2 ALGEBRA – Further Maths

Section 2.1 – 2.5

Q1. $f(x) = 2x^2 + 7$ for all values of x.

(a) What is the value of f(—1)? **(1 mark)**

(b) What is the range of f(x)? **(1 mark)**

Q2. The function g is given by g(x) = $x^2 - 4$ with domain $-1 < x < 3$

Work out the range of the function. **(2 marks)**

Q3. f(x) = 10 − x^2 for all values of x. g(x) = (x + 2a)(x + 3) for all values of x.

(a) Circle the correct value of f(−4)

26 −6 36 196 **(1 mark)**

(b) Write down the range of f(x). **(1 mark)**

(c) g(0) = 24

Show that a = 4 **(1 mark)**

(d) Hence solve f(x) = g(x) **(4 marks)**

Q4. The graph of y = f(x) is a straight line.

The domain of f(x) is $1 \le x \le 5$

The range of f(x) is $3 \le f(x) \le 11$

Work out **one** possible expression for f(x). **(4 marks)**

Q5. h(x) = 5x – 3 The **range** of h(x) is $-2 < h(x) < 1$

Work out the domain of h(x). **(2 marks)**

Q6. f(x) = x^4

The domain of f(x) is $x \ge 2$

Work out the range of f(x). **(1 mark)**

Q7. The function f is given by $f(x) = \sqrt{2x - 5}$

(a) Which of these inequalities is a possible domain for f(x)?
Circle the inequality.

$x \geqslant 0$ $x \geqslant \frac{2}{5}$ $x \geqslant 2$ $x \geqslant \frac{5}{2}$

(1 mark)

(b) Work out x when f(x) = 1.2 **(2 marks)**

(c) Work out the value of $f(2\frac{5}{8})$

Give your answer as a fraction in its simplest form. **(3 marks)**

Q8. f(x) = $x^2 - 7$ for all values of x g(x) = $1 - 3x$ for $-4 \leq x \leq 4$

(a) Work out the range of f(x). Give your answer as an inequality. **(1 mark)**

(b) Work out the range of g(x). Give your answer as an inequality. **(2 marks)**

(c) Solve $2f(x) = g(x)$ You **must** show your working.

Give your answers to 3 decimal places. **(4 marks)**

Q9. g(x) = $5 - x^2$ The domain of g(x) is $-2 \leq x \leq 1$

Work out the range of g(x). **(2 marks)**

Q10. f(x) is a function with domain all values of x.

$f(x) = \sqrt{x^2 + 6x - a}$ where a is a constant.

Work out the possible values of a. Give your answer as an inequality. **(4 marks)**

Q11. $f(x) = \left(\frac{9x}{2}\right)^{-1}$ $g(x) = \sqrt{1 - px^3}$ where p is a constant.

Given that $f\left(\frac{1}{3}\right) = g\left(\frac{1}{3}\right)$ work out the value of p. **(5 marks)**

Q12. f(x) = $x^3 - 2$

The domain of f(x) is $x \leq 3$ Work out the range of f(x). **(2 marks)**

Q13. $f(x) = 3x^2 + 6$ for all x $g(x) = \sqrt{x-5}$ $x \geq 5$

(a) Work out the value of $gf(4)$ **(2 marks)**

(b) Show that $fg(x)$ can be written in the form $a(x - a)$ where a is an integer.

(2 marks)

Q14. $h(x) = x^2 + 5$

k is a function such that $hk(x) = 4x^2 + 5$ Work out an expression for $kh(x)$ **(2 marks)**

Q15. $h(x) = 5x(x - 4)$ Solve $3h(x) = h(2x)$ **(4 marks)**

Q16. $f(x) = \frac{x-3}{6x-5}$ Which value of x can **not** be in the domain of $f(x)$?

Circle your answer. 0 $\frac{5}{6}$ $\frac{6}{5}$ 3 **(1 mark)**

Q17. $g(x) = \frac{6}{x}$ $h(x) = x - 5$ Solve $gh(x) = x$ **(4 marks)**

Q18. $f(x) = (x + 4)^3$ Work out $f^{-1}(-8)$ **(2 marks)**

Q19. $f(x) = (x + 2)^3$ g is a function such that $gf(x) = (x + 2)^{12}$

Work out an expression for $g(x)$ **(1 mark)**

Q20. The function h is given by $h(x) = \frac{3+x}{2}$ Work out $h^{-1}(x)$ **(2 marks)**

Section 2.6

Q1. $(3x + a)(5x - 4) \equiv 15x^2 - 2x + b$ Work out the values of a and b. **(3 marks)**

Q2. Expand and simplify $(3w + 2y)(w - 4y)$ **(3 marks)**

Q3. Expand and simplify $(y^2 - 5y + 2)(2y - 3)$ **(3 marks)**

Q4. Expand and simplify $\frac{3}{x^2}\left(\frac{x}{3}+3x^2-1\right)$ **(3 marks)**

Q5. $2x^2 - 2bx + 7a \equiv 2(x - a)^2 + 3$
Work out the **two** possible pairs of values of a and b. **(6 marks)**

Q6. $x^2 + 2ax + b \equiv (x - 5)^2 - a$
Work out the values of a and b. **(3 marks)**

Q7. $3x^3 - 2x^2 - 147x + 98 \equiv (ax - c)(bx + d)(bx - d)$
where a, b, c and d are positive integers. Work out the values of a, b, c and d. **(3 marks)**

Q8. $(x + 4)(x^2 - kx - 5)$ is expanded and simplified.
The coefficient of the x^2 term is twice the coefficient of the x term. Work out the value of k.
(3 marks)

Q9. The x^2 term in the expansion of $(3x + 4)(x^2 + px + 5)$ is $-23x^2$
Work out the value of p. **(3 marks)**

Q10. Expand and simplify fully $(x + 2)(x + 3)(x + 4)$ **(3 marks)**

Q11. $A = 2 - 5x$ $B = 3x - 1$ $C = x^2$ Show that $(2A + 3B)^2 \equiv A + B + C$
(4 marks)

Q12. $p(x - 1) + 2(3x + k) \equiv 4(x + 2)$ where p and k are integers.

Work out the values of p and k. **(4 marks)**

Q13. Simplify $(n + 2)^3 - n^2(n - 5)$

Give your answer in the form $an^2 + bn + c$ where a, b and c are integers. **(4 marks)**

Q14. Show that $(x + 1)(x + 3)(x + 4) - x(x^2 + 7x + 11)$
can be written in the form $(x + a)(x + b)$ where a and b are positive integers. **(5 marks)**

Q15. $3x^2 + 2bx + 8a$ can be written in the form $3(x + a)^2 + b + 2$

Work out the **two** possible pairs of values of a and b. **(6 marks)**

Q16. Expand and simplify fully $(5x + 3y^2)(4x - y^2)$ **(3 marks)**

Q17. Expand and simplify fully $(3x + 4)(2x - 3)(5x - 2)$ **(3 marks)**

Section 2.7

Q1. Expand and simplify fully $(3 + 2x)^5$ **(4 marks)**

Q2. The coefficient of x^4 in the expansion of $(a + 2x)^6$ is 1500
Work out the **two** possible values of a. **(3 marks)**

Q3. The coefficient of the x^4 term in the expansion of $(2x + a)^6$ is 60.
Work out the possible values of a. **(4 marks)**

Q4. The coefficient of x^2 in the expansion of $(3 + ax)^4$ is 150
Work out the **two** possible values of a. **(3 marks)**

Section 2.8

Q1. Factorise fully $(x + y)^2 + (x + y)(2x + 5y)$ **(3 marks)**

Q2.

(a) Factorise fully $5m^2 - 20p^2$ **(3 marks)**

(b) You are given that $p = 15$ and $5m^2 - 20p^2 = 0$
Using your answer to part (a), or otherwise, work out the values of m. **(2 marks)**

Q3. Factorise fully $12c^2d - 9d^2$ **(2 marks)**

Q4.

(a) Show that $(x + 7)^2 - (x - 3)^2$ simplifies to $20(x + 2)$ **(3 marks)**

(b) Hence, or otherwise, work out $107^2 - 97^2$ **(2 marks)**

Q5. Factorise fully $(w + 4)^3 - (w + 4)^2(w + 1)$ **(3 marks)**

Q6. Factorise fully $3x^2 - 12$ **(2 marks)**

Q7. Factorise $5x^2 + 4xy - 12y^2$ **(3 marks)**

Q8. Factorise fully $(x + 6)^4 + (x + 6)^3(3x + 4)$
Do **not** attempt to expand the brackets. **(3 marks)**

Q9. Factorise fully $48 - 75x^2$ **(2 marks)**

Q10. Factorise fully $6(y + 3)^5 + 4(y + 3)^4$
Give your answer in its simplest form.
Do **not** attempt to expand $(y + 3)^5$ or $(y + 3)^4$ **(3 marks)**

Q11. Factorise fully $12pq^3r - 18pq^2r^2 + 24pq^2r$ **(2 marks)**

Q12. Factorise fully $6x^2 + 26xy - 20y^2$ **(3 marks)**

Q13. Factorise fully $x^4y + 3x^2y^3$ **(2 marks)**

Q14. Factorise fully $x^6 - 9x^4$ **(2 marks)**

Q15. Factorise fully $x^4 - 81$ **(2 marks)**

Section 2.9

Q1. Simplify fully $\dfrac{5x}{(x+4)(x-6)} - \dfrac{3}{(x-6)}$ **(4 marks)**

Q2. Simplify $\dfrac{x^2+4x-12}{x^2-25} \div \dfrac{x+6}{x^2-5x}$ **(5 marks)**

Q3. Simplify fully $\dfrac{4x^2+19x-5}{9x^2-16} \div \dfrac{x+5}{3x-4}$ **(5 marks)**

Q4. (a) Show that $\dfrac{4}{x} + \dfrac{2}{x-1}$ simplifies to $\dfrac{6x-4}{x(x-1)}$ **(2 marks)**

(b) Hence, or otherwise, solve $\dfrac{4}{x} + \dfrac{2}{x-1} = 3$
Give your solutions to 3 significant figures. **(5 marks)**

Q5. Solve $\frac{4}{x-2}+\frac{1}{x+3}=5$ **(7 marks)**

Q6. Simplify fully $\frac{8c^7}{15d^6}\div\frac{6c^2}{5d^3}$ **(3 marks)**

Q7. (a) Show that $\frac{c^2+5c+4}{3c+3}$ simplifies to $\frac{c+4}{3}$ **(2 marks)**

(b) Hence, or otherwise, simplify fully $\frac{c^2+5c+4}{3c+3}+\frac{3-2c}{6}$ **(3 marks)**

Q8. Write as a single fraction $\frac{5}{m+1}+\frac{6}{m-4}$
Give your answer in its simplest form. **(4 marks)**

Q9. By factorising fully, simplify $\frac{x^4-x^3-2x^2}{x^4-5x^2+4}$ **(5 marks)**

Q10. Solve $\frac{3}{x-2}+\frac{2}{x-1}=5$ Write your solutions to 3 significant figures. **(6 marks)**

Q11. Simplify fully $\left(\frac{x}{2}+\frac{3x}{5}\right)\div\sqrt{\frac{x^6}{4}}$ **(5 marks)**

Q12. Simplify fully $\frac{8a}{3a+6}\times\frac{5a+10}{3a^2}\div\frac{4}{15a^3}$ **(3 marks)**

Q13. Simplify fully $\frac{x}{x-3}+\frac{6}{(x-3)(x-5)}$ **(4 marks)**

Q14. Write $\frac{7}{9x}+\frac{2}{3x^2}$ as a single fraction in its simplest form. **(3 marks)**

Q15.

Show that $\frac{x^4}{x+4}\times\frac{x+2}{x}\div\frac{x^2}{3x+12}$

simplifies to the form $ax^2 + bx$ where a and b are integers. **(4 marks)**

Q16. Simplify fully $\frac{x-x^3}{2x+2x^2}$ You **must** show your working. **(4 marks)**

Q17. Simplify fully $\frac{15x^2y-5xy^2}{12x-4y}$ **(3 marks)**

Q18. Write $\frac{5}{6a}+\frac{a}{4}$ as a single fraction. Give your answer in its simplest form. **(2 marks)**

Q19. Simplify $\frac{8x^3-50x}{2x(6x^2-x-35)}$

Give your answer in the form $\frac{ax+b}{cx+d}$ where a, b, c and d are integers. **(5 marks)**

Q20. Work out $\frac{7}{2x^2}+\frac{4}{3x}$

Give your answer as a single fraction in its simplest form. **(2 marks)**

Q21. Simplify fully $\frac{6c^4-c^3}{36c^2-1}$ **(3 marks)**

Q22. Simplify fully $\frac{10x-2y}{3y-15x}$ **(2 marks)**

Section 2.10

Q1. Rearrange $ef=\frac{5e+4}{3}$ to make e the subject. **(3 marks)**

Q2. Rearrange $y=\sqrt{\frac{x+2w}{3}}$ to make w the subject. **(3 marks)**

Q3. $5t + 3 = 4w(t + 2)$

(a) Rearrange the formula to make t the subject. **(4 marks)**

(b) Work out the exact value of t when $w = -\frac{1}{8}$

Give your answer in its simplest form. **(3 marks)**

Q4. $S = \frac{a}{1-r}$

(a) Show that $r = \frac{S-a}{S}$ **(3 marks)**

(b) Work out the value of r when $S = 10a$ **(2 marks)**

Q5. Rearrange $x = \frac{2w+1}{5-3w}$ to make w the subject. **(4 marks)**

Q6. Make y the subject of $\sqrt{\frac{3xy}{x+y}} = 4$ **(4 marks)**

Q7. Rearrange $y = \frac{8(w-x)}{x}$ to make x the subject. **(4 marks)**

Q8. Rearrange $\frac{1}{xy} = 4 - \frac{3}{y}$ to make x the subject. **(3 marks)**

Q9. Rearrange $t = \frac{3w^3+a}{w^3-2}$ to make w the subject. **(5 marks)**

Q10. Rearrange $m = \frac{2p+1}{p} + \frac{p+5}{3p}$ to make p the subject. **(4 marks)**

Section 2.11

Q1. $f(x) = 2x^3 + 11x^2 + 12x - 9$

(a) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. **(2 marks)**

(b) Show that $f(x) = 0$ has **exactly two** solutions. **(4 marks)**

Q2.

(a) Use the factor theorem to show that $(x - 1)$ and $(x - 4)$ are factors of $x^3 - 21x + 20$ **(2 marks)**

(b) Show that $(x - 1)$ and $(x - 4)$ are also factors of $x^3 - 10x^2 + 29x - 20$ **(2 marks)**

(c) Hence, simplify fully $\dfrac{x^3 - 21x + 20}{x^3 - 10x^2 + 29x - 20}$ **(3 marks)**

Q3. $f(x) = x^3 + ax^2 + bx + 24$ for all values of x.

Two of the factors of $f(x)$ are $(x - 2)$ and $(x + 3)$. Work out the values of a and b. **(5 marks)**

Q4. $(x - a)$ is a factor of $2x^3 - 7ax + 3a$

Work out the **largest** possible value of a. **(4 marks)**

Q5. $(x - a)$ is a factor of $x^3 + 2ax^2 - a^2x - 16$

(a) Show that $a = 2$ **(2 marks)**

(b) Solve $x^3 + 4x^2 - 4x - 16 = 0$ **(4 marks)**

Q6.

(a) $(x - 3)$ is a factor of $x^3 - 8x^2 + ax + 42$ where a is an integer.

Show that the value of a is 1 **(2 marks)**

(b) Hence, factorise fully $x^3 - 8x^2 + x + 42$ **(3 marks)**

Q7. $f(x) = 200x^3 + 100x^2 - 18x - 9$

(a) Use the factor theorem to show that $(2x + 1)$ is a factor of $f(x)$. **(2 marks)**

(b) Hence solve $f(x) = 0$ **(3 marks)**

Q8. $f(x) = x^3 - 10x - c$ where c is a positive integer. $(x + c)$ is a factor of $f(x)$.

Use the factor theorem to work out the value of c. **(3 marks)**

Q9. $f(x) = 3x^3 - 2x^2 - 7x - 2$

(a) Use the factor theorem to show that $(3x + 1)$ is a factor of $f(x)$. **(2 marks)**

(b) Factorise $f(x)$ fully. **(3 marks)**

Section 2.12

Q1. Write $6x^2 - 24x + 17$ in the form $a(x + b)^2 + c$ where a, b and c are integers. **(3 marks)**

Q2. Write $2x^2 - 16x + 13$ in the form $a(x + b)^2 + c$ where a, b and c are integers. **(4 marks)**

Q3.

Write $7 - 12x - 18x^2$ in the form $a - 2(bx + c)^2$

where a, b and c are positive integers. **(3 marks)**

Q4. The nth term of a sequence is $n^2 - 6n + 14$

By completing the square, or otherwise, show that every term is positive. **(3 marks)**

Q5. You are given that $x^2 + 6x + 2 \equiv (x + h)^2 + k$

(a) Work out the values of h and k. **(2 marks)**

(b) Write down the coordinates of the minimum point on the curve $y = x^2 + 6x + 2$

(1 mark)

(c) Solve the equation $x^2 + 6x + 2 = 0$ Give your answers in the form $a \pm \sqrt{b}$ **(1 mark)**

Q6. Write $12x^2 - 60x + 5$ in the form $a(bx + c)^2 + d$ where a, b, c and d are integers.

(5 marks)

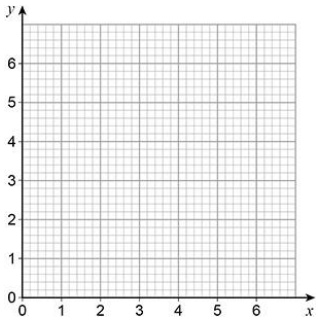
Section 2.13

Q1. (a) $f(x) = 4 - x \quad 0 \leqslant x < 1$

$= 4x - x^2 \quad 1 \leqslant x < 4$

$= 2x - 8 \quad 4 \leqslant x \leqslant 6$

On the grid, draw the graph of $y = f(x)$



(4 marks)

(b) $g(x) = 6 - 3x$ Work out $g^{-1}(x)$. **(2 marks)**

Q2.

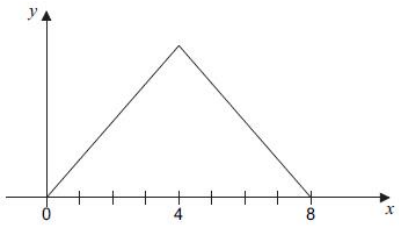
$y = f(x)$ is the graph of a cubic function.

$y < 0$ for $x < 5$
$y \geqslant 0$ for $x \geqslant 5$

The function is
increasing for $x < -1$
decreasing for $-1 < x < 2$
increasing for $x > 2$

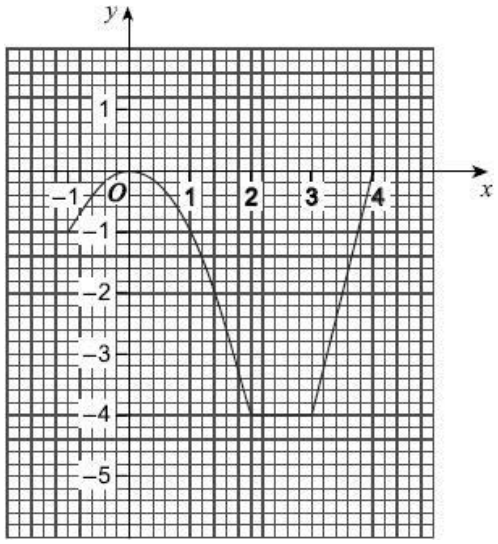
Draw a possible sketch of $y = f(x)$ for values of x from –2 to 6 **(4 marks)**

Q3. A sketch of $y = g(x)$ for domain $0 \leq x \leq 8$ is shown.



The graph is symmetrical about $x = 4$ The range of $g(x)$ is $0 \leq g(x) \leq 12$
Work out the function $g(x)$. **(5 marks)**

Q4. Here is the graph of $y = f(x)$
It consists of a quadratic curve and two straight lines.



Define $f(x)$, stating clearly the domain for each part. **(4 marks)**

Q5. A function f(x) is defined as

$$\begin{aligned} f(x) &= x + 3 && -3 \le x < 0 \\ &= 3 && 0 \le x < 1 \\ &= 5 - 2x && 1 \le x \le 2 \end{aligned}$$

Draw the graph of $y = f(x)$ for $-3 \le x \le 2$ **(3 marks)**

Q6. A function f(x) is defined as

$$\begin{aligned} f(x) &= 3 - 2x && -2 \le x < 0 \\ &= (1 + x)(3 - x) && 0 \le x < 4 \\ &= 5x - 25 && 4 \le x \le 5 \end{aligned}$$

(a) Draw the graph of $y = f(x)$ **(4 marks)**

(b) State the range of f(x) **(2 marks)**

Q7. Draw the graph of $y = f(x)$

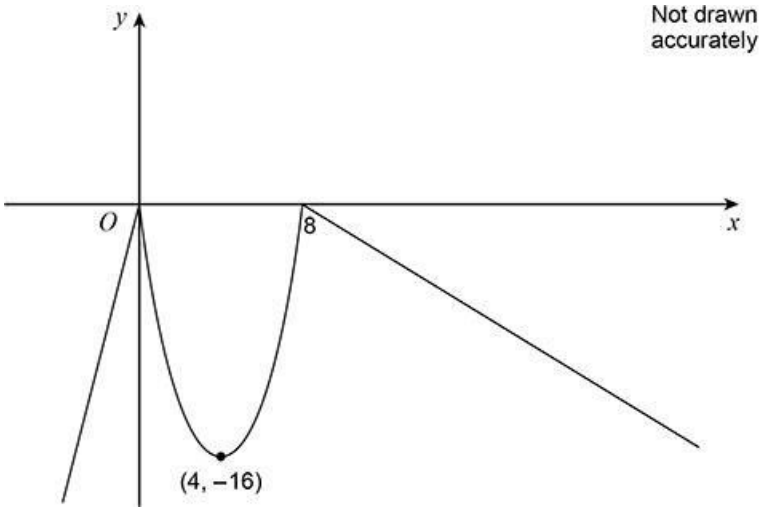
$$\begin{aligned} f(x) &= x + 4 && -4 \le x < 0 \\ &= 4 - 3x && 0 \le x < 2 \\ &= -2 && 2 \le x \le 5 \end{aligned}$$

(4 marks)

Q8. A function f is given by

$$\begin{aligned} f(x) &= 4x && x < 0 \\ &= x^2 - 8x && 0 \leqslant x \leqslant 8 \\ &= 16 - 2x && x > 8 \end{aligned}$$

A sketch of $y = f(x)$ is shown.



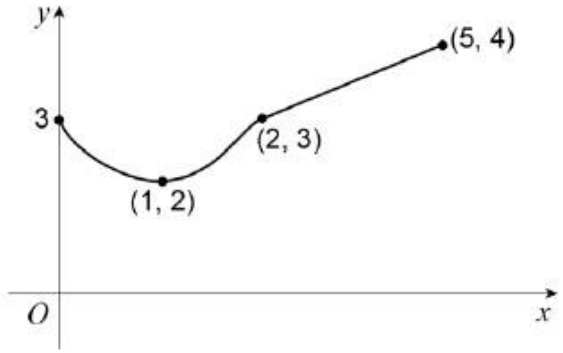
Work out **all** the values of x for which $f(x) = -12$ **(4 marks)**

Q9. $f(x) = (x - a)^2 + b \quad 0 \leq x < 2$

$= cx + d \quad 2 \leq x \leq 5$

a, b, c and d are constants.

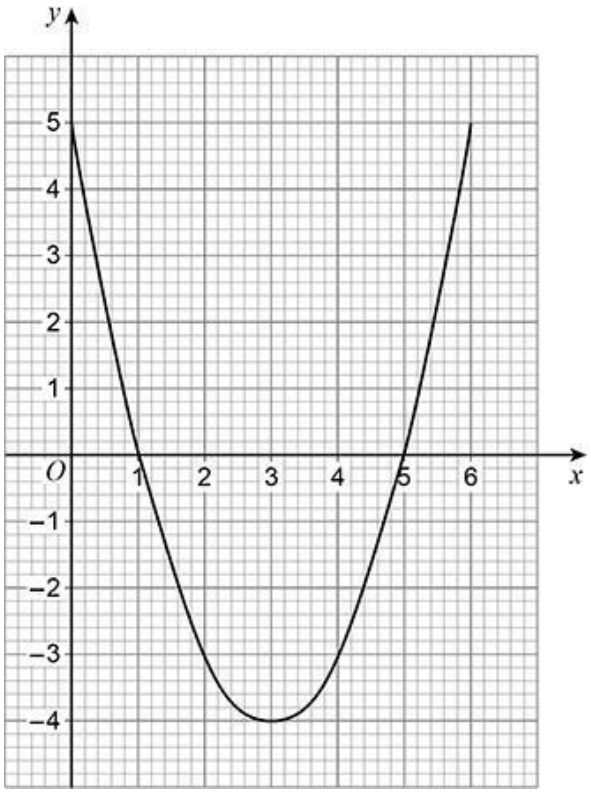
A sketch of $y = f(x)$ is shown.



Work out the values of a, b, c and d. **(4 marks)**

Section 2.14 – 2.15

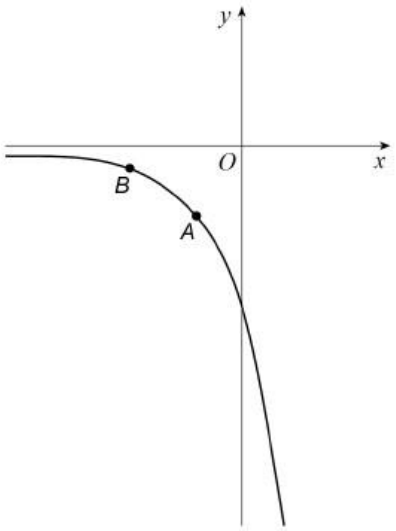
Q1. Here is the graph of $y = x^2 - 6x + 5$ for values of x between 0 and 6



By drawing a suitable **linear** graph on the grid, work out approximate solutions to $x^2 - 7x + 9 = 0$

(3 marks)

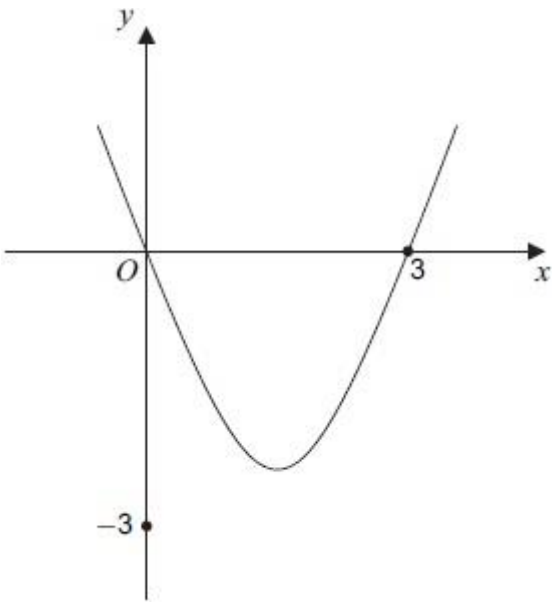
Q2. The points A (–1, –4) and $B\left(-2, -\frac{4}{3}\right)$ lie on the curve $y = ab^x$ as shown.



Not drawn accurately

Work out the values of a and b. **(4 marks)**

Q3. The diagram shows a sketch of $y = x^2 - 3x$



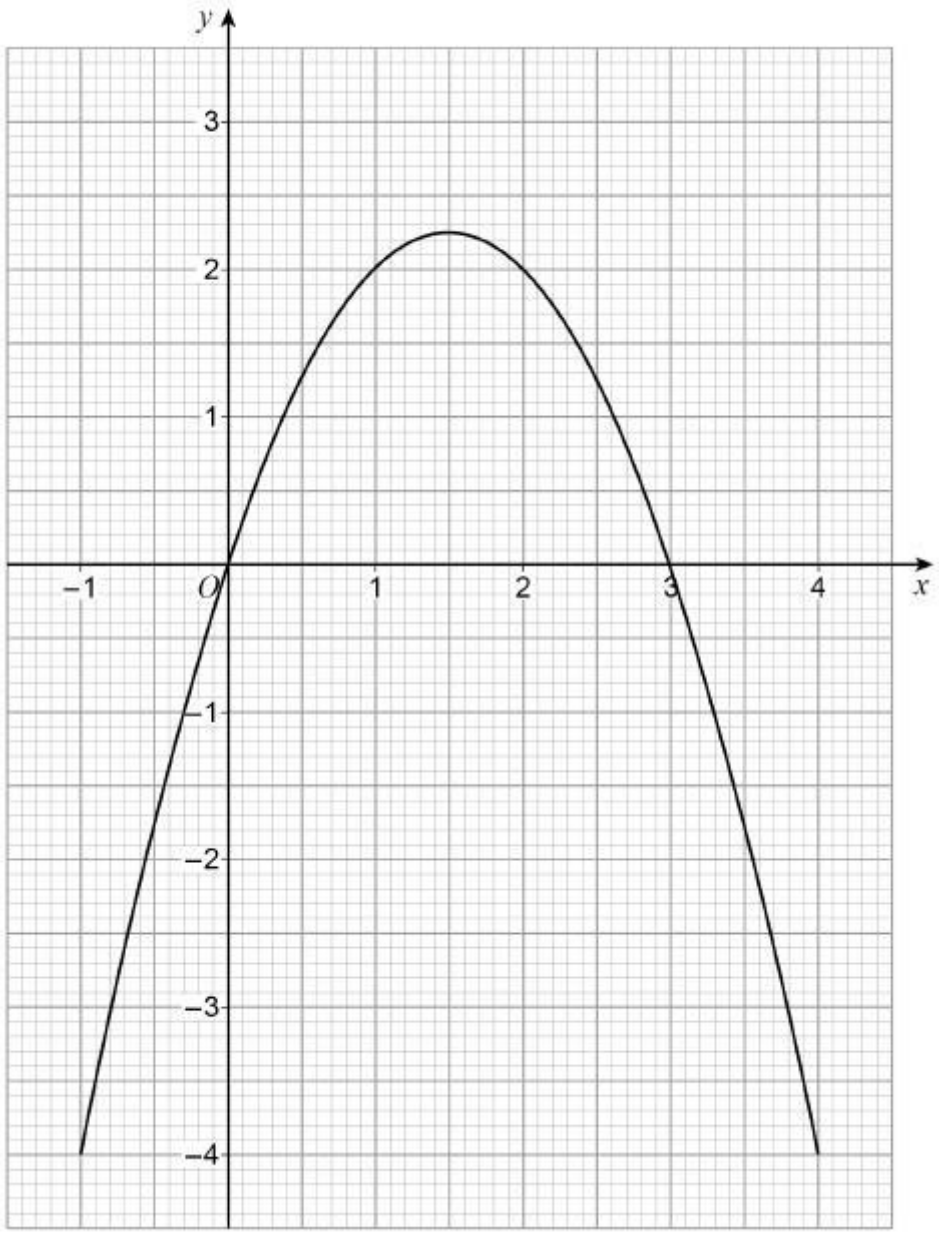
(a) Sketch the line $y = \frac{1}{2}(x - 3)$ on the diagram.
Mark the value where this line crosses the y-axis. **(2 marks)**

(b) By factorising $x^2 - 3x$, or otherwise, work out the smaller solution of

$x^2 - 3x = \frac{1}{2}(x - 3)$ **(2 marks)**

Q4.

Here is the graph of $y = 3x - x^2$ for values of x from −1 to 4



By drawing a suitable **linear** graph on the grid, work out approximate solutions to

$x^2 - 4x + 2 = 0$ **(4 marks)**

Q5. Solve the simultaneous equations

$$x - y = \frac{19}{4}$$

$$xy = -3$$

Do **not** use trial and improvement. You **must** show your working. **(6 marks)**

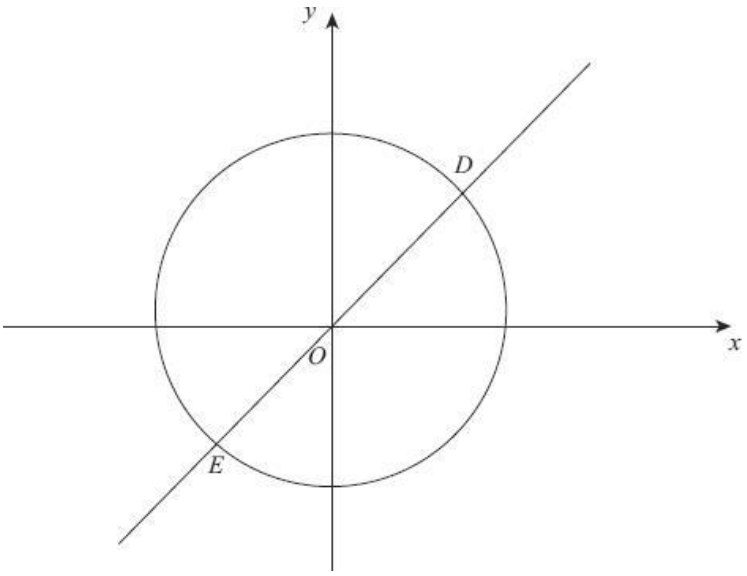
Q6. Solve the simultaneous equations

$$10x^2 + 5xy - 7y^2 + 23 = 0$$
$$x - y = 2$$

Do **not** use trial and improvement. You **must** show your working. **(5 marks)**

Q7. The circle $x^2 + y^2 = 20$ and the line $y = 2x$ intersect at points D and E.

Not drawn accurately



Work out the coordinates of D and E. You **must** show your working. **(5 marks)**

Q8. Solve the simultaneous equations $\frac{x-1}{y-2} = 3$ $\frac{x+6}{y-1} = 4$

You **must** show your working. **(5 marks)**

Q9. Solve the simultaneous equations

$xy = 2$ and $y = 3x + 5$ **(6 marks)**

Q10. Solve the simultaneous equations. Do not use trial & improvement.

$x + y = 4$

$y^2 = 4x + 5$ **(6 marks)**

Q11. The equation of a circle is $(x - 2)^2 + (y - 1)^2 = 16$

The equation of a line is $y = 2x + 1$

The circle and the line intersect at two points.

Work out the coordinates of the two points. **(5 marks)**

Section 2.16

Q1. Solve the simultaneous equations

$a + 3b - 2c = 4$

$4a - 3b + 5c = -5$

$2a + b + 3c = 9$

You **must** show your working. **(5 marks)**

Q2. Solve the simultaneous equations.

$4a - b + 3c = 27$

$3a + 2b - c = 5$

$2a - 5c = -7$

You **must** show your working. **(5 marks)**

Q3. Solve the simultaneous equations

$2a + b - c = 8$

$4a - 3b - 2c = -9$

$6a + 3b + c = 0$ **(5 marks)**

Section 2.17

Q1. Work out all the **negative** integer values of x for which $3x^2 < 48$ **(3 marks)**

Q2. Work out the range of values of x for which $x^2 - 11x + 28 > 0$

You **must** show your working. **(3 marks)**

Q3. Work out the integer values of x for which $x^2 - 20x + 96 < 0$ **(3 marks)**

Q4. Solve $2x^2 + 4 > (2x - 3)(x + 1)$ **(3 marks)**

Q5. Work out the **smallest** integer value of x that satisfies the inequality $8 - 5x < 26$

(2 marks)

Q6. $-11 < 5x \leq 5$ and $6x + 7 \leq 4x + 4$

Show that there is **exactly** one integer that x can be. **(5 marks)**

Q7.

w is an integer such that $6 \leq 3w < 18$

x is an integer such that $-4 \leq x \leq 3$

(a) Work out **all** the possible integer values of w. **(3 marks)**

(b) Write down the **highest** possible value of x^2 **(1 mark)**

(c) Work out the **lowest** possible value of $w - x$ **(2 marks)**

Q8. $a^2 < 4$ and $a + 2b = 8$

Work out the range of possible values of b. Give your answer as an inequality. **(4 marks)**

Section 2.18

Q1. $w^3x^2y^5 = w^{13}x^7$

Write y in terms of w and x. Give your answer in its simplest form. **(2 marks)**

Q2. Simplify fully $\left(\frac{2}{3}x^3y\right)^3$ **(2 marks)**

Q3. $p^{-2} = q^6 \times r^4$

Write p in terms of q and r. Give your answer in its simplest form. **(2 marks)**

Q4. $(c^5)^p = (c^2)^6$ Work out the value of p. **(2 marks)**

Q5. $y = \dfrac{a^{\frac{3}{4}} \times a^{\frac{7}{12}}}{\sqrt{a}}$ Show that y^6 can be written in the form a^k where k is an integer. **(3 marks)**

Q6. Simplify fully $\dfrac{(4cd^2)^3}{2cd^4}$ **(3 marks)**

Q7. Simplify fully $(6x^3y^{-2} + 9x^5y) \div 3x^2y^{-3}$ **(3 marks)**

Q8. Solve $y^{-3} = 125$ **(2 marks)**

Q9. Solve $x^{-\frac{2}{3}} = 7\frac{1}{9}$ Write your answer as a proper fraction. **(5 marks)**

Q10. Solve $\sqrt{(33 + \sqrt{x})} = 6$ **(3 marks)**

Q11. Work out the values of a when $2^{a^2} = 8^a \times 16$

Do **not** use trial and improvement. You **must** show your working. **(4 marks)**

Q12. Simplify $\sqrt{x^5 \times x^9}$ Give your answer in the form x^p where p is an integer. **(2 marks)**

Q13. Solve $(3-\sqrt{x})^{\frac{1}{3}} = -2$ **(3 marks)**

Q14. Write $\dfrac{15x^8 - 18x^7}{3x^2}$ in the form $ax^n - nx^a$ where a and n are integers. **(2 marks)**

Q15. Solve $\dfrac{56}{\sqrt[3]{x}} = 4$ **(2 marks)**

Q16. Work out the value of p when $9^{0.5p} \times 81 = 27^{2p} - 1$ **(4 marks)**

Q17. Work out the value of $\left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)^2$ You **must** show your working. **(3 marks)**

Q18. Solve $\sqrt[3]{\left(2\sqrt{x} - 10\right)} = 2$ **(3 marks)**

Q19. By multiplying both sides of the equation by $x^{\frac{1}{2}}$

Solve $2x^{\frac{3}{2}} - 3x^{\frac{1}{2}} = 7x^{-\frac{1}{2}}$ for $x > 0$ Give your answer to 3 s.f. **(4 marks)**

Q20. Simplify fully $\left(\dfrac{x}{2} + \dfrac{3x}{5}\right) \div \sqrt{\dfrac{x^6}{4}}$ **(5 marks)**

Q21. By multiplying throughout by $x^{\frac{1}{3}}$, or otherwise, solve $x^{\frac{2}{3}} + x^{-\frac{1}{3}} = 6x^{\frac{5}{3}}$ **(3 marks)**

Q22. Using powers of 2 or otherwise, work out the non-zero value of x for which $\left(16^x\right)^x = \dfrac{1}{2^{3x}}$

You **must** show your working. **(4 marks)**

Q23. a is a value greater than 1
Work out the value of m for which $(a^m)^4 = (a^5)^{2m}$ **(2 marks)**

Section 2.19

Q1. Prove that $(5n + 3)(n - 1) + n(n + 2)$ is a multiple of 3 for all integer values of n. **(4 marks)**

Q2. Prove algebraically that when n is an integer $\frac{(2n+1)^2-(2n-1)^2}{4}$ is always even. **(3 marks)**

Q3. Show that $(2n + 3)^3 + n^3$ is divisible by 9 for all integer values of n. **(4 marks)**

Q4. The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$
A new sequence is formed by squaring each term of the linear sequence and adding 1.
Prove algebraically that **all** the terms in the new sequence are multiples of 5. **(4 marks)**

Q5. Use algebra to prove that the value of $\frac{8c^2+16}{3c^2+6}+\frac{1}{3}$ is an integer for all values of c. **(3 marks)**

Q6. Prove that $(3x + 5)^2 - 5x(x + 10) \geq 0$ for all values of x. **(4 marks)**

Q7. $A = 2 - 5x$ $B = 3x - 1$ $C = x^2$ Show that $(2A + 3B)^2 \equiv A + B + C$ **(4 marks)**

Section 2.20 – 2.21

Q1. The nth term of a sequence is $\frac{3n^2}{n^2+2}$

(a) One term in the sequence is $\frac{32}{11}$ Work out the value of n. **(2 marks)**

(b) Write down the limiting value of the sequence as $n \to \infty$ **(1 mark)**

Q2. The nth term of a sequence is $\frac{1420-5n}{1420+5n}$

(a) Work out the **position** of the term that has the value zero. **(2 marks)**

(b) Write down the limiting value of the sequence as $n \to \infty$ **(1 mark)**

Q3. The nth term of a sequence is $\frac{2n^2+7}{3n^2-2}$

(a) Work out the 7th term. Give your answer as a fraction in its simplest form. **(2 marks)**

(b) Show that the limiting value of $\frac{2n^2+7}{3n^2-2}$ as $n \longrightarrow \infty$ is $\frac{2}{3}$. **(2 marks)**

Q4. The nth term of a sequence is T_n

$$T_n = \frac{32n}{3n-7}$$

(a) Work out the largest value of n for which $T_n > 11$ **(3 marks)**

(b) Write down the limiting value of T_n as $n \longrightarrow \infty$ **(1 mark)**

Q5. The nth term of a sequence is $n^2 - 6n + 14$

By completing the square, or otherwise, show that every term is positive. **(3 marks)**

Q6. The first term of a sequence is $1 - a$

The term-to-term rule of a sequence is add $2a$ then multiply by 3

(a) Show that the second term is $3 + 3a$ **(1 mark)**

(b) The third term is 16 Work out the value of a. **(3 marks)**

Q7. A linear sequence has first term $7 + 12\sqrt{5}$

The term-to-term rule is add $9 - 2\sqrt{5}$

One term of the sequence is an integer. Work out the value of this integer. **(2 marks)**

Q8. Here are the first four terms of a sequence. $4a$ $9a$ $14a$ $19a$

The nth term of the sequence is $\frac{10n-2}{3}$ Work out the value of a. **(2 marks)**

Q9. For sequence A, nth term = $\frac{n}{14n+30}$ For sequence B, nth term = $\frac{2}{n}$

The kth term of sequence A equals the kth term of sequence B.

Work out the value of k. You **must** show your working. **(4 marks)**

Q10. The first three terms of a linear sequence are

30 $\quad 30 + 4k \quad 30 + 8k$ where k is a constant.

(a) Work out an expression, in terms of k, for the 4th term.
Give your answer in its simplest form. **(1 mark)**

(b) The 100th term of the sequence is 525
Work out the value of k. **(3 marks)**

Section 2.22

Q1. The first four terms of a quadratic sequence are 10 33 64 103 ...
Work out an expression for the *n*th term. **(4 marks)**

Q2. The first four terms of a quadratic sequence are 0 1 0 –3
Work out an expression for the *n*th term. **(3 marks)**

Q3. A quadratic sequence starts –2 –1 4 13

(a) Work out an expression for the *n*th term. **(3 marks)**

(b) A different quadratic sequence has *n*th term $n2 + 10n$
Use an algebraic method to work out how many terms in the sequence are less than 2000
Do **not** use trial and improvement. You **must** show your working. **(3 marks)**

Q4. A quadratic sequence starts 302 600 894 1184 .....

(a) Work out an expression for the n^{th} term. **(3 marks)**

(b) A term in the sequence has value 0
Find the position of this term. **(2 marks)**