4 CALCULUS – Further Maths

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Section 4.1 – 4.3

Mark schemes

Q1.

Answer	Mark	Comments
$y = \frac{6x^5 - 14x^3}{x}$	M1	or other sensible first step eg $y = 2x(3x^3 - 7x)$ or $y = 2x^2(3x^2 - 7)$ Allow one error
$y = 6x^4 - 14x^2$	A1	
$\frac{dy}{dx} = 24x^3 - 28x$	B2ft	B1ft for each term ft their $y =$… if there are two terms

Q2.

Answer	Mark	Comments
$5x^4$ or 1	M1	
$x^4 = \frac{80}{5}$		oe

or $x^4 = 16$ or $\sqrt[4]{16}$	M1dep	$x^4 = \frac{81-1}{5}$
2	A1	

Q3.

Answer	Mark	Comments
$\frac{6x^5}{2}$ or $3x^5$ or $\frac{4x^3}{4}$ or x^3	M1	oe eg $\frac{12x^5}{4}$
$3x^5 + x^3$	A1	or a correct factorised version eg $x^3(3x^2 + 1)$

Additional Guidance
Do not ignore further work, eg correct answer followed by $4x^8$ scores M1 A0 They must use the powers of x as given in the question, so no misread possible here

Q4.

Answer	Mark	Comments
$-3x^{-2}$	M1	
$20x^9$ or $+ 6x^{-3}$	M1	
$20x^9 + 6x^{-3}$	A1	no additional terms

Q5.

Answer	Mark	Comments
$2x^5 - 7x^4$	M1	
$10x^4$ or (–) $28x^3$	M1	oe eg $5 \times 2x^{5-1}$

$\left(\frac{dy}{dx} =\right)$ $10x^4 - 28x^3$ with no additional terms	A1	do not award for $y =$ or $\frac{d^2y}{dx^2}$ = on the answer line SC2 $2x^4 - 7x^3 + 8x^4 - 21x^3$ SC1 $2x^4 - 7x^3 + x(8x^3 - 21x^2)$

Additional Guidance	
Allow $y =$ ….. for M marks but must be recovered for A1 $\left(\frac{dy}{dx} =\right)$ $10x^4 - 28x^3 + c$	M2A0

Q6.

Answer	Mark	Comments
$25x^2 - 15x - 15x + 9$	M1	4 terms with 3 correct including a term in x^2
$25x^2 - 15x - 15x + 9$ **or** $25x^2 - 30x + 9$	A1	Fully correct
Correctly differentiates their quadratic $50x - 15 - 15$ **or** $50x - 30$	M1	ft their $25x^2 - 15x - 15x + 9$
$10(5x - 3)$ **or** $5(10x - 6)$ **or** $2(25x - 15)$	A1ft	ft M1 A0 M1 if their $50x - 30$ factorises to $a(bx - c)$ where a, b and c are integers > 1

Alternative method		
$2(5x - 3) \times 5$	M2	
$10(5x - 3)$ **or** $5(10x - 6)$ **or** $2(25x - 15)$	A2	

Q7.

Answer	Mark	Comments
$3x$ or $-2x^{-1}$ or $0.75x^{-2}$	M1	oe must have powers of x simplified eg $\frac{12x}{4}$ or $-\frac{2}{x}$ or $\frac{3}{4x^2}$
$3x$ and $-2x^{-1}$ and $0.75x^{-2}$	M1dep	oe must have powers of x simplified oe eg $\frac{12x}{4}$ or $-\frac{2}{x}$ or $\frac{3}{4x^2}$
Any one of $3x$ **and** $3(x^0)$ or $-2x^{-1}$ **and** $2x^{-2}$ or $0.75x^{-2}$ **and** $-1.5x^{-3}$	M1	oe eg $\frac{12x}{4}$ and $\frac{12}{4}x^{1-1}$ or $-\frac{2}{x}$ and $\frac{2}{x^2}$ or $-\frac{2}{x}$ and $--2x^{-2}$ or $\frac{3}{4x^2}$ and $-\frac{3}{2x^3}$ implies 1st M1 for the derivatives x may be (–1)
At least two of $3x$ **and** $3(x^0)$ or $-2x^{-1}$ **and** $2x^{-2}$ or $0.75x^{-2}$ **and** $-1.5x^{-3}$	M1dep	oe dep on 3rd M1 for the derivatives x may be (–1)
All three terms and their derivatives correct and 6.5	A1	oe eg all three terms and their derivatives correct and $\frac{13}{2}$ for the derivatives x may be (–1) SC3 104

Additional Guidance	
Up to M4 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
$\frac{3}{4x^2}$ seen but subsequently incorrectly simplified eg $12x^{-2}$ (subsequent marks may be scored)	(1st) M1
Correct answer after correct use of quotient rule or product rule	M4A1
Incorrect answer after use of quotient rule or product rule	Zero

Condone $y = 3 + 2x^{-2}$… etc	
All three terms and their derivatives correct and 6.5 in working but different answer eg $y = 6.5x$ …	M4A0
SC3 is for multiplying the numerator by $4x^{-2}$ with no subsequent errors	

Q8.

Answer	Mark	Comments
$2ax$ **or** $+3$	M1	either term correct
their $2a(-1) + 3 = -5$	M1dep	oe two terms needed here ... an x term with -1 substituted and a constant term
$(a =)\ 4$	A1	

Additional Guidance
If $dy/dx = 5$ is used (misread) then $-2a + 3 = 5$ scores M1 M1 A0
A 1st line of $2a + 3$ followed by $2a + 3 = -5$ can only score M1 M0 A0
Condone $y = 2ax + 3$ for the 1st M1 ... they have differentiated but used the wrong notation

Q9.

Answer	Mark	Comments
$(y =)\ 2x^7 + 4x^4 - 6x^3$	M1	for any 2 terms correct
$\left(\frac{dy}{dx} =\right)\ 14x^6 + 16x^3 - 18x^2$	A2	oe eg $2x^2(7x^4 + 8x - 9)$ A1 for any correct term correctly differentiated

Alternative method 2 (product rule)		
$\left(\frac{dy}{dx} =\right)$ $8x^3(x^3 + 2 - \frac{3}{x}) + 2x^4\ (3x^2 +$	M1	for either $2x^4$ differentiated correctly multiplied by the bracket or the bracket differentiated correctly multiplied by $2x^4$

$3x^{-2}$)		eg $8x^3(x^3 + 2 - \frac{3}{x})$
$\left(\frac{dy}{dx} =\right)$ $14x^6 + 16x^3 - 18x^2$	A2	eg $2x^2(7x^4 + 8x - 9)$ A1 for any term correct

Additional Guidance	
Ignore subsequent incorrect factorisation Condone incorrect use of y = on the answer line	

Q10.

Answer	Mark	Comments
$\left(\frac{dy}{dx} =\right)$ $6x^2 + a$	M1	Allow one error
$x = -1$ $6 + a$	A1	
$x = 2$ $24 + a$	A1	
Their $(24 + a) = 2 \times$ their $(6 + a)$	M1	Must follow from their $\frac{dy}{dx}$ and must be an equation in a
$a = 12$	A1	$a = -3$ from $\frac{dy}{dx} = 6x^2 + ax$ scores SC3

Q11.

Answer	Mark	Comments
$\frac{3}{2} \times (-2) - k \times (-2)^4 + k$ or $-3 - 16k + k$ or $-3 - 15k$	M1	oe Allow missing brackets even if not recovered eg $\frac{3}{2} \times -2 - k \times -2^4 + k$ or $-3 + 16k + k$ or $-3 + 17k$
$-3 - 16k + k = 12$ or $-3 - 15k = 12$ or $-15k = 15$		oe correct equation (brackets may be recovered) $\frac{3}{2} \times (-2)$ and $(-2)^4$ must be

	A1	evaluated Implied by $k = -1$
–1	A1	SC2 $\frac{15}{17}$ or 0.88... or 0.9

Additional Guidance	
–1 with no errors seen (recovered bracket is not an error)	M1 A2
Substituting $x = 2$	M0 A0

Q12.

Answer	Mark	Comments
$\frac{2x^6}{3}$ or $\frac{2}{3}x^6$ or $\frac{15x}{3}$ or $5x$	M1	implied by $\frac{2x^6+a}{3}$ or $\frac{b+15x}{3}$ a can be numerical or algebraic b can be numerical or algebraic allow 0.66... or 0.67 for $\frac{2}{3}$
$6 \times \frac{2x^5}{3}$ or $\frac{12x^5}{3}$ or $4x^5$ or $\frac{15}{3}$ or 5	M1dep	correct differentiation of one correct term implied by $\frac{6\times 2x^5+a}{3}$ or $\frac{b+15}{3}$
$4x^5 + 5 = 133$ or $4x^5 = 128$ or $x^5 = 32$ or $\sqrt[5]{32}$	A1	oe both correct terms differentiated and simplified correctly and equated to 133
2	A1	

Additional Guidance	
$\frac{14x^6+30x}{3}$	Zero

Q13.

Answer	Mark	Comments
$3x^2 + 2ax$	M1	allow a derivative with at least one term correct and a term in a eg $3x^2 + 2ax + 7$ or $3x^2 + 2a$
$3(4)^2 + 2a(4)$ or $48 + 8a$	M1	
$3(-1)^2 + 2a(-1)$ or $3 - 2a$	M1	
$48 + 8a = 2(3 - 2a)$	M1dep	oe ft if first M1 earned
$(a =)\ -3.5$	A1	oe

Additional Guidance	
Minimum expected working is to see the correct derivative in the first M mark. If no working seen then no marks can be awarded	
If the word "twice" is interpreted the wrong way round ie equation becomes $2(48 + 8a) = 3 - 2a$ this gives an answer of $a = -5\frac{1}{6}$ or $-5.1666...$	M1, A1, A1, M0, A0

Q14.

	Answer	Mark	Comments
(a)	$3x^2$ or $-10x$	M1	oe eg $3 \times x^{3-1}$ or $-2 \times 5x^1$
	$3x^2 - 10x - 4 = 0$ or $-3x^2 + 10x + 4 = 0$	A1	must show = 0

Additional Guidance	
M1 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
Ignore extra terms eg $3x^2 - 10x + c$	M1
$3x^2 - 10x = 4$ (even if $3x^2 - 10x - 4 = 0$ in (b))	M1A0
$3x^2 - 10x - 4$ (even if $3x^2 - 10x - 4 = 0$ in (b))	M1A0
$3x^2 - 10x - 4 = 0$ seen in working with $3x^2 - 10x - 4$ on answer line	M1A1

Condone for M1 $y = 3x^2$ … etc (may still score A1 if recovered)	
Answer $y = 3x^2 - 10x - 4 = 0$	M1A0

(b)

$\frac{--10 \pm \sqrt{(-10)^2 - 4 \times 3 \times -4}}{2 \times 3}$ or $\frac{10 \pm \sqrt{148}}{6}$ or $\frac{5}{3} \pm \sqrt{\frac{37}{9}}$ or two correct solutions with at least one not to 3 sf	M1	oe eg $\frac{5 \pm \sqrt{37}}{3}$ correct attempt to solve their $ax^2 + bx + c$ (= 0) from (a) a, b and c all non-zero eg 3.69(4…) **and** –0.36(09…) or 3.7 **and** –0.36(09…)
3.69 and –0.361	A1ft	correct or ft any answers that have at least 4 sf must be rounded to 3 sf at least one answer must have at least 4 sf

Additional Guidance	
-10^2 used for $(-10)^2$ is M0 unless recovered	
10^2 is equivalent to $(-10)^2$	
Not using ± is M0 unless recovered	
A short dividing line or a short square root symbol is M0 unless recovered	
$\sqrt{((-10)^2 - 4 \times 3 \times -4)}$ is correct for $\sqrt{(-10)^2 - 4 \times 3 \times -4}$	
Correct factorisation of their $ax^2 + bx + c$ (= 0) from (a) scores at least M1	
(a) $3x^2 - 10x + 4 = 0$ (b) $\frac{--10 \pm \sqrt{(-10)^2 - 4 \times 3 \times 4}}{2 \times 3}$ 2.87 and 0.465	M1A1ft
(a) $3x^2 - 10x = 4$ (b) up to 2 marks can be scored if using $3x^2 - 10x - 4 = 0$	
(a) $3x^2 - 10x - 8$ (b) up to 2 marks can be scored if using $3x^2 - 10x - 8 = 0$	
One solution correct does not imply M1	
Both solutions seen in working but only one on answer line	M1A0
3.69 and –0.361 in working with –3.69 and 0.361 on answer line	M1A0

Section 4.4
Mark schemes

Q1.

	Answer	Mark	Comments
(a)	$4x^3 - 10x$ (+ 0)	B2	Accept $4 \times x^3 - 10 \times x$ B1 for $4x^3$ or $4 \times x^3$ B1 for $-10x$ or $-10 \times x$ $4x^3 - 10x$ + something extra scores B1 eg $4x^3 - 10x + 9$
(b)	(when $x = 2$) (gradient =) 12	B1ft	ft their answer to (a)
	(when $x = 2$) (y =) 5	B1	
	their 5 = their 12 × 2 + c or $y - 5 = 12(x - 2)$	M1	oe
	$y = 12x - 19$	A1ft	ft their m and their 5

Q2.

Answer	Mark	Comments
$14 - 3x^{-3}$	M1	oe
$14 - 3 \times \left(\frac{1}{2}\right)^{-3}$ or 14 – 24 or –10	M1	oe substitution of $x = \frac{1}{2}$ into their derivative their derivative must have a negative power of x
–1 ÷ their –10 or $\frac{1}{10}$	M1dep	dep on 2nd M1

$y - 13 =$ their $\frac{1}{10}\left(x - \frac{1}{2}\right)$	M1	oe
$20y - 2x - 259 = 0$ or $2x - 20y + 259 = 0$	A1	

Q3.

Answer	Mark	Comments
$2x + 4$	M1	
−2	A1	
$\frac{1}{2}$	M1	$\frac{-1}{\text{their } -2}$
$y = 2$	B1	
$y - 2 = \frac{1}{2}(x + 3)$	A1ft	oe eg $y = \frac{1}{2}x + \frac{7}{2}$ ft their $\frac{1}{2}$ and their 2 if M2 gained

Q4.

Answer	Mark	Comments
$3ax^2$ or $20x$	M1	oe eg $3 \times ax^{3-1}$ or $2 \times 10x^{2-1}$
$3a \times 2^2 + 20 \times 2$ or $12a + 40$	M1	ft substitution of $x = 2$ into their derivative must have attempted differentiation and have two terms with one involving a may be seen in a denominator
their $(12a + 40) = -1 \div -\frac{1}{4}$ or their $(12a + 40) = 4$	M1dep	oe eg $-\frac{1}{\text{their }(12a+40)} = -\frac{1}{4}$ dep on 2nd M1
– 3	A1	

Additional Guidance	
Only substituting $x = 2$ into y	Zero
$ax^2 + 10x$ $4a + 20 = 4$	M0 M1M1
$3x^2 + 20x$ $12 + 20$	M1 M0M0

Q5.

	Answer	Mark	Comments
(a)	$x^3 - 2x^2$	B2	B1 for x^3 B1 for $-2x^2$
(b)	$3x^2$ or $-4x$	M1	At least one term of their $x^3 - 2x^2$ differentiated correctly
	$3(3)^2 - 4(3)$ or $27 - 12$	M1dep	oe Substitutes $x = 3$ in their $\frac{dy}{dx}$ their $\frac{dy}{dx}$ must be an expression in x Allow even if their (a) has only one term
	15	A1ft	ft M2 and their (a) Only ft if their (a) has at least two terms of different order and all of their terms are differentiated correctly
(c)	$y - 9 =$ their $15(x - 3)$ or $y =$ their $15x + c$ and substitutes (3, 9)	M1	oe e.g. $\frac{9-y}{3-x} =$ their 15 their 15 from (b) Allow $y - 9 = \frac{-1}{\text{their } 15}(x - 3)$ or $y = \frac{-1}{\text{their } 15}$ and substitutes (3,

		9) for M1 A0 only
$y = 15x - 36$	A1ft	ft their 15 from (b) $15x - 36$ is M1 A0 unless $y = 15x - 36$ seen in working

Q6.

Answer	Mark	Comments
$4x + 3$ or gradient = −5 seen	M1	
$4x + 3 = -5$	M1dep	
$x = -2$	A1	
$y = -7$	A1ft	ft their x only if M2 earned

Q7.

Answer	Mark	Comments
$(y =)\ \frac{3}{2}x\ldots$ or $(y =)\ 1.5x\ldots$ or $\frac{3}{2}$ or 1.5	M1	oe eg $(y =)\ \frac{3x-9}{2}$ 1.5
$\frac{x^5 - 17}{10} = \frac{3}{2}$	M1dep	oe implies M2
$x^5 = \frac{3}{2} \times 10 + 17$ or $\sqrt[5]{32}$ or correctly rearranges $\frac{x^5 - 17}{10} = k$ to the form $x^5 =$ (k any non-zero value)	M1	oe eg $x^5 = 15 + 17$ or $x^5 = 32$ or $\sqrt[5]{15+17}$ must rearrange to the form $x^5 =$
2	A1	

Additional Guidance

Condone error seen in rearrangement of $3x - 2y = 9$ if gradient is $\frac{3}{2}$ May go on to score M3 A1	
$\frac{x^5 - 17}{10} = \frac{3}{2}x$	M1, M0, M0, A0
(gradient =) 3 $\frac{x^5 - 17}{10} = 3$ $x^5 = 30 + 17$ (3rd M is not dependent) 2.16	M0, M0dep M1 A0
$\frac{3}{2}$ $\frac{x^5 - 17}{10} = -\frac{2}{3}$ $x^5 = -\frac{2}{3} \times 10 + 17$ (3rd M is not dependent) 1.595	M1 M0 M1 A0
Condone answer (2, …)	
2 embedded	M3, A0

Q8.

	Answer	Mark	Comments
(a)	$(-4)^2 + 5 \times -4 + 8$ or 4	M1	oe
	$2x + 5$	M1	$\frac{dy}{dx}$
	$2 \times -4 + 5$ or -3	M1dep	gradient of tangent
	$-\frac{1}{\text{their } -3}$ or $\frac{1}{3}$	M1dep	dep on 2nd and 3rd M1
	$y - 4 = \frac{1}{3}(x + 4)$ and $3y = x + 16$	A1	must see correct working leading to $3y = x + 16$

(b)	$x + 16 = 3(x^2 + 5x + 8)$	M1	oe
	$3x^2 + 14x + 8$ (= 0)	A1	
	$(3x + 2)(x + 4)$ (= 0) or $\frac{-14 \pm \sqrt{14^2 - 4 \times 3 \times 8}}{2 \times 3}$ or $-\frac{7}{3} \pm \sqrt{\frac{25}{9}}$	M1	oe correct attempt to solve their 3-term quadratic
	$-\frac{2}{3}$	A1	

Section 4.5
Mark schemes

Q1.

Answer	Mark	Comments
$150 - 6x^2$	B1	
their $150 - 6x^2 > 0$ **or** their $150 - 6x^2 = 0$	M1	their $150 - 6x^2$ must be in terms of x Must be > 0 or = 0
$\frac{150}{6} > x^2$ or $(6)(5 - x)(5 + x)$ (> 0) **or** $\frac{150}{6} = x^2$ or $(6)(5 - x)(5 + x)$ (= 0)	M1Dep	ft Their inequality **only if a quadratic** either simplified to $k > x^2$ or factorised correctly **or** ft Their equation **only if a quadratic** either simplified to $k = x^2$ or factorised correctly
$-5 < x < 5$	A1	Allow $x > -5$ and $x > 5$ (must have both inequalities as well as the 'and')

Q2.

Answer	Mark	Comments

$2x^2$ or $7x$	M1	oe eg $3 \times \frac{2}{3}x^{3-1}$
$2x^2 + 7x$	A1	
their $2x^2 + 7x < 0$ or their $2x^2 + 7x \leq 0$	M1dep	may be implied by final inequality must be a two-term quadratic dep on first M1
$x(2x + 7)$ or $x = 0$ and $x = -\frac{7}{2}$	M1dep	factorises or solves their two-term quadratic derivative dep on M2
$-\frac{7}{2} < x < 0$ or $-\frac{7}{2} \leq x \leq 0$	A1	oe single inequality in x

Additional Guidance	
$2x^2 + 7 < 0$	M1A0M1M0A0
$x^2 + 7x < 0$ $x(x + 7)$ $-7 < x < 0$	M1A0M1 M1 A0

Q3.

Answer	Mark	Comments
$6x^2 - 24x + 25$	M1	allow one error
$6(x^2 - 4x)$ …	M1dep	ft their $6x^2 - 24x + 25$ must have 3 term quadratic
$6(x - 2)^2$ ...	M1dep	ft their $6(x^2 - 4x)$ ...
$6(x - 2)^2 + 1$ and valid argument that this is > 0	A1	

Q4.

	Answer	Mark	Comments
(a)	$3x^2 - 4x - 4$ = 0 or < 0 or ≤ 0	M1	
	$(3x + 2)(x - 2)$ (= 0 or < 0 or ≤ 0)	M1	$(3x \pm a)(x \pm b)$ where $ab = \pm 4$ scores M1
	$-\frac{2}{3}$ and 2 seen as solutions	A1	
	$-\frac{2}{3} < x < 2$	A1	condone $-\frac{2}{3} \le x \le 2$ SC1 for either $x < 2$ or $x \le 2$ seen

Additional Guidance
The 2nd M1 is for an attempt to factorise, they must have $3x$ and x but can have 1 and 4 for the values of a and b
Seeing solutions to the quadratic (whether correct or not) implies the first M mark ... they might not formally state $3x^2 - 4x - 4 = 0$

	Answer	Mark	Comments
(b)	substitutes $x = 1$ correctly into the expression for $\frac{dy}{dx}$	M1	
	$(\frac{dy}{dx}=)\ -5$	A1	
	gradient normal = $\frac{1}{5}$	M1	ft their −5 if first M1 earned
	$y - -2 = \frac{1}{5}(x - 1)$ or $-2 = \frac{1}{5}(1) + c$	M1dep	ft their gradient of the normal dep on both previous M marks earned
	$y = \frac{1}{5}x - 2\frac{1}{5}$	A1ft	oe … it need not be in $y = mx + c$ form

Additional Guidance
If they do not get −5 for the gradient of the tangent, they can still score 4 of the 5 marks if they follow through correctly with their value for the gradient of the normal, but it must be their gradient of the normal, not the gradient of the tangent.
If you see $y = -5x + 3$, they have given us the equation of the tangent and

they score M1 A1 only.

Section 4.6

Mark schemes

Q1.

Answer	Mark	Comments
$12x^3$ or $6x^{-2}$	M1	oe eg $- -6x^{-1-1}$
$36x^2$ or $-12x^{-3}$	M1	ft their term(s) for $\frac{dy}{dx}$ oe eg $-2 \times 6x^{-2-1}$
$36x^2 - 12x^{-3}$	A1	
$\frac{291}{2}$ or 145.5	A1	oe value

Q2.

Answer	Mark	Comments
$\frac{6x^9}{2x^4} + \frac{x^8}{2x^4}$ or $3x^5$ or $\frac{1}{2}x^4$	M1	
$3x^5 + \frac{1}{2}x^4$	A1	
$15x^4$ or $2x^3$	M1dep	differentiates at least one term correctly
$60x^3 + 6x^2$	M1dep	differentiates their 2-term $\frac{dy}{dx}$ correctly
9	A1	

Section 4.7 – 4.8

Mark schemes

Q1.

Answer	Mark	Comments
$\left(\frac{dy}{dx} =\right)$ $4x^3 - 36x$	M1	either term correct
their $\frac{dy}{dx} = 0$	M1dep	could be written as $x(x^2 - 9) = 0$ or $4x(x^2 - 9) = 0$ follow through an incorrect differentiation as long as it has at least one term correct
$4x(x + 3)(x - 3)$ (= 0)	M1dep	oe $x(x + 3)(x - 3)$ (= 0) solutions could be gained by using the factor theorem
(–3, –81) (0, 0) (3, –81)	A1	may be seen in calculation rather than put in coordinates at this stage
$\left(\frac{d^2y}{dx^2} =\right)$ $12x^2 - 36$ and when $x = -3$ $\left(\frac{d^2y}{dx^2}\right)$ = 72 and/or positive or when $x = 0$ $\left(\frac{d^2y}{dx^2}\right)$ = –36 and/or negative or when $x = 3$ $\left(\frac{d^2y}{dx^2}\right)$ = 72 and/or positive or any check to both sides of one of their solutions to give one side with a negative gradient and one side with a positive gradient	M1dep	dependent on M3 oe correct y coordinates not required for this M mark any one point assessed correctly (don't need to state max or min at this stage) but if the value of f''(x) is worked out incorrectly then penalise. The value of f''(x) may not be shown and then the correct statement will suffice. eg $x = -4$ $\frac{dy}{dx} < 0$ $x = -1$ $\frac{dy}{dx} > 0$ $x = 1$ $\frac{dy}{dx} < 0$ $x = 4$ $\frac{dy}{dx} > 0$

(–3, –81) Minimum (0, 0) Maximum (3, –81) Minimum	A1	all three points must have been determined correctly to gain this mark this could imply the previous mark by use of a correct sketch graph or a statement that says a positive quartic has these stationary points

Additional Guidance	
Condone incorrect writing of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ even if it's just y = as long as it's recovered to get the correct nature of the turning points	

Q2.

Answer	Mark	Comments
Alternative method 1		
12 or – $3x^{-2}$	M1	oe eg $12x^0$ or 3 × – $1x^{-1-1}$ or $-\frac{3}{x^2}$
12 and – $3x^{-2}$	M1dep	oe eg $12 - \frac{3}{x^2}$ or $12x^0$ and 3 × – $1x^{-1-1}$
$12 - 3x^{-2} = 0$ and $x = 0.5$ or $12 - 3 \times 0.5^{-2} = 0$	M1dep	oe = 0 must be seen condone inclusion of x = – 0.5
$6x^{-3}$	M1	oe eg – 2 × – $3x^{-2-1}$ ft differentiation of their first derivative if it involves a negative power of x
M4 and 6 × 0.5^{-3} (= 48) which is positive (so minimum)	A1	oe do not allow if $\frac{6}{0.5^3}$ is evaluated incorrectly

Alternative method 2		
12 or – $3x^{-2}$	M1	oe eg $12x^0$ or 3 × – $1x^{-1-1}$

12 and – $3x^{-2}$	M1dep	oe eg $12 - \frac{3}{x^2}$ or $12x^0$ and $3 \times -1x^{-1-1}$
$12 - 3x^{-2} = 0$ and $x = 0.5$ or $12 - 3 \times 0.5^{-2} = 0$	M1dep	oe = 0 must be seen condone inclusion of $x = -0.5$
Substitutes one x value in range (0, 0.5) into $12 - 3x^{-2}$ and substitutes one x value > 0.5 into $12 - 3x^{-2}$	M1	eg $12 - 3 \times 0.25^{-2}$ and $12 - 3 \times 1^{-2}$ ft substitution into their first derivative if it involves a negative power of x
M4 and two correct evaluations (so minimum) or M4 and two correct signs shown with no incorrect evaluations (so minimum)	A1	eg M4 and $12 - 3 \times 0.25^{-2} = -36$ and $12 - 3 \times 1^{-2} = 9$ (so minimum) or M4 and $12 - 3 \times 0.25^{-2}$ is negative and $12 - 3 \times 1^{-2}$ is positive (so minimum)

Additional Guidance	
Alt 1 $12 + 3x^{-2} = 0$ $-6x^{-3}$	M1M0M0 M1A0
Alt 2 $12 - 3x^{-2}$ $6x^{-3}$ $12 - 3 \times 0.25^{-2} = -36$ $\quad$ $12 - 3 \times 1^{-2} = 9$ so minimum (A1 only possible after awarding M4)	M1M1M0 M1 A0
Ignore any testing of the stationary point at $x = -0.5$	

Q3.

	Answer	Mark	Comments
(a)	$\frac{dy}{dx} = 6x^2 - 24x + 24$	M1	Allow one error
	$6(x^2 - 4x + 4)$	M1	oe eg $(6x - 12)(x - 2)$ or $(3x - 6)(2x - 4)$
	$6(x - 2)^2$	A1	

(b)	$\frac{dy}{dx} = 0$ when $x = 2$	M1	ft their answer to part (a) if in the form $a(x - b)^2$
	(2, 5)	A1ft	ft their answer to part (a)

Q4.

Answer	Mark	Comments
Alternative method 1		
Substitutes a value $0 < x < 3$ and obtains a correct expression in k e.g. $x = 2 \longrightarrow 2k\,(2 - 3)^3$ or $2k\,(-1)^3$ and substitutes a value $x > 3$ and obtains a correct expression in k e.g. $x = 4 \longrightarrow 4k(4 - 3)^3$ or $4k(1)^3$	M1	oe
Obtains correct expressions for both and correctly indicates whether they are positive or negative e.g. $-2k$ positive and $4k$ negative	M1dep	
Max(imum point)	A1	Must see the working for M1 M1

Alternative method 2		
Correct second derivative with $x = 3$ substituted in leading to 0	M1	oe e.g. $3kx(x - 3)^2 + k(x - 3)^3$

i.e. $4kx^3 - 27kx^2 + 54kx - 27k$ and $x = 3 \longrightarrow 0$		and $x = 3 \longrightarrow 0$
Correct third derivative with $x = 3$ substituted in leading to 0 and correct fourth derivative with $x = 3$ substituted in leading to < 0 i.e. $12kx^2 - 54kx + 54k$ and $x = 3 \longrightarrow 0$ and $24kx - 54k$ and $x = 3 \longrightarrow 18k$ negative	M1dep	
Max(imum point)	A1	Must see the working for M1 M1

Q5.

Answer	Mark	Comments
$3x^4$ or $4x^3$	M1	oe eg $5 \times \frac{3}{5}x^{5-1}$
$3x^4 + 4x^3$	A1	
$x^3(3x + 4)$ (= 0)	M1dep	allow partial factorisation of their $3x^4 + 4x^3$ if at least x is taken as a factor ft their two terms if M1 scored
$x^3(3x + 4)$ (= 0) and (x =) 0 and $(x =) -\frac{4}{3}$ with no other solutions	A1	allow partial factorisation if at least x is taken as a factor

Additional Guidance	
$3x^4 + 4x^3 = 0$	M1, A1
$x = 0$ and $x = -\frac{4}{3}$	M0, A0

Condone $y = 3x^4 + 4x^3$	M1, A1
Ignore higher derivatives	
Condone (0, …) and $\left(-\frac{4}{3}, \ldots\right)$ for (x =) 0 and $(x =) -\frac{4}{3}$	
Allow −1.33… for $-\frac{4}{3}$ (ignore any incorrect conversion attempt after $-\frac{4}{3}$ seen)	

Q6.

	Answer	Mark	Comments
(a)	$30x + 20x + 15x + 10x + 15x + y + y = 252$ or $90x + 2y = 252$	M1	oe
	$y = \frac{252 - 90x}{2}$ and $y = 126 - 45x$	A1	must see working for M1

(b)	$30x \times 15x + 20x \times (126 - 45x)$ or $15x \times 10x + 20x \times (126 - 45x + 15x)$ or $15x \times 10x + 20x \times (126 - 30x)$	M1	oe
	$450x^2 + 2520x - 900x^2 = 2520x - 450x^2$ or $150x^2 + 2520x - 900x^2 + 300x^2 = 2520x - 450x^2$ or $150x^2 + 2520x - 600x^2 = 2520x - 450x^2$	A1	must see correct expansion of brackets

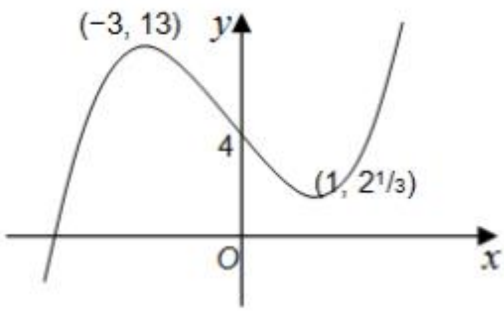
(c)	2520 − 900x	M1	
	their (2520 − 900x) = 0 or x = 2.8	M1dep	oe
	3528	A1	

Section 4.9
Mark schemes

Q1.

	Answer	Mark	Comments
(a)	C	B1	
(b)	D	B1	
(c)	A	B1	

Q2.

Answer	Mark	Comments
factorising to get (x + 3)(x − 1) (= 0) or completing the square and getting as far as x + 1 = ± 2 or using the quadratic formula and getting as far as $x = \frac{-2 \pm 4}{2}$	M1	
x = −3 and x = 1	A1	
(−3, 13) as a maximum point and (1, $2\frac{1}{3}$) as a minimum point, plotted	M1	

Smooth correct curve which must have the stationary points plotted in the correct quadrants and **must** cross the negative x-axis	A1	

Additional Guidance
SC1 for a fully correct sketch with the stationary points in the correct quadrants but lacking any detail in terms of the x coordinates of the stationary points, or with incorrect values of the stationary points, and with no evidence of a valid method to obtain $x = -3$ and $x = 1$

Q3.

Answer	Mark	Comments
Straight line with gradient > 0	B1	mark intention

Additional Guidance	
Ignore any attempt at an equation	
Mark the entire graph on the grid	
Ignore any graph not on the grid	
Vertical line	B0
A straight line joined to another line with a different gradient	B0
Line does not need to start at (0, 0)	
Ignore any points plotted	

Q4.

Answer	Mark	Comments
Horizontal straight line	B1	mark intention

Additional Guidance	
Ignore any attempt at an equation	
Mark the entire graph on the grid	
Ignore any graph not on the grid	
Line clearly drawn on the x-axis	B1

Line does not need to start from the y-axis	
Ignore any points plotted	