13\right)$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. **(5 marks)**

Q3. Work out the equation of the normal to the curve $y = x^2 + 4x + 5$ at the point where $x = -3$

(5 marks)

Q4. P is the point on the curve $y = ax^3 + 10x^2$ where $x = 2$

The gradient of the **normal** to the curve at P is $-\frac{1}{4}$ Work out the value of a. **(4 marks)**

Q5.

(a) Expand $x^2(x - 2)$ **(2 marks)**

(b) A curve has equation $y = x^2(x - 2)$
Work out the gradient of the curve at the point (3, 9). **(3 marks)**

(c) Line L is the tangent to the curve $y = x^2(x - 2)$ at the point (3, 9).
Work out the equation of L. Give your answer in the form $y = mx + c$ **(2 marks)**

Q6. A curve has equation $y = 2x^2 + 3x - 9$
At a point P on the curve, the tangent is parallel to the line $y = 4 - 5x$

Work out the coordinates of P. **(4 marks)**

Q7. P is a point on a curve. The curve has **gradient function** $\frac{x^5 - 17}{10}$

The tangent to the curve at P is parallel to the line $3x - 2y = 9$
Work out the x-coordinate of P. **(4 marks)**

Q8. Point A lies on the curve $y = x^2 + 5x + 8$

The x-coordinate of A is -4

(a) Show that the equation of the normal to the curve at A is $3y = x + 16$ **(5 marks)**

(b) The normal at A also intersects the curve at B.
Work out the x-coordinate of B. **(4 marks)**

Section 4.5

Q1. For what values of x is $y = 150x - 2x^3$ an increasing function? **(4 marks)**

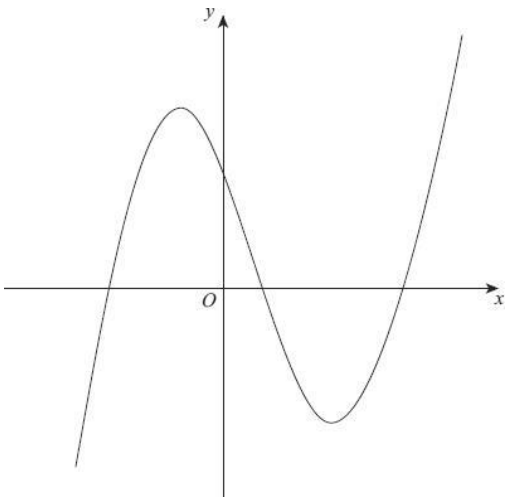
Q2. Work out the values of x for which $f(x) = \frac{2}{3}x^3 + \frac{7}{2}x^2$ is a decreasing function.

Give your answer as an inequality. **(5 marks)**

Q3. f(x) = $2x^3 - 12x^2 + 25x - 11$

Use differentiation to show that f(x) is an increasing function for all values of x. **(4 marks)**

Q4. This is a sketch of the curve $y = f(x)$



(a) For this curve $\frac{dy}{dx} = 3x^2 - 4x - 4$

Work out the range of values of x for which f(x) is a decreasing function.
Write your answer as an inequality. **(4 marks)**

(b) Work out the equation of the normal to the curve at the point (1, −2)
Give your answer in the form $y = mx + c$ **(5 marks)**

Section 4.6

Q1. $y = 3x^4 - \frac{6}{x}$ Work out the value of $\frac{d^2y}{dx^2}$ when $x = -2$ **(4 marks)**

Q2. $y = \frac{6x^9 + x^8}{2x^4}$ Work out the value of $\frac{d^2y}{dx^2}$ when $x = 0.5$ **(5 marks)**

Section 4.7 – 4.8

Q1. The curve $y = x^4 - 18x^2$ has three stationary points.

Work out the coordinates of the three stationary points and determine their nature. **(6 marks)**

Q2. $y = 12x + \frac{3}{x}$ Show that y has a minimum value when $x = 0.5$ **(5 marks)**

Q3. $y = 2x^3 - 12x^2 + 24x - 11$

(a) Work out $\frac{dy}{dx}$

Give your answer in the form $\frac{dy}{dx} = a(x - b)^2$, where a and b are integers. **(3 marks)**

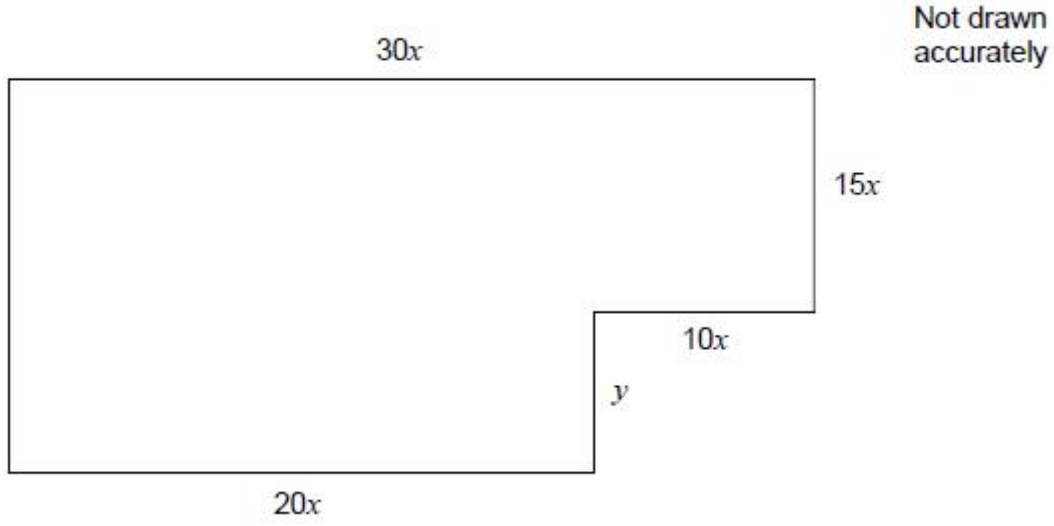
(b) Hence, or otherwise, work out the coordinates of the stationary point of $y = 2x^3 - 12x^2 + 24x - 11$ **(2 marks)**

Q4. The curve $y = f(x)$ has $\frac{dy}{dx} = kx(x - 3)^3$ where k is a **negative** constant.

There is a stationary point at $x = 3$. Determine the nature of this stationary point.
You **must** show your working. **(3 marks)**

Q5. Show that the curve $y = \frac{3}{5}x^5 + x^4$ has **exactly two** stationary points. **(4 marks)**

Q6. This shape is made from two rectangles. All dimensions are in centimetres.



(a) The perimeter of the shape is 252 cm
Show that $y = 126 - 45x$ **(2 marks)**

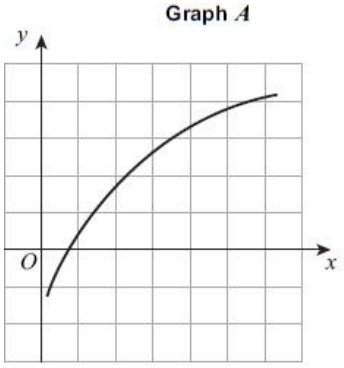
(b) The area of the shape is A cm^2
Show that $A = 2520x - 450x^2$ **(2 marks)**

(c) Use differentiation to work out the maximum value of A as x varies. **(3 marks)**

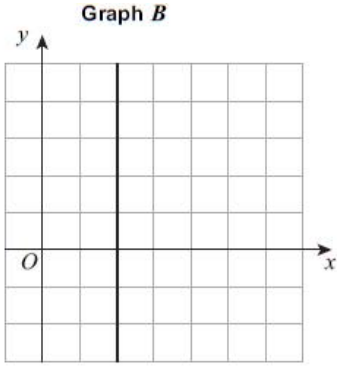
Section 4.9

Q1. Here are five graphs.

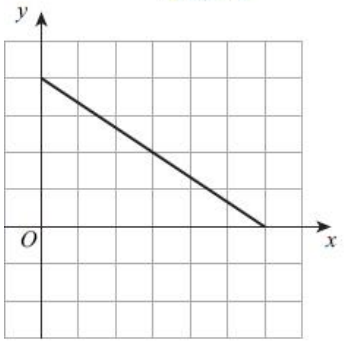
Graph *A*



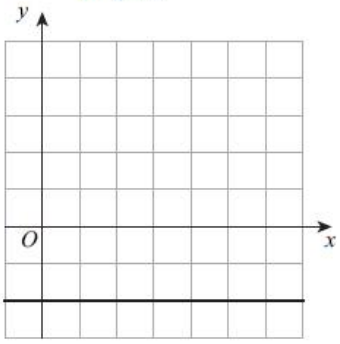
Graph *B*



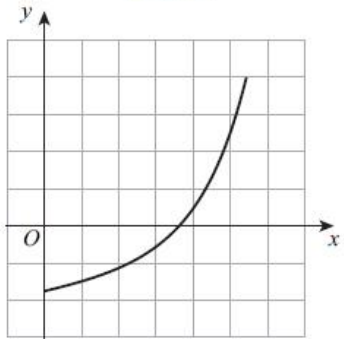
Graph *C*



Graph *D*



Graph *E*



For each of the following statements, decide which graph is being described.
Circle your answer each time.

(a) The rate of change of y with respect to x is always negative.

A B C D E **(1 mark)**

(b) The rate of change of y with respect to x is always zero.

A B C D E **(1 mark)**

(c) As x increases, the rate of change of y with respect to x decreases.

A *B* *C* *D* *E* **(1 mark)**

Q2. $y = f(x)$ is a cubic curve with a maximum and a minimum stationary point.

$$\frac{dy}{dx} = x^2 + 2x - 3$$

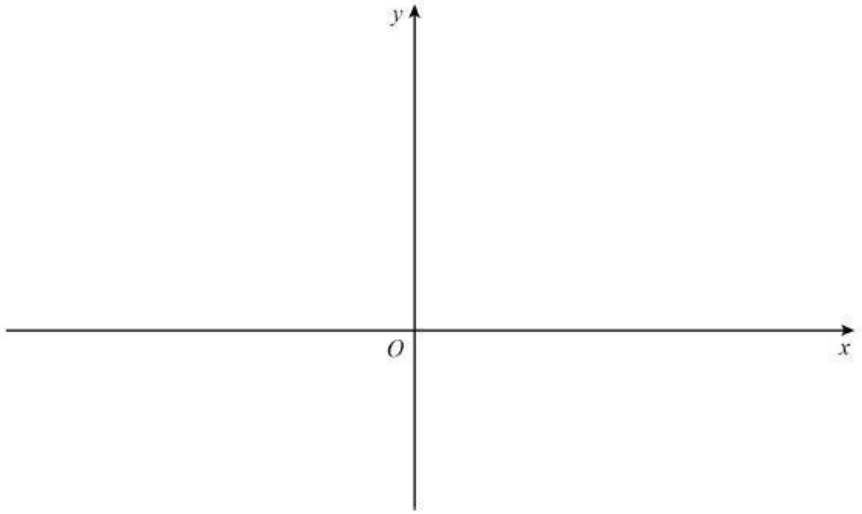
The y-coordinate of the minimum point is $2\frac{1}{3}$

The y-coordinate of the maximum point is 13

(0, 4) is a point on the curve.

The tangent at (0, 4) has a negative gradient.

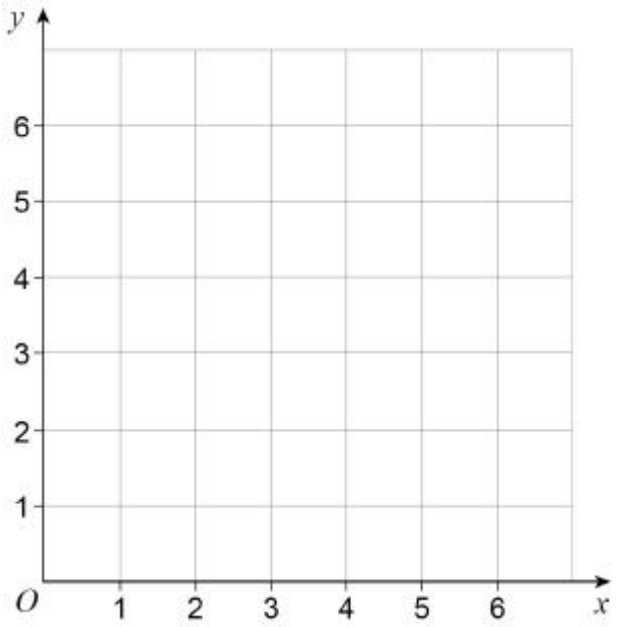
Sketch the curve on the grid below. Show the coordinates of the stationary points.



(4 marks)

Q3.

On the grid, sketch a graph for which
the rate of change of y with respect to x is always a positive constant.

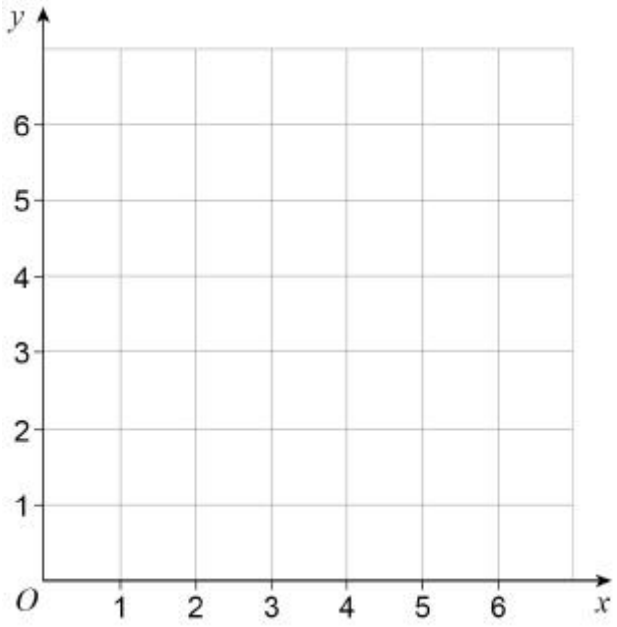


(1 mark)

Q4.

On the grid, sketch a graph for which

the rate of change of y with respect to x is always zero.



(1 mark)