2 ALGEBRA – Further Maths

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Mark schemes

Q1.

	Answer	Mark	Comments
(a)	9	B1	
(b)	$f(x) \geq 7$	B1	Allow $y \geq 7$

Q2.

Answer	Mark	Comments
$-4 \leqslant g(x) < 5$	B2	oe eg $5 > g(x) \geqslant -4$

or $g(x) < 5$ and $g(x) \geqslant -4$		word ‘and’ must be included if writing two inequalities for B2 or B1 or SC1 B1 $-4 < g(x) < 5$ or $-4 < g(x) \leqslant 5$ or $-4 \leqslant g(x) \leqslant 5$ or $g(x) < 5$ and $g(x) > -4$ or $g(x) \leqslant 5$ and $g(x) > -4$ or $g(x) \leqslant 5$ and $g(x) \geqslant -4$ or $k < g(x) < 5$ where k is less than 5 or $k \leqslant g(x) < 5$ where k is less than 5 or $-4 \leqslant g(x) < m$ where m is greater than -4 SC1 $-4 \leqslant x < 5$ or $x < 5$ and $x \geqslant -4$ or only -4 and 5 seen (condone 9 given as a range in this case)

Additional Guidance	
Condone $g(x)$ replaced by eg y or g or gx or f or fx or G or Gx or $x^2 - 4$ eg1 $-4 \leqslant f(x) < 5$ eg2 $-4 \leqslant f(x) < 5$	 B2 B1
$[-4, 5)$	B2
$(-4, 5)$ or $(-4, 5]$ or $[-4, 5]$	B1
Condone eg $g(x) = -4 \leqslant g(x) < 5$	B2
Condone eg $g(x) = -4 < g(x) < 5$	B1
B2 response with a list of integers on answer line	B1
B1 response with a list of integers on answer line	B0
Only a list of integers	B0

Q3.

	Answer	Mark	Comments
(a)	−6	B1	
(b)	f(x) ≤ 10 or 10 ≥ f(x)	B1	Condone y ≤ 10 or 10 ≥ y
(c)	6a = 24 (so a = 4)	B1	B1 for 2a × 3 = 24 B1 for 24 = (0 + 8)(0 + 3) 8 × 3 = 24… on its own … is B0
(d)	$10 - x^2 = (x + 8)(x + 3)$ or $10 - x^2 = x^2 + 2ax + 3x + 6a$	M1	oe
	$2x^2 + 11x + 14$ (= 0)	M1dep	oe allow one error
	$(2x + c)(x + d)$ (= 0)	M1dep	cd = 14 or $c + 2d$ = 11 ft from their quadratic (factorising or correct substitution in quadratic formula)
	−3.5 and −2	A1	oe

Q4.

Answer	Mark	Comments
Identifies (1, 3) **or** (5, 11)	B1	May be implied by M1 or seen in a table of values or on a graph or as a mapping (eg 1→ 3)
$\frac{\text{their } 11 - \text{their } 3}{\text{their } 5 - \text{their } 1}$ (= 2)	M1	oe
y − their 3 = their 2(x − their 1) **or** y − their 11 = their 2(x − their 5)	M1	y = their 2x + c **and** substitutes their (1, 3) or their (5, 11)
(y =) 2x + 1	A1	

Alternative method 1		
Identifies (1, 11) **or** (5, 3)	B1	May be implied by M1 or seen in a table of values or on a graph or as a mapping (eg 3→ 1)

$\frac{\text{their } 11 - \text{their } 3}{\text{their } 1 - \text{their } 5}$ (= −2)	M1	oe
y − their 11 = their −2(x − their 1) **or** y − their 3 = their −2(x − their 5)	M1	y = their − 2x + c **and** substitutes their (1, 11) or their (5, 3)
(y =) − 2x + 13	A1	

Alternative method 2		
$m + c = 3$ **or** $5m + c = 11$	B1	$m + c = 11$ **or** $5m + c = 3$
Eliminates a letter from their 2 equations eg $5m - m = 11 - 3$	M1	Eliminates a letter from their 2 equations eg $5m - m = 3 - 11$
$m = 2$ **or** $c = 1$	A1	$m = -2$ **or** $c = 13$
(y =) 2x + 1	A1	(y =) − 2x + 13

Q5.

Answer	**Mark**	**Comments**
$5x - 3 < 1$ or $-2 < 5x - 3$ or $-2 < 5x - 3 < 1$	M1	oe eg $x < \frac{4}{5}$ or $\frac{1}{5} < x$ or $1 < 5x < 4$
$\frac{1}{5} < x < \frac{4}{5}$ or $0.2 < x < 0.8$	A1	oe SC1 $\frac{1}{5} < h(x) < \frac{4}{5}$ (condone absence of (x) or absence of brackets) or $\frac{1}{5} < y < \frac{4}{5}$ or $\frac{1}{5} \le x \le \frac{4}{5}$

Additional Guidance	
Both inequalities $x < \frac{4}{5}$ and $\frac{1}{5} < x$ given as their answer	M1 A1

M1 Must use correct inequality symbol unless recovered in the A mark $5x - 3 \le 1$ or $5x - 3 > 1$ (answer not correct)	M0 A0
M1 If using equations award M0 unless recovered in the A mark $5x - 3 = 1$ $5x - 3 = -2$ $0.2 < x < 0.8$	M1 A1

Q6.

Answer	Mark	Comments
$f(x) \ge 16$ or $y \ge 16$	B1	Condone absence of (x) or absence of brackets

Additional Guidance	
$x \ge 16$	B0
$f(x) > 16$ or $f(x) \le 16$ or $f(x) < 16$	B0
16	B0

Q7.

	Answer	Mark	Comments
(a)	$x \geqslant \frac{5}{2}$	B1	
(b)	1. $2^2 = 2x - 5$ **or** $1.44 = 2x - 5$	M1	oe
	(x =) 3.22	A1	oe eg $\frac{161}{50}$
(c)	$\sqrt{5\frac{1}{4} - 5}$ **or** $\sqrt{\frac{21}{4} - 5}$	M1	oe $\sqrt{\frac{2(21)}{8} - 5}$ $\sqrt{\frac{42}{8} - 5}$ $\sqrt{2\left(2\frac{5}{8}\right) - 5}$ $\sqrt{5.25 - 5}$ $\sqrt{2(2.625) - 5}$
	$\sqrt{\frac{1}{4}}$ **or** $\sqrt{(0.25)}$	A1	oe

$\frac{1}{2}$ **or** 0.5	A1	Condone $\pm\frac{1}{2}$ but not $-\frac{1}{2}$ on its own

Additional Guidance
Condone decimals throughout An answer of $\frac{\sqrt{1}}{2}$ is M1 M1 A0

Q8.

(a)

Answer	Mark	Comments
$f(x) \geq -7$ or $-7 \leq f(x)$	B1	

Additional Guidance	
$f(x)$ may be replaced by y or f or fx or $g(x)$ or g or gx or $x^2 - 7$	
$x \geq -7$	B0
≥ -7	B0
Condone $-7 \leq f(x) < \infty$ or $-7 \leq f(x) \leq \infty$ or $-7 \leq f(x) <$ or $-7 \leq f(x) \leq$	B1
$[-7, \infty)$ or $[-7, \infty]$	B0

(b)

$-11 \leq g(x) \leq 13$ or $13 \geq g(x) \geq -11$	B2	B1 $g(x) \geq -11$ or $g(x) \leq 13$ on their own or embedded within an inequality or $-11 < g(x) < 13$ or $[-11, 13]$ or $-11 \leq x \leq 13$

Additional Guidance	
$g(x)$ may be replaced by y or g or gx or $f(x)$ or f or fx or $1 - 3x$ in B2 or B1 responses	
$g(x) \geq -11$ $g(x) \leq 13$	B1

−11 to 13 inclusive (‘inclusive’ must be seen) Do not allow if 24 also seen	B1
B1 may be seen with an incorrect inequality eg1 $-11 < g(x) \le 13$ eg2 $-11 \le g(x) < 13$ eg3 $0 < g(x) \le 13$ eg4 $13 \le g(x) \ge -11$	B1 B1 B1 B1
[−11, 13) or (−11, 13] or (−11, 13)	B0
$-11 < x \le 13$ or $-11 \le x < 13$ or $-11 < x < 13$	B0
{−11, −10, −9, ... 0, 1, 2, 3, ..., 12, 13}	B0

(c)

$2x^2 - 14$	M1	
$2x^2 + 3x - 15$ (= 0) or $-2x^2 - 3x + 15$ (= 0) or $2x^2 + 3x = 15$ or $-2x^2 - 3x = -15$	A1	
$\dfrac{-3 \pm \sqrt{3^2 - 4 \times 2 \times -15}}{2 \times 2}$ or $\dfrac{-3 \pm \sqrt{9 + 120}}{4}$ or $\dfrac{-3 \pm \sqrt{129}}{4}$	M1	oe eg $-\dfrac{3}{4} \pm \sqrt{\dfrac{15}{2} + \left(\dfrac{3}{4}\right)^2}$ correct method to solve their 3-term quadratic implied by correct solutions to their 3-term quadratic to at least 2 dp
2.089 −3.589	A1ft	correct or ft M1A0M1 or M0A0M1 must both be rounded to 3 decimal places

Additional Guidance	
2nd M1 Allow correct factorisation of their 3-term quadratic if it does factorise	
2nd M1 Allow correct use of formula even if discriminant is negative	
Two ‘correct’ solutions to at least 2 decimal places implies M1A1M1	M1A1 M1A0

eg 2.09 and −3.59	
2.089 and −3.589 in working but only one on answer line	M1A1 M1A0
Answers only 2.089 −3.589	M1A1 M1A1
Answer only 2.089	Zero
Answer only −3.589	Zero
$2x^2 - 7$ from incorrect expansion leading to 1.386 −2.886	M0A0 M1A1ft
$x^2 - 14$ from incorrect expansion leading to 2.653 −5.653	M0A0 M1A1ft
$2x^2 - 14$ and $2x^2 + 3x - 13$ (= 0) Answers 1.908 −3.408	M1A0 M1A1ft

Q9.

Answer	Mark	Comments
$1 \le g(x) \le 5$ or $5 \ge g(x) \ge 1$	B2	B1 $1 \le g(x) < 5$ or $1 < g(x) \le 5$ or $1 < g(x) < 5$ or $g(x) \ge 1$ and $g(x) \le 5$ or $1 \le g(x) \le k$ where k is a constant > 1 or $p \le g(x) \le 5$ where p is a constant < 5 SC1 $1 \le x \le 5$

Additional Guidance	
Condone $g(x)$ replaced by eg y or g or gx or $f(x)$ or f or fx or $5 - x^2$ in B2 or B1 responses	
Equivalent inequalities may be seen eg $5 \ge g(x) > 1$	B1

Only $g(x) \geq 1$ given as the answer	B0
Only $g(x) \leq 5$ given as the answer	B0
$1 \leq g(x) \leq 4$	B1
$1 \leq g(x) < 4$	B0
$0 \leq g(x) \leq 5$	B1
$0 < g(x) \leq 5$	B0
Invalid statements do not score eg1 $1 \leq g(x) \geq 5$ eg2 $1 \geq g(x) \leq 5$ eg3 $6 \leq g(x) \leq 5$	B0 B0 B0
[1, 5]	B1
[1, 5) or (1, 5] or (1, 5) or 1 − 5 or 5 − 1	B0
$1 \leq g(x) \leq 5$ in working with list of integers on answer line	B1
Only a list of integers	B0

Q10.

Answer	Mark	Comments
Alternative method 1		
$(x + 3)^2$….	M1	
$(x + 3)^2 - 3^2 - a$ or $(x + 3)^2 - 3^2 \geq a$ or $(x + 3)^2 \geq a + 3^2$	M1dep	oe expression or inequality eg $(x + 3)^2 \geq 9 + a$ allow ≥ to be any inequality symbol or = eg allow $(x + 3)^2 - 9 = a$ implies M2
$-3^2 - a \geq 0$ or $-3^2 - a > 0$	M1dep	oe inequality eg $-9 - a \geq 0$ or $-9 - a > 0$ or $a < -9$ implies M3
$a \leq -9$ or $-9 \geq a$	A1	SC1 $x^2 + 6x - a \geq 0$ oe inequality

		(may be seen in working lines)

Alternative method 2		
$2x + 6 = 0$	M1	must have = 0
(minimum at) $x = -3$	M1dep	implies M2 $x = -3$ must be the only value or be clearly chosen
$(-3)^2 + 6 \times (-3) - a \geq 0$ or $(-3)^2 + 6 \times (-3) - a > 0$	M1dep	oe inequality eg $9 - 18 - a \geq 0$ or $9 - 18 - a > 0$ or $a < -9$ implies M3
$a \leq -9$ or $-9 \geq a$	A1	SC1 $x^2 + 6x - a \geq 0$ oe inequality (may be seen in working lines)

Alternative method 3		
$6^2 - 4 \times 1 \times -a$	M1	$b2 - 4ac$ must be selected if seen in quadratic formula
$6^2 - 4 \times 1 \times -a \leq 0$ or $6^2 - 4 \times 1 \times -a < 0$	M1dep	oe inequality implies M2
$36 + 4a \leq 0$ or $36 + 4a < 0$	M1dep	oe inequality eg $4a \leq -36$ implies M3
$a \leq -9$ or $-9 \geq a$	A1	SC1 $x^2 + 6x - a \geq 0$ oe inequality (may be seen in working lines)

Additional Guidance
Alt 1 2nd M1 Any inequality symbol or = allowed 3rd M1 Only the inequality symbols shown are allowed (do not allow =)
Allow $(x + 3)(x + 3)$ for $(x + 3)^2$

Q11.

Answer	Mark	Comments
$\frac{9}{2} \times \frac{1}{3}$ or $\frac{3}{2}$ or $\frac{2}{9x}$	M1	oe
$\frac{2}{3}$	A1	
their $\frac{2}{3} = \sqrt{1 - p \times \left(\frac{1}{3}\right)^3}$	M1dep	oe
$\left(\text{their}\,\frac{2}{3}\right)^2 = 1 - p \times \left(\frac{1}{3}\right)^3$	M1dep	oe
15	A1	

Q12.

Answer	Mark	Comments
$f(x) \leq 25$ or $25 \geq f(x)$	B2	B1 $f(x) < 25$ or $k \leq f(x) \leq 25$ or $k < f(x) \leq 25$ where k is any number < 25 SC1 ≤ 25 or $x \leq 25$

Additional Guidance	
Condone f(x) replaced by eg y or f or fx or F(x) or F or Fx or $x^3 - 2$ in B2 or B1 responses	
Equivalent inequalities may be seen $25 > f(x)$	B1
Allow $-\infty < f(x) \leq 25$	B2
Condone $-\infty \leq f(x) \leq 25$	B2
$-\infty < f(x) < 25$ or $-\infty \leq f(x) < 25$	B1
$[-\infty, 25]$ or $(-\infty, 25]$	B1

(−∞, 25)	B0
Condone f(x) = ≤ 25	B2
Condone f(x) = < 25	B1
Condone f(x) = x ≤ 25	SC1
f(x) ≤ 25 in working with list of integers on answer line	B1
Only a list of integers	B0

Q13.

(a)

Answer	Mark	Comments
$3 \times 4^2 + 6$ or $3 \times 16 + 6$ or 54 or $\sqrt{3x^2+6-5}$ or $\sqrt{3x^2+1}$	M1	oe
7	A1	

(b)

$3(x - 5) + 6$	M1	oe
$3x - 9 = 3(x - 3)$	A1	

Q14.

Answer	Mark	Comments
$2x^2 + 10$ or $2(x^2 + 5)$	B2	B1 $k(x) = 2x$ or $(k(x))^2 = 4x^2$ or $h(2x) = 4x^2 + 5$ or $(2x)^2 + 5$

Additional Guidance	
$2(x^2 + 5)$ in working with answer $2x^2 + 5$	B1

Q15.

Answer	Mark	Comments
$15x(x – 4)$	M1	oe
$10x(2x – 4)$	M1	oe

$15x^2 - 60x = 20x^2 - 40x$ or $5x^2 + 20x = 0$	M1dep	oe brackets expanded dep on M2
0 and –4	A1	

Q16.

Answer	Mark	Comments
$\frac{5}{6}$	B1	

Q17.

Answer	Mark	Comments
$\frac{6}{x-5}$	B1	
$6 = x(x - 5)$	M1	oe eg $x^2 - 5x - 6$ (= 0) ft their $\frac{6}{x-5}$ = x with fractions eliminated
$(x + 1)(x - 6)$ or $\frac{--5 \pm \sqrt{(-5)^2 - 4 \times 1 \times -6}}{2 \times 1}$ or $\frac{5}{2} \pm \sqrt{\frac{49}{4}}$	M1	oe correct factorisation or correct formula for their 3-term quadratic
–1 and 6	A1	

Additional Guidance	
$\frac{6}{x} - 5 = x$	B0
$6 - 5x = x^2$ $x^2 + 5x - 6 = 0$	M1

$(x + 6)(x - 1)$ –6 and 1	M1 A0

Q18.

Answer	Mark	Comments
$\sqrt[3]{x}\ldots$ or $\sqrt[3]{-8}$	M1	oe eg $\sqrt[3]{y}\ldots$
–6	A1	

Q19.

Answer	Mark	Comments
x^4	B1	

Q20.

Answer	Mark	Comments
Alternative method 1		
$x = 2\text{h}(x) - 3$ or $x = 2y - 3$	M1	oe
$2x - 3$	A1	
Alternative method 2		
$x = \dfrac{3 + h^{-1}(x)}{2}$ or $x = \dfrac{3 + y}{2}$	M1	oe
$2x - 3$	A1	

Additional Guidance	
Answer left as $y = 2x - 3$	M1A0

Section 2.6

Mark schemes

Q1.

Answer	Mark	Comments
$15x^2 - 12x + 5ax - 4a$ or $5ax - 12x = -2x$ or $5a - 12 = -2$ or $b = -4a$	M1	oe
$(a =)$ 2	A1	
$(b =)$ –4 × their a	A1ft	–8, but do not award -8 unless it comes from $a = 2$

Additional Guidance	
Candidates who use substitutions for x are likely to use $x = 0$ and gain M1. Award M1 for any number substituted in correctly to gain an equation in a and b	

Q2.

Answer	Mark	Comments
$3w^2 + 2wy - 12wy - 8y^2$	M1	oe 4 terms with 3 correct Terms may be seen in a grid May be implied eg1 $3w^2 - 10wy + 8y^2$ eg2 $w^2 - 10wy - 8y^2$
$3w^2 + 2wy - 12wy - 8y^2$	A1	Fully correct Do not allow if only seen in a grid
$3w^2 - 10wy - 8y^2$	A1ft	ft M1 A0

Additional Guidance	
Accept yw for wy throughout	
A correct term must include a − sign if it is negative	
$3w^2 + 2wy - 12wy - 8y$ $3w^2 - 10wy - 8y$	M1 A0 A1ft

$3w^2 + 2wy + 12wy - 8y^2$ $3w^2 + 14wy - 8y$ (does not ft from previous line)	M1 A0 A0ft
$3w - 10wy - 8y^2$ (implied M1 and A1ft as terms collected)	M1 A0 A1ft
$3w^2 + 2wy - 12wy - 8wy$ $3w^2 - 18wy$	M1 A0 A1ft
$3w^2 + 10wy - 8y^2$	M0 A0 A0ft
Penalise the 2nd A1 if further work seen $3w^2 - 10wy - 8y^2 = 3w^2 - 18wy^2$	M1 A1 A0ft

Q3.

Answer	Mark	Comments
$2y^3 - 10y^2 + 4y - 3y^2 + 15y - 6$	M1	Must have at least five terms with at least four correct
$2y^3 - 10y^2 + 4y - 3y^2 + 15y - 6$	A1	
$2y^3 - 13y^2 + 19y - 6$	A1ft	ft from M1 A0

Q4.

Answer	Mark	Comments
$\frac{3x}{3x^2}$ or $\frac{9x^2}{x^2}$ or $(-)\frac{3}{x^2}$	M1	oe eg1 $\frac{3 \times x}{x^2 \times 3}$ eg2 9 One correct product, unsimplified or simplified

$\frac{3x}{3x^2} + \frac{9x^2}{x^2} - \frac{3}{x^2}$ or $\frac{1}{x} + \frac{9x^2}{x^2} - 3x^{-2}$ or $\frac{3x + 27x^2}{3x^2} - \frac{3}{x^2}$ or $\frac{x}{x^2} + \frac{9x^2 - 3}{x^2}$ or $\frac{9x^2}{x^2} + \frac{3(x-3)}{3x^2}$ or $\frac{3x + 27x^2 - 9}{3x^2}$	A1	oe Fully correct expansion of given expression that requires further simplification Multiplication signs not allowed unless recovered eg $\frac{3 \times x}{x^2 \times 3} + \frac{9x^2}{x^2} - \frac{3}{x^2}$ M1 A0
$\frac{1}{x} + 9 - \frac{3}{x^2}$ or $x^{-1} + 9 - 3x^{-2}$ or $\frac{1}{x} + \frac{9x^2 - 3}{x^2}$ or $x^{-1} + \frac{9x^2 - 3}{x^2}$ or $\frac{x-3}{x^2} + 9$ or $\frac{1+9x}{x} - \frac{3}{x^2}$ or $\frac{x + 9x^2 - 3}{x^2}$	A1	oe Any of these answers implies M1 A1 A1 Do not allow $\frac{9}{1}$ for 9 Multiplication signs or brackets that require expansion not allowed unless recovered After M1 A1 A1 penalise further work eg $\frac{x + 9x^2 - 3}{x^2}$ followed by $\frac{3x + 27x^2 - 9}{3x^2}$ M1 A1 A0

Additional Guidance
3 mark responses with fractions must have fractions in their simplest form

Q5.

Answer	Mark	Comments
Alternative method 1		
$2x^2 - 4ax + 2a^2$ (+ 3)	M1	or $2(x^2 - 2ax + a^2)$ (+ 3) allow one error

$2a^2 + 3 = 7a$ or $2a^2 - 7a + 3 = 0$	M1	oe for equating constant terms
$(2a - 1)(a - 3)$ (= 0)	A1	
$a = \frac{1}{2}$ and $a = 3$	A1	
$-2bx = -4ax$ or $2b = 4a$ or $b = 2a$	M1	oe for equating x terms
$b = 1$ when $a = \frac{1}{2}$ and $b = 6$ when $a = 3$	A1ft	ft their a values
Alternative method 2		
when $x = 0$ $7a = 2a^2 + 3$ or $2a^2 - 7a + 3 = 0$	M1	oe
$(2a - 1)(a - 3)$ (= 0)	A1	
$a = \frac{1}{2}$ and $a = 3$	A1	
when $x = 1$ $2 - 2b + 7a = 2(1 - a)^2 + 3$ or $2b = 7a - 1 - 2(1 - a)^2$	M1	oe
substituting $a = \frac{1}{2}$ and $a = 3$ in the expression for $2b$ (or b)	M1	
$b = 1$ when $a = \frac{1}{2}$ and $b = 6$ when $a = 3$	A1ft	ft their a values

Alternative method 3		
when $x = 0$ $7a = 2a^2 + 3$ or $2a^2 - 7a + 3 = 0$	M1	oe
$(2a - 1)(a - 3)$ (= 0)	A1	
$a = \frac{1}{2}$ and $a = 3$	A1	

Correctly substitute a second value of x into the identity	M1	eg if $x = 2$, $8 - 4b + 7a = 2(2 - a)^2 + 3$
Correctly substitute a third value of x into the identity	M1	eg if $x = 3$, $18 - 6b + 7a = 2(3 - a)^2 + 3$
$b = 1$ when $a = \frac{1}{2}$ and $b = 6$ when $a = 3$	A1ft	ft their a values
Alternative method 4		
$2[(x - b/2)^2 - b^2/4 + 7a/2]$ or $2(x - b/2)^2 - b^2/2 + 7a$	M1	
$a = b/2$ or $3 = -b^2/2 + 7a$	M1	
$2a^2 - 7a + 3 = 0$ or $b^2 - 7b + 6 = 0$	M1	oe
$(2a - 1)(a - 3)$ (= 0) or $(b - 1)(b - 6)$ (= 0)	A1	
$a = \frac{1}{2}$ and $a = 3$ or $b = 1$ and $b = 6$	A1	
$b = 1$ when $a = \frac{1}{2}$ and $b = 6$ when $a = 3$	A1ft	ft from the values they calculate first

Q6.

Answer	Mark	Comments
Alternative method 1		
$a = -5$	B1	
$b = 25 - a$ or $x^2 - 10x + 25$ seen or $x^2 - 5x - 5x + 25$ seen	M1	
$b = 30$	A1ft	ft using $b = 25 - a$ if M1 earned

Alternative method 2		
$a = -5$	B1	
$(x + a)^2 - a^2\ (+b)$ or $b - a^2 = -a$ or $b = a^2 - a$	M1	
$b = 30$	A1ft	ft using b = their $(a^2 - a)$

Alternative method 3		
$a = -5$	B1	
Substituting one value of x into the identity, correctly, to give an equation connecting a and b	A1	eg $x = 0$, $a + b = 25$ $x = 1$, $3a + b = 15$ $x = 2$, $5a + b = 5$ $x = 3$, $7a + b = -5$
$b = 30$	A1	

Alternative method 4		
Substituting two values of x into the identity, correctly, to give two simultaneous equations	M1	eg $x = 0$, $a + b = 25$ $x = 1$, $3a + b = 15$ $x = 2$, $5a + b = 5$ $x = 3$, $7a + b = -5$
$a = -5$	A1	
$b = 30$	A1	

Q7.

Answer	Mark	Comments
Sight of ab^2 or cb^2 or ad^2 or or cd^2 or (3x......)(x ......)(x.......)	M1	
Two or three correct coefficients	A1	which may be embedded
$a = 3$, $b = 1$, $c = 2$, $d = 7$	A1	which may be embedded SC2 for $(3x - 2)(x^2 - 49)$

		SC1 for $(3x......)(x^2 - 49)$

Q8.

Answer	Mark	Comments
$(x^3 +)\ 4x^2 - kx^2 - 4kx - 5x\ (-20)$	M1	or $4 - k$ and $-4k - 5$ seen as coefficients
$4 - k = 2(-4k - 5)$	M1dep	ft their expansion if first M mark earned
$(k =) -2$	A1	

Additional Guidance
Condone one **sign** error in the first two steps Ignore errors in x^3 and -20 for the first M1

Q9.

Answer	Mark	Comments
$4x^2$ or $3px^2$ or $4 + 3p$	M1	May be seen in an expansion or a grid Allow unsimplified eg $3x \times px$
their $4(x^2)$ + their $3p(x^2) = -23(x^2)$	M1dep	Correct or ft their expansion ft is equating their terms in x^2 to $-23x^2$ Must be at least two terms with at least one linear term in p Allow unsimplified eg $3x \times px + 4x^2 = -23x^2$
–9	A1	

Additional Guidance	
In this question, only consider terms in x^2	
If only one term in x^2 the maximum mark is M1	
$4 + 3p = -23$ followed by $7p = -23$	M1 M1 A0

Q10.

Answer	Mark	Comments
Alternative method 1 expands $(x + 2)(x + 3)$ first		
$x^2 + 3x + 2x + 6$ or $x^2 + 5x + 6$	M1	oe must have a term in x^2 allow one error but no omissions or extras implied by $x^2 + 5x + k$ or $ax^2 + 5x + 6$
$x^3 + 5x^2 + 6x + 4x^2 + 20x + 24$	M1dep	oe eg $x^3 + 3x^2 + 2x^2 + 6x + 4x^2 + 12x + 8x + 24$ allow one further error but no omissions or extras
$x^3 + 9x^2 + 26x + 24$	A1	

Alternative method 2 expands $(x + 3)(x + 4)$ first		
$x^2 + 3x + 4x + 12$ or $x^2 + 7x + 12$	M1	oe must have a term in x^2 allow one error but no omissions or extras implied by $x^2 + 7x + k$ or $ax^2 + 7x + 12$
$x^3 + 7x^2 + 12x + 2x^2 + 14x + 24$	M1dep	oe eg $x^3 + 3x^2 + 4x^2 + 12x + 2x^2 + 6x + 8x + 24$ allow one further error but no omissions or extras
$x^3 + 9x^2 + 26x + 24$	A1	

Alternative method 3 expands $(x + 2)(x + 4)$ first		
$x^2 + 4x + 2x + 8$ or $x^2 + 6x + 8$	M1	oe must have a term in x^2 allow one error but no omissions

		or extras implied by $x^2 + 6x + k$ or $ax^2 + 6x + 8$
$x^3 + 6x^2 + 8x + 3x^2 + 18x + 24$	M1dep	oe eg $x^3 + 4x^2 + 2x^2 + 8x + 3x^2 + 12x + 6x + 24$ allow one further error but no omissions or extras
$x^3 + 9x^2 + 26x + 24$	A1	

Additional Guidance	
For M marks terms may be seen in a grid (+ signs not needed)	
Correct answer followed by further work	M2A0
Ignore further simplification after 4 terms seen eg Alt 1 $x^2 + 3x + 2x + 6 = x^2 + 6x + 6$ $(x^2 + 6x + 6)(x + 4) \longrightarrow x^3 + 4x^2 + 6x^2 + 24x + 6x + 18$ (error)	M1 M1depA0
Second M1 Must be the product of a two term bracket and a three or four term bracket	
Missing brackets may be recovered	

Q11.

Answer	Mark	Comments
Alternative method 1		
$2(2 - 5x) + 3(3x - 1)$ or $4 - 10x$ or $9x - 3$	M1	
$4 - 10x + 9x - 3 = 1 - x$	M1dep	
$(1 - x)^2 = 1 - 2x + x^2$	A1	must see working for M2
$2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$	B1	

Alternative method 2		
$4(2 - 5x)^2 + 6(2 - 5x)(3x - 1)$	M1	oe

$+ 6(2 - 5x)(3x - 1) + 9(3x - 1)^2$		allow $+ 12(2 - 5x)(3x - 1)$ for $+ 6(2 - 5x)(3x - 1) + 6(2 - 5x)(3x - 1)$
$4(4 - 10x - 10x + 25x^2)$ $+ 6(6x - 2 - 15x^2 + 5x)$ $+ 6(6x - 2 - 15x^2 + 5x)$ $+ 9(9x^2 - 3x - 3x + 1)$ $= 16 - 40x - 40x + 100x^2 + 36x - 12$ $- 90x^2 + 30x + 36x - 12 - 90x^2$ $+ 30x + 81x^2 - 27x - 27x + 9$	M1dep	oe must see expansions must see working for 1st M1 allow $+ 12(6x - 2 - 15x^2 + 5x)$ for $+ 6(6x - 2 - 15x^2 + 5x)$ $+ 6(6x - 2 - 15x^2 + 5x)$
$1 - 2x + x^2$	A1	must see working for M2
$2 - 5x + 3x - 1 + x2 = 1 - 2x + x^2$	B1	

Alternative method 3		
$2(2 - 5x) + 3(3x - 1)$ or $4 - 10x$ or $9x - 3$	M1	oe
$(4 - 10x + 9x - 3)^2$ $= 16 - 40x + 36x - 12 - 40x + 100x^2$ $- 90x^2 + 30x + 36x - 90x^2 + 81x^2$ $- 27x - 12 + 30x - 27x + 9$	M1dep	oe must see expansions
$1 - 2x + x^2$	A1	must see working for M2
$2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$	B1	

Additional Guidance	
Allow working down both sides of an equation/identity	
M2A1 is for working on $(2A + 3B)^2$	
B1 is for working on $A + B + C$	

$1 - 2x + x^2$ with working for M2 seen and $2 - 5x + 3x - 1 + x^2 = x^2 - 2x + 1$	4 marks
$1 - x^2 = 1 - 2x + x^2$ (do not allow missing brackets even if recovered)	

Q12.

Answer	Mark	Comments
Alternative method 1		
$px - p + 6x + 2k = 4x + 8$ or $px + 6x = 4x$ or $p + 6 = 4$	M1	oe
$p = -2$	A1	This could imply first M mark if not seen
$2k -$ their $p = 8$ or $2k =$ their $p + 8$	M1	oe could be awarded by substituting a value of x with $p = -2$
$k = 3$	A1ft	need to check back for ft mark

Alternative method 2		
A correct equation obtained by substituting a value for x in the identity	M1	eg $x = 0$ $2k - p = 8$ $x = 1$ $p - p + 6 + 2k = 12$ $x = 2$ $2p - p + 12 + 2k = 16$
A second correct equation obtained by substituting a value for x in the identity	M1	oe could go back to equating coefficients at this stage
$p = -2$	A1	
$k = 3$	A1	may come from one equation by substituting $x = 1$

Additional Guidance	
Correct expansion, then $p + 6 = 4$ followed by $p = 2$ (incorrect) would give $k = 5$ on ft ... allow ft mark for k	M1, A0 M1, A1ft
In Alt 2 substituting $x = 1$ leads to $k = 3$ (a second equation	M1, A1

would be needed to gain further marks)	

Q13.

Answer	Mark	Comments
$n^3 + 2n^2 + 2n^2 + 4n + 2n^2 + 4n + 4n + 8$ or $n^3 + 4n^2 + 2n^2 + 4n + 8n + 8$ or $n^3 + 6n^2 + 12n + 8$	B2	oe eg $n^3 + 3 \times 2n^2 + 3 \times 2^2n + 8$ B1 $n^2 + 2n + 2n + 4$ or $n^2 + 4n + 4$
their $n^3 + 6n^2 + 12n + 8$ $-n^3 + 5n^2$	M1	
$11n^2 + 12n + 8$	A1	

Additional Guidance	
$n^3 + 8 - n^3 + 5n^2$	B0M1A0

Q14.

Answer	Mark	Comments
$x^2 + 3x + x + 3$ with three terms correct or $x^2 + 4x + k$ where k is a non-zero constant	M1	oe expansion attempt of one pair of brackets eg1 $x^2 + 4x + 3x + 12$ with three terms correct or $x^2 + 7x + k$ where k is a non-zero constant eg2 $x^2 + 4x + x + 4$ with three terms correct or $x^2 + 5x + k$ where k is a non-zero constant
$x^3 + 3x^2 + x^2 + 3x$ or $x^3 + 4x^2 + 3x$	M1dep	attempt at a full expansion with correct multiplication of their 3 or 4 terms by one of the terms in

or $4x^2 + 12x + 4x + 12$ or $4x^2 + 16x + 12$		the remaining bracket oe eg $x^3 + 4x^2 + 3x^2 + 12x$ or $x^3 + 7x^2 + 12x$ or $x^2 + 4x + 3x + 12$ or $x^2 + 7x + 12$ ($x^2 + 7x + 12$ must be from an attempt at a full expansion) or $x^3 + 4x^2 + x^2 + 4x$ or $x^3 + 5x^2 + 4x$ or $3x^2 + 12x + 3x + 12$ or $3x^2 + 15x + 12$
$x^3 + 8x^2 + 19x + 12$	A1	fully correct expansion allow if terms not collected eg $x^3 + 3x^2 + x^2 + 3x + 4x^2 + 12x + 4x + 12$ or $x^3 + 4x^2 + 3x + 4x^2 + 16x + 12$
$x^2 + 8x + 12$	A1ft	ft M2A0 full simplification of their $(x^3 + 8x^2 + 19x + 12) - x^3 - 7x^2 - 11x$ their $(x^3 + 8x^2 + 19x + 12)$ must be a cubic
$x^2 + 8x + 12$ and $(x + 6)(x + 2)$ or $(x + 2)(x + 6)$	A1	oe product of brackets

Additional Guidance	
1st M1 Do not allow omissions or extras eg1 $x^2 + 3x + 3$ eg2 $x^2 + 3x + x + 3 + x^2$	 M0 M0

For the first 2 marks terms may be seen in a grid	
If 1st A1 has been awarded with terms not collected, A1ft can still be awarded using their simplified cubic eg $x^3 + 4x^2 + 3x + 4x^2 + 16x + 12$ $= x^3 + 8x^2 + 18x + 12$ $x^3 + 8x^2 + 18x + 12 - x^3 - 7x^2 - 11x$ $= x^2 + 7x + 12$	M1M1A1 A1ftA0
First A1 may be seen embedded eg $x^3 + 8x^2 + 19x + 12 - x^3 + 7x^2 - 11x$	M1, M1, A1
If an attempt at the expansion of all three brackets in one go is made it must be fully correct to gain M2A1, otherwise M0M0A0 eg $x^2 + 3x + x + 3 + x^2 + 4x$	M0, M0, A0
Allow recovery of missing brackets when subtracting $x^3 + 7x^2 + 11x$ from their cubic	
For final A1 allow $x^2 + 8x + 12$ and $a = 6$ $b = 2$ or $x^2 + 8x + 12$ and $a = 2$ $b = 6$	
Ignore equating to zero and/or any 'solving' of an equation	

Q15.

Answer	Mark	Comments
Alternative method 1		
$3(x^2 + ax + ax + a^2)$ … or $3(x^2 + 2ax + a^2)$ … or $3\left(x + \frac{b}{3}\right)^2 \dots$ or $2b = 6a$ or $8a = 3a^2 + b + 2$	M1	oe eg $3x^2 + 6ax + 3a^2$ … or $\frac{b}{3} = a$ or $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
$2b = 6a$ and $8a = 3a^2 + b + 2$	M1dep	oe equations eg $\frac{b}{3} = a$ and $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$

$3a^2 + 3a - 8a + 2$ (= 0) or $3a^2 - 5a + 2$ (= 0)	M1dep	oe quadratic equation in a
$(3a - 2)(a - 1)$ or $\frac{--5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 2}}{2 \times 3}$	M1	oe eg $\frac{5}{6} \pm \sqrt{\frac{25}{36} - \frac{2}{3}}$ ft their 3-term quadratic
$a = \frac{2}{3}$ and $a = 1$ or $a = \frac{2}{3}$ and $b = 2$ or $a = 1$ and $b = 3$	A1	
$a = \frac{2}{3}$ and $b = 2$ and $a = 1$ and $b = 3$	A1	

Alternative method 2		
$3(x^2 + ax + ax + a^2)$ … or $3(x^2 + 2ax + a^2)$ … or $3\left(x + \frac{b}{3}\right)^2$ … or $2b = 6a$ or $8a = 3a^2 + b + 2$	M1	oe eg $3x^2 + 6ax + 3a^2$ … or $\frac{b}{3} = a$ or $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
$2b = 6a$ and $8a = 3a^2 + b + 2$	M1dep	oe equations eg $\frac{b}{3} = a$ and $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
$\frac{8b}{3} = 3\left(\frac{b}{3}\right)^2 + b + 2$ or $b^2 - 5b + 6$ (= 0)	M1dep	oe quadratic equation in b
$(b - 2)(b - 3)$	M1	oe eg $\frac{5}{2} \pm \sqrt{\frac{25}{4} - 6}$ ft their 3-term quadratic

or $\frac{--5\pm\sqrt{(-5)^2-4\times1\times6}}{2\times1}$		
b = 2 and b = 3 or $a = \frac{2}{3}$ and b = 2 or a = 1 and b = 3	A1	
$a = \frac{2}{3}$ and b = 2 and a = 1 and b = 3	A1	

Additional Guidance	
Allow $0.\dot{6}$ for $\frac{2}{3}$	
Allow 0.67 for $\frac{2}{3}$ for first A1	
In quadratic formula allow 5^2 for $(-5)^2$ but use of -5^2 must be recovered	

Q16.

Answer	**Mark**	**Comments**
3 terms from $20x^2 - 5xy^2\ (+)12xy^2 - 3y^4$	M1	may be seen in a grid
$20x^2 - 5xy^2 + 12xy^2 - 3y^4$	A1	four correct terms in any order may be seen in a grid implied by correct answer
$20x^2 + 7xy^2 - 3y^4$	A1	terms may be in any order

Additional Guidance	
Terms seen in a grid must have the correct signs	
Terms must be fully processed eg do not allow $4x3y^2$ unless recovered	

xy^2 may be y^2x throughout	
$20x^2 + 7xy^2 – 3y^4$ followed by incorrect further work	M1A1A0

Q17.

Answer	Mark	Comments
Alternative method 1 Expands $(3x + 4)(2x – 3)$ first		
$6x^2 – 9x + 8x – 12$ or $6x^2 – x – 12$	M1	oe 4 terms with at least 3 correct implied by $6x^2 – x + k$ or $px^2 – x – 12$ where k and p are non-zero constants may be seen in a grid
$30x^3 – 45x^2 + 40x^2 – 60x – 12x^2 + 18x – 16x + 24$ or $30x^3 – 5x^2 – 60x – 12x^2 + 2x + 24$	M1	oe full expansion with correct multiplication of their 3 or 4 terms by $5x$ or $–2$ may be seen in a grid
$30x^3 – 17x^2 – 58x + 24$	A1	terms in any order

Alternative method 2 Expands $(2x – 3)(5x – 2)$ first		
$10x^2 – 4x – 15x + 6$ or $10x^2 – 19x + 6$	M1	oe 4 terms with at least 3 correct implied by $10x^2 – 19x + k$ or $px^2 – 19x + 6$ where k and p are non-zero constants may be seen in a grid
$30x^3 – 12x^2 – 45x^2 + 18x + 40x^2 –16x – 60x + 24$ or $30x^3 – 57x^2 + 18x + 40x^2 – 76x + 24$	M1	oe full expansion with correct multiplication of their 3 or 4 terms by $3x$ or 4 may be seen in a grid
$30x^3 – 17x^2 – 58x + 24$	A1	terms in any order

Alternative method 3 Expands $(3x + 4)(5x – 2)$ first		
$15x^2 – 6x + 20x – 8$ or $15x^2 + 14x – 8$	M1	oe 4 terms with at least 3 correct implied by $15x^2 + 14x + k$ or $px^2 + 14x – 8$ where k and p are non-zero constants may be seen in a grid
$30x^3 – 12x^2 + 40x^2 – 16x – 45x^2$ $+ 18x – 60x + 24$ or $30x^3 + 28x^2 – 16x – 45x^2 – 42x + 24$	M1	oe full expansion with correct multiplication of their 3 or 4 terms by $2x$ or –3 may be seen in a grid
$30x^3 – 17x^2 – 58x + 24$	A1	terms in any order

Additional Guidance	
For terms seen in a grid accept $8x$ for $+8x$ etc	
2nd M1 A full expansion will be 8 terms if 4 terms are used in first expansion A full expansion will be 6 terms if 3 terms are used in first expansion	
Alt 1 $6x^2 + 9x – 8x – 12$ only 2 terms correct $(6x^2 + 9x – 8x – 12)(5x – 2)$ $= 30x^3 + 45x^2 – 40x^2 – 60x – 12x^2 + 18x – 16x + 24$ 8 terms with correct multiplication of their 4 terms by $5x$	M0 M1A0
Alt 2 $10x^2 – 19x – 5$ implied 4 terms with 3 correct $= 30x^3 + 45x^2 – 40x^2 – 60x – 12x^2 + 18x – 16x + 24$ 8 terms with correct multiplication of their 4 terms by $5x$ 6 terms with correct multiplication of their 3 terms by 4	M1 M1A0

1st M1 with a 4-term expansion followed by incorrect simplification to 3 terms can still score the 2nd M1 using their 3 terms	
One single expansion is full marks or zero	

Section 2.7

Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		
Evidence of 1 5 10 10 5 1 **used** for all six coefficients (terms could be written incorrectly)	M1	the 1s can be ignored but 5 10 10 5 must be seen and **used** (don't accept it just being written in Pascal's triangle)
$(3)^5 + 5(3)^4(2x) + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5(3)(2x)^4 + (2x)^5$	M1dep	oe eg $(3)^5(2x)^0$ written for first term at least 4 terms correct (could already be simplified and missing brackets recovered)
$(3)^5 + 5(3)^4(2x) + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5\ (3)(2x)^4 + (2x)^5$	M1dep	oe eg $(3)^5(2x)^0$ written for first term all correct
$243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	A1	

Alternative method 2		
$(3 + 2x)^2 = 9 + 12x + 4x^2$	M1	
$(3 + 2x)^3 = 27 + 54x + 36x^2 + 8x^3$	M1dep	oe the terms may not have been collected could do $(3 + 2x)^2 \times (3 + 2x)^2$. If they use this method (doesn't refer to $(3 + 2x)^3$) then award this mark for answer expanded correctly but with one numerical error. Terms must be collected
$(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$	M1dep	terms must be collected could do $(3 + 2x)^2 \times (3 + 2x)^3$. If they use this method (doesn't refer to $(3 + 2x)^4$) then award this mark for answer expanded correctly but with one numerical error. Terms

		must be collected would imply first 2 M marks if done correctly
$243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	A1	

Alternative method 3		
Evidence of 1 5 10 10 5 1 used for all six coefficients (could be written incorrectly)	M1	the 1s can be ignored but 5 10 10 5 must be seen and used
$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	M1dep	from using a general expansion of $(a + b)^5$
$(3)^5 + 5(3)^4(2x) + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5(3)(2x)^4 + (2x)^5$	M1dep	oe all correct
$243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	A1	

Additional Guidance	
Working could be seen as a list or a grid. This can be awarded full marks if done correctly Candidates could use a combination of methods. Use whichever alt method works best (probably alt 2) Missing brackets must be recovered	M3A1

Q2.

Answer	Mark	Comments
15×2^4 or 15×16 or 240	M1	oe eg $\binom{6}{4}2^4$ or $2^6 \times \frac{6 \times 5}{2} \times \left(\frac{1}{2}\right)^2$ may include a^2 and/or x^4 allow embedded eg ${}^6C_4\, a^2(2x)^4$
$240a^2 = 1500$ or $a^2 = \frac{1500}{240}$ or $(\pm)\sqrt{\frac{1500}{240}}$ or $\frac{5}{2}$ or $-\frac{5}{2}$	M1dep	must evaluate $\binom{6}{4}$ oe eg $15 \times 2^4a^2 = 1500$ or $(\pm)\sqrt{\frac{1500}{15 \times 2^4}}$ may include x^4 on both sides of an

		equation
$\frac{5}{2}$ and $-\frac{5}{2}$ with no other values	A1	oe eg 2.5 and –2.5 SC2 [2.236, 2.24] and [–2.24, –2.236] SC1 [2.236, 2.24] or [–2.24, –2.236]

Additional Guidance	
The relevant term must be selected from a full expansion but the other terms can be ignored	
Allow $\begin{pmatrix}6\\4\end{pmatrix}$ to be $\begin{pmatrix}6\\2\end{pmatrix}$	
$240a^2x^4 = 1500x^4$	M1M1
$240a^2x^4 = 1500$ recovered to (±) $\sqrt{\frac{1500}{240}}$ oe	M1M1
$240a^2x^4 = 1500$ not recovered to (±) $\sqrt{\frac{1500}{240}}$ oe	M1M0

Q3.

Answer	Mark	Comments
$15(2x)^4(a)^2$	M1	
$15 \times 16a^2 = 60$ or $240a^2 = 60$	M1dep	oe
$\sqrt{\frac{\text{their } 60}{\text{their } 240}}$ or $\frac{1}{2}$ or $-\frac{1}{2}$	M1dep	oe
$\frac{1}{2}$ and $-\frac{1}{2}$	A1	oe

Q4.

Answer	Mark	Comments
$6 \times 3^2 \times (ax)^2$ or $54a^2x^2$ or $6 \times 3^2 \times a^2$ or $54a^2$	M1	oe

$a^2 = \frac{150}{54}$ or $a^2 = \frac{25}{9}$ or $\sqrt{\frac{150}{54}}$ or $\sqrt{\frac{25}{9}}$ or $\frac{5}{3}$ or $-\frac{5}{3}$	M1	oe
$\frac{5}{3}$ and $-\frac{5}{3}$	A1	oe values eg $\pm\frac{5}{3}$

Section 2.8
Mark schemes

Q1.

Answer	Mark	Comments
$(x + y)[(x + y) + (2x + 5y)]$	M1	
$(x + y)(3x + 6y)$	A1	
$3(x + y)(x + 2y)$	A1	$(x + y)(x + 2y)$ scores SC2

Alternative method		
$x^2 + xy + xy + y^2 + 2x^2 + 2xy + 5xy + 5y^2$ or $3x^2 + 9xy + 6y^2$	M1	Condone two errors
$(x + y)(3x + 6y)$ or $(3x + 3y)(x + 2y)$ or $3(x^2 + 3xy + 2y^2)$	A1	
$3(x + y)(x + 2y)$	A1	$(x + y)(x + 2y)$ scores SC2

Q2.

Answer	Mark	Comments
(a) $5(m + 2p)(m - 2p)$	B3	B2 $(5m + 10p)(m - 2p)$ **or** $(5m - 10p)(m + 2p)$ B1 $5(m^2 - 4p^2)$ **or** $(5m + ap)(m + bp)$ where $ab = \pm 20$

(b) Their $(m + 2p) = 0$ **or** Their $(m - 2p) = 0$	M1	oe eg $m = -2p$ **or** $m = 2p$ May substitute for p at this stage
−30 **and** 30	A1	

Alternative method		
$5m^2 – 20 \times 15 \times 15 = 0$	M1	oe eg $5m^2 = 4500$
–30 **and** 30	A1	

Q3.

Answer	Mark	Comments
$3d(4c^2 - 3d)$	B2	B1 $d(12c^2 - 9d)$ or $3(4c^2d - 3d^2)$

Q4.

Answer	Mark	Comments
(a) $(x + 7 + x - 3)(x + 7 - x + 3)$	M1	Allow one sign error
$(2x + 4) \times 10$	A1	oe
$10 \times 2(x + 2)$ or $20x + 40$	A1	

Alternative method		
$x^2 + 7x + 7x + 49$ $(-)\ x^2 - 3x - 3x + 9$	M1	oe Allow one error
$x^2 + 7x + 7x + 49$ $- (x^2 - 3x - 3x + 9)$	A1	oe All terms correct
$x^2 + 7x + 7x + 49$	A1	oe

$- x^2 + 3x + 3x - 9 = 20x + 40$		

(b)	20(100 + 2) or 204 × 10	M1	11449 or 9409 seen
	2040	A1	

Q5.

Answer	Mark	Comments
Alternative method 1		
$(w + 4)^2$ as a factor	M1	Allow $(w + 4)(w + 4)$
$(w + 4)^2(w + 4 - (w + 1))$ or $(w + 4)^2(w + 4 - w + 1)$ or $(w + 4)^2(w + 4 - w - 1)$	M1dep	Allow $(w + 4)(w + 4)$ for $(w + 4)^2$
$3(w + 4)^2$	A1	Allow $3(w + 4)(w + 4)$

Alternative method 2		
$(w + 4)[(w + 4)^2 - (w + 4)(w + 1)]$	M1	
$(w + 4)(aw + b)$	M1dep	a and b both non-zero
$3(w + 4)^2$	A1	Allow $3(w + 4)(w + 4)$

Alternative method 3		
$w^3 + 12w^2 + 48w + 64$ or $w^3 + 9w^2 + 24w + 16$ or $-w^3 - 9w^2 - 24w - 16$ or $-w^3 + 9w^2 + 24w + 16$ or	M1	Must collect terms

$3w^2 + 24w + 48$ or $3(w^2 + 8w + 16)$		
$(3w + 12)(w + 4)$	M1dep	Correctly factorises their three term quadratic
$3(w + 4)^2$	A1	Accept $3(w + 4)(w + 4)$

Q6.

Answer	Mark	Comments
$3(x + 2)(x - 2)$	B2	B1 for $3(x^2 - 4)$ or $(3x + 6)(x - 2)$ or $(x + 2)(3x - 6)$

Q7.

Answer	Mark	Comments
$(5x + ay)(x + by)$	M1	where $ab = \pm 12$ or $a + 5b = \pm 4$
$(5x \pm 6y)(x \pm 2y)$	A1	for correct y terms in correct brackets, but with a sign error
$(5x - 6y)(x + 2y)$	A1	

Q8.

Answer	Mark	Comments
$(x + 6)^3\,[x + 6 + 3x + 4]$ **or** $(x + 6)^2[(x + 6)^2 + (x + 6)(3x + 4)]$ **or** $(x + 6)[(x + 6)^3 + (x + 6)^3(3x + 4)]$	M1	for sight of $(x + 6)^3$, $(x + 6)^2$ or $(x + 6)$ taken out as a common factor
$(x + 6)^3[4x + 10]$	A1	
$2(x + 6)^3(2x + 5)$	A1	

Additional Guidance
$(x + 6)^3(x + 6)(3x + 4)$ implies M1

SC1 for all correct factors seen in working but never written as a product of terms
An attempt to expand brackets will be M0 unless the expansion leads to a correct solution worth 2 or 3 marks
$(x + 6)^3$ $[x + 6 + 4x + 3]$ scores M1 ... ignore the error in the 2nd bracket

Q9.

Answer	Mark	Comments
$3(4 + 5x)(4 - 5x)$ or $3(-4 - 5x)(5x - 4)$ or $-3(4 + 5x)(5x - 4)$ or $-3(-4 - 5x)(4 - 5x)$	B2	B1 Partial factorisation eg $3(16 - 25x^2)$ or $-3(25x^2 - 16)$ or $(12 + 15x)(4 - 5x)$ or $(12 - 15x)(4 + 5x)$

Additional Guidance	
Brackets in either order for B2 or B1	
$-(75x^2 - 48)$	B0
$(-5x + 4)$ is equivalent to $(4 - 5x)$ etc	
Incorrect notation eg $(4 + 5x)3(4 - 5x)$	B1
Use of surds eg $(\sqrt{48} + \sqrt{75}\,x)(\sqrt{48} - \sqrt{75}\,x)$ or $(4\sqrt{3} + 5\sqrt{3}\,x)(4\sqrt{3} - 5\sqrt{3}\,x)$	B1
Use of multiplication signs scores a maximum of B1 eg $3 \times (4 + 5x)(4 - 5x)$	B1
B2 answer followed by further work	B1
B1 answer followed by further work	B1
Missing brackets must be recovered eg $3 \times 16 - 25x^2$	B0

Q10.

Answer	Mark	Comments
Correct factorised expression with a common factor	M1	eg $(y + 3)$ $[6(y + 3)^4 + 4(y + 3)^3]$

		or $2[3(y + 3)^5 + 2(y + 3)^4]$ or $2(y + 3)^2 [3(y + 3)^3 + 2(y + 3)^2]$
$2(y + 3)^4 [3(y + 3) + 2]$ or $2(y + 3)^4 (3y + 9 + 2)$ or $(y + 3)^4 [6(y + 3) + 4]$ or $(y + 3)^4 (6y + 18 + 4)$ or $(y + 3)^4 (6y + 22)$	A1	
$2(y + 3)^4 (3y + 11)$	A1	

Additional Guidance	
Use of multiplication signs scores a maximum of M1A1A0	
Any combination of bracket shape may be used	
Correct answer followed by further work	M1A1A0
Incorrect notation eg $(y + 3)^4\ 2(3y + 11)$	M1A1A0
$(2)(y + 3)^4 (3y + 11)$ or $(2(y + 3)^4)(3y + 11)$	M1A1A1
Allow substitution eg $n = (y + 3)$ for M1A1 but must revert to $(y + 3)$ for final mark	
Missing brackets must be recovered eg $(y + 3)^4\ 6y + 22$ with M1 not seen	Zero

Q11.

Answer	Mark	Comments
$6pq^2r (2q - 3r + 4)$	B2	B1 correct factorised expression with a common factor involving at least two variables eg $pq(12q^2r - 18qr^2 + 24qr)$ or $2q^2r (6pq - 9pr + 12p)$ or common factor $6pq^2r$ with two out of the three terms in the bracket correct eg $6pq^2r (2q - 3r + 4p)$

Additional Guidance	
B2 answer followed by further work	B1
$6pq^2r$ $(2q - 3r + 4)$ in working with $6qp^2r$ $(2q - 3r + 4)$ on answer line	B1
B1 answer followed by further work	B1
$2q^2r$ $(6pq - 9pr + 12p)$ in working with $2p^2r$ $(6pq - 9pr + 12p)$ on answer line	B1
Use of multiplication signs scores a maximum of B1	
$qpq(12qr - 18r^2 + 24r)$	B1
$6pqrq$ $(2q - 3r + 4)$	B1

Q12.

Answer	Mark	Comments
Alternative method 1		
$(6x + ay)(x + by)$	M1	$ab = -20$ or $a + 6b = 26$
$(6x - 4y)(x + 5y)$	A1	
$2(3x - 2y)(x + 5y)$	A1	oe but must have 3 correct factors

Alternative method 2		
$(3x + ay)(2x + by)$	M1	$ab = -20$ or $2a + 3b = 26$
$(3x - 2y)(2x + 10y)$	A1	
$2(3x - 2y)(x + 5y)$	A1	oe but must have 3 correct factors

Alternative method 3		
$2(3x^2 + 13xy - 10y^2)$	M1	
$2(3x - 2y)(x + 5y)$	A2	oe but must have 3 correct factors A1 for correct answer with signs wrong way round ie $2(3x + 2y)(x - 5y)$

Alternative method 4 using $(3x^2 + 13xy - 10y^2)$

$(3x + ay)(x + by)$	M1	$ab = -10$ or $a + 3b = 13$
$(3x - 2y)(x + 5y)$	A1	
$2(3x - 2y)(x + 5y)$	A1	oe but must have 3 correct factors

Additional Guidance	
Candidates who remove x or y, factorise correctly and then replace the letter to gain correct answer	M1, A2
Candidates who remove x or y, factorise correctly and then **don't** replace the letter	M0, A0
Condone further working in an attempt to solve an equation	

Q13.

Answer	Mark	Comments
$x^2y(x^2 + 3y^2)$	B2	B1 correct partial factorisation eg $x^2(x^2y + 3y^3)$ or $xy(x^3 + 3xy^2)$ or $y(x^4 + 3xy^3)$ or $x(x^3y + 3xy^3)$

Additional Guidance	
Only common factor removed is 1	B0

Q14.

Answer	Mark	Comments
$x^4(x + 3)(x – 3)$	B2	B1 $x^4(x^2 – 9)$

Q15.

Answer	Mark	Comments
$(x^2 - 9)(x^2 + 9)$ or $(x + 3)(x^3 - 3x^2 + 9x - 27)$ or $(x - 3)(x^3 + 3x^2 + 9x - 27)$	M1	
$(x + 3)(x - 3)(x^2 + 9)$	A1	Do not award A1 if further working

Section 2.9

Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		
common denominator $(x + 4)(x - 6)$	M1	oe allow $(x + 4)(x - 6)^2$
(numerator) $5x - 3(x + 4)$	M1	oe allow $5x\ (x - 6) - 3(x + 4)(x - 6)$
$\frac{2x-12}{(x+4)(x-6)}$	A1	$\frac{(2x-12)(x-6)}{(x+4)(x-6)^2}$
$\frac{2}{(x+4)}$	A1	

Alternative method 2		
remove common factor of $\frac{1}{(x-6)}$ **and** common denominator $(x + 4)$	M1	
numerator $5x - 3(x + 4)$	M1	
$\frac{2x-12}{(x+4)(x-6)}$	A1	
$\frac{2}{(x+4)}$	A1	

Q2.

Answer	Mark	Comments
$(x + 6)(x - 2)$	B1	
$(x + 5)(x - 5)$	B1	

$x(x-5)$	B1	
$\dfrac{\text{their } (x+6)(x-2)}{\text{their } (x+5)(x-5)} \times \dfrac{\text{their } x(x-5)}{x+6}$	M1	Must have attempted to factorise at least two of the above
$\dfrac{x(x-2)}{x+5}$ or $\dfrac{x^2-2x}{x+5}$	A1	A0 if incorrect further work seen

Q3.

Answer	Mark	Comments
$(ax + b)(cx + d)$	M1	Where ac = 4 and bd = ±5 or ad + bc = ±19
$(4x - 1)(x + 5)$	A1	
$(3x - 4)(3x + 4)$	B1	
their $\dfrac{(4x-1)(x+5)}{(3x-4)(3x+4)} \times \dfrac{(3x-4)}{(x+5)}$	M1	Inverting the 2nd fraction and multiplying Must have attempted to factorise both expressions (allow max one error in each)
$\dfrac{4x-1}{3x+4}$	A1	

Q4.

(a)

Answer	Mark	Comments
$\dfrac{4(x-1)+2x}{x(x-1)}$	M1	oe eg two separate fractions Condone absence of brackets only if recovered
$\dfrac{4x-4+2x}{x(x-1)}$ $\left(= \dfrac{6x-4}{x(x-1)}\right)$	A1	Do not condone absence of brackets even if recovered

(b)

$6x - 4 = 3x(x - 1)$	M1	oe eg $4(x - 1) + 2x = 3x\,(x - 1)$
$3x^2 - 9x + 4$ (= 0)	A1	$-3x^2 + 9x - 4$ (= 0)
$\dfrac{--9 \pm \sqrt{(-9)^2 - 4\times3\times4}}{2\times3}$	M2	Correct use of formula for their quadratic M1 Allow one sign error (must

$(= \frac{9 \pm \sqrt{33}}{6})$		have square root and numerator all over 2*a*) Allow M2 for correct factorisation of their quadratic M2 $(x - \frac{3}{2})^2 = \frac{9}{4} - \frac{4}{3}$ oe M1 $(x - \frac{3}{2})^2 - \frac{9}{4} + \frac{4}{3} = 0$ oe
2.46 **and** 0.543	A1	Must both be to 3 significant figures

Q5.

Answer	Mark	Comments
$4(x + 3) + x - 2$ **or** $\frac{4(x+3)}{(x-2)(x+3)} + \frac{x-2}{(x-2)(x+3)}$	M1	Must be correct
$4x + 12 + x - 2$ $(= 5x + 10)$ **or** $\frac{4x+12}{(x-2)(x+3)} + \frac{x-2}{(x-2)(x+3)}$	A1	
$5(x - 2)(x + 3)$	M1	Must have 5 and be correct Must be in an equation **and** not a denominator oe eg $(5x - 10)(x + 3)$
$(5)(x^2 + 3x - 2x - 6)$	M1	5 may be missing Must be in an equation **and** not a denominator 4 terms including term in x^2 with 3 correct oe eg 1 $x^2 + x - 6$ eg 2 $5x^2 + 15x - 10x - 6$ (1 error)
$5x^2 = 40$	A1	oe eg $5x^2 - 40 = 0$ Must collect all terms and have an equation

Correct attempt at solution of their quadratic eg $x = \sqrt{\frac{40}{5}}$	M1dep	dep on M3 Quadratic formula must have no errors in substitution If completing square must have no errors up to $p(x - q)^2 = r$ $p(x - q)^2 - r = 0$
[2.8, 2.83] **and** [–2.83, –2.8]	A1ft	oe eg (+) $\sqrt{8}$ **and** – $\sqrt{8}$ **or** ± $\sqrt{8}$ ft their quadratic equation if M4 SC7 Both solutions correct (no valid method) SC3 One solution correct (no valid method)

Q6.

Answer	Mark	Comments
$\frac{4c^5}{9d^3}$ or $\frac{4c^5d^{-3}}{9}$ or $\frac{0.\dot{4}c^5}{d^3}$ or $0.\dot{4}c^5d^{-3}$	B3	B2 Any two of these three components • numerator having c^5 (no c in denominator) • denominator having d^3 (no d in numerator) or numerator having d^{-3} (no d in denominator) • number $\frac{4}{9}$ or $0.\dot{4}$ B1 Any one of these three components • numerator having c^5 (no c in denominator) • denominator having d^3 (no d in numerator) or numerator having d^{-3} (no d in denominator) • number $\frac{4}{9}$ or $0.\dot{4}$ or

		$\frac{40c^7d^3}{90d^6c^2}$ or $\frac{20c^7d^3}{45d^6c^2}$ or $\frac{8c^7d^3}{18d^6c^2}$ or $\frac{1.\dot{3}c^7d^3}{3d^6c^2}$ or $\frac{\frac{4}{3}c^7d^3}{3d^6c^2}$ SC1 $\frac{9d^3}{4c^5}$ or $\frac{2.25d^3}{c^5}$ Always award SC1 if this is their final answer even if $\frac{4c^5}{9d^3}$ seen in working

Q7.

(a)

Answer	Mark	Comments
$(c + 4)(c + 1)$ **or** $3(c + 1)$	M1	Correct factorisation
$\frac{(c+4)(c+1)}{3(c+1)} = \frac{c+4}{3}$	A1	Must be a fraction and completed to $\frac{c+4}{3}$
Correctly converts to a common denominator eg 1 $\frac{2(c+4)}{6} + \frac{3-2c}{6}$ eg 2 $\frac{6(c+4)}{18} + \frac{3(3-2c)}{18}$	M1	M2 $\frac{2c}{6} + \frac{8}{6} + \frac{3}{6} - \frac{2c}{6}$

(b)

Correctly expands their brackets (must have common denominator) $\frac{2c+8+3-2c}{6}$ **or** $\frac{2c+8}{6} + \frac{3-2c}{6}$	M1	Allow M1 if their first line of working is $\frac{2c+4+3-2c}{6}$ **or** $\frac{2c+4}{6} + \frac{3-2c}{6}$
$\frac{11}{6}$ **or** $1\frac{5}{6}$ **or** 1.833(.....)	A1	$\frac{33}{18}$ A0 $\frac{5.5}{3}$ A0 $\frac{8+3}{6}$ A0

Alternative method		
Correctly converts to a common	M1	oe

denominator, eg $\frac{6(c^2+5c+4)}{6(3c+3)}+\frac{(3-2c)(3c+3)}{6(3c+3)}$		May also expand the denominator
Correctly expands their brackets (must have common denominator) $\frac{6c^2+30c+24+9c+9-6c^2-6c}{6(3c+3)}$ **or** $\frac{6c^2+30c+24}{6(3c+3)}+\frac{9c+9-6c^2-6c}{6(3c+3)}$	M1	oe May also expand the denominator
$\frac{11}{6}$ **or** $1\frac{5}{6}$ **or** 1.833(….).	A1	$\frac{33}{18}$ A0 $\frac{5.5}{3}$ A0 $\frac{8+3}{6}$ A0

Q8.

Answer	Mark	Comments
$(m + 1)(m - 4)$ or $m^2 - 3m - 4$ seen as a common denominator	B1	oe
$5(m - 4) + 6(m + 1)$	M1	Allow one error in expansion if not showing brackets e.g. Allow $5m - 20 + m + 6$
$\frac{5m-20+6m+6}{\text{their common denominator}}$ or $\frac{5m-20}{\text{their common denominator}}+\frac{6m+6}{\text{their common denominator}}$	M1	Allow one error in expansion of numerator(s) their common denominator must be a quadratic
$\frac{11m-14}{(m+1)(m-4)}$ or $\frac{11m-14}{m^2-3m-4}$	A1	

Q9.

Answer	Mark	Comments
$x^2(x^2 - x - 2)$ or $(x - 2)(x^3 + x^2)$ or $(x + 1)(x^3 - 2x^2)$ or $(x^2 + x)(x^2 - 2x)$	M1	
$x^2(x + 1)(x - 2)$ seen in numerator	M1	allow $x(x + 1)x(x - 2)$ or $(x + 1)x^2(x - 2)$
$(x^2 - 1)(x^2 - 4)$ seen in denominator	M1	
$(x + 1)(x - 1)$ **or** $(x + 2)(x - 2)$	M1dep	dep on previous M mark
$\dfrac{x^2}{(x-1)(x+2)}$	A1	accept $\dfrac{x^2}{x^2+x-2}$

Additional Guidance
… any incorrect fw will lose the A mark

Q10.

Answer	Mark	Comments
$3(x – 1)$ or $3x – 3$ or $2(x – 2)$ or $2x – 4$	M1	
$3x – 3$ and $2x – 4$ or $5x – 7$	M1	Implies M1 M1
$5(x – 1)(x – 2)$ or $5(x^2 – 2x – x + 2)$ or $5x^2 – 15x + 10$ or $(x – 1)(x – 2)$ expanded and multiplied by 5	M1	oe Allow one error in four term expansion of $5(x – 1)(x – 2)$ Implied by $5(x^2 – 3x + k)$ or $5(ax^2 – 3x + 2)$
$5x^2 – 20x + 17$ (= 0)	M1dep	dep on 3rd M1 oe 3-term quadratic equation eg $5x^2 – 20x = –17$ Correctly collects terms in their expansion
$\dfrac{--20 \pm \sqrt{(-20)^2 - 4 \times 5 \times 17}}{2 \times 5}$		oe Correct use of quadratic formula for their 3-term quadratic eg (–

or $\frac{10 \pm \sqrt{15}}{5}$ or $(x - 2)^2 - 4 = -\frac{17}{5}$ or $5[(x - 2)^2 - 4] = -17$	M1	$20)^2$ can be 20^2 or correct factorisation of their 3-term quadratic or attempt to complete the square for their 3-term quadratic Must be correct up to form $(x - a)^2 + b = c$ or $k[(x - d)^2 + e] = f$
1.23 and 2.77	A1	Must be 3 significant figures

Additional Guidance	
For A1, the word ‘and’ is not needed eg 1.23, 2.77 (with method seen)	M5 A1
Brackets may be recovered throughout	
5th M1 may be implied by solutions of their quadratic equation seen	
M0 M0 M0 M0 M1 A0 is possible if they have a 3-term quadratic equation	
Answers only	Zero

Q11.

Answer	Mark	Comments
Alternative method 1 Processes the brackets then divides		
$\frac{5x}{10} + \frac{6x}{10}$	M1	oe valid common denominator with both numerators correct eg $\frac{10x}{20} + \frac{12x}{20}$
$\frac{11x}{10}$	A1	oe single term eg $\frac{22x}{20}$ or $1.1x$ may be implied eg by single term with roots evaluated that is equivalent to $\frac{11}{5x^2}$
$\frac{x^{6\div2}}{2}$ or $\frac{x^3}{2}$	M1	may be implied

		eg by multiplication by $\frac{2}{x^3}$
their $\frac{11x}{10} \times \frac{2}{x^3}$ or $\frac{22x}{10x^3}$ or $\frac{22}{10x^2}$ or $\frac{11x}{5x^3}$ or $\frac{22}{10}x^{-2}$	M1dep	oe multiplication eg $\frac{11x}{10} \times 2x^{-3}$ their $\frac{11x}{10}$ can be unprocessed dep on 2nd M1
$\frac{11}{5x^2}$ or $\frac{11}{5}x^{-2}$ or $2.2x^{-2}$	A1	allow $2\frac{1}{5}x^{-2}$ or $\frac{2.2}{x^2}$

Alternative method 2 Divides then expands the brackets		
$\frac{x^{6 \div 2}}{2}$ or $\frac{x^3}{2}$	M1	may be implied eg by multiplication by $\frac{2}{x^3}$
$\left(\frac{x}{2}+\frac{3x}{5}\right) \times \frac{2}{x^3}$	M1dep	oe multiplication eg $\left(\frac{x}{2}+\frac{3x}{5}\right) \times 2x^{-3}$
$\frac{2x}{2x^3} + \frac{6x}{5x^3}$ or $\frac{1}{x^2} + \frac{6}{5x^2}$	M1dep	oe expansion of brackets
$\frac{10x}{10x^3} + \frac{12x}{10x^3}$ or $\frac{5}{5x^2} + \frac{6}{5x^2}$ or $\frac{22x}{10x^3}$ or $\frac{22}{10x^2}$ or $\frac{11x}{5x^3}$ or $\frac{22}{10}x^{-2}$	M1dep	oe valid common denominator with both numerators correct eg $\frac{10x^4}{10x^6} + \frac{12x^4}{10x^6}$ or $\frac{22x^4}{10x^6}$ roots must be processed
$\frac{11}{5x^2}$ or $\frac{11}{5}x^{-2}$ or $2.2x^{-2}$	A1	allow $2\frac{1}{5}x^{-2}$ or $\frac{2.2}{x^2}$

Additional Guidance	
Any single fraction with roots evaluated that is equivalent to $\frac{11}{5x^2}$	4 marks
Allow inclusion of ± from the square root for up to 4 marks	
$\frac{11}{5x^2}$ in working with answer $\frac{11}{5}x^2$	4 marks

Alt 1 $\frac{11x}{10}$ subsequently squared and not recovered	M1A1 M0M0A0

Q12.

Answer	Mark	Comments
Single correct fraction with terms processed	M1	eg1 $\frac{600a^5 + 1200a^4}{36a^3 + 72a^2}$ eg2 $\frac{50a^3 + 100a^2}{3a + 6}$ Only bracket allowed is (a + 2) eg $\frac{50a^4(a+2)}{3a^3 + 6a^2}$ (scores M2)
Factorises correctly using (a + 2)	M1	Only needs to be seen once eg1 $\frac{8a}{3a+6} \times \frac{5(a+2)}{3a^2} \div \frac{4}{15a^3}$ eg2 $\frac{8a}{3(a+2)} \times \frac{5a+10}{3a^2} \times \frac{15a^3}{4}$ Award M2 for fully correct unprocessed expression with full cancelling seen, eg $\frac{{}^{2}\cancel{8}a}{3\cancel{(a+2)}} \times \frac{5\cancel{(a+2)}}{\cancel{3}\cancel{a^2}} \times \frac{{}^{5}\cancel{15}a\cancel{{}^{3}}{}^{1}}{\cancel{4}}$ or $\frac{2a}{3} \times 5 \times 5a$ oe
$\frac{50a^2}{3}$ or $16\frac{2}{3}a^2$ or $16.\dot{6}a^2$	A1	

Additional Guidance	
$\frac{50 \times a \times a}{3}$	M2A0
A correct single fraction with (a + 2) cancelled will be M2 eg1 $\frac{250a^2}{15}$ eg2 $\frac{50a^4}{3a^2}$	M2A0

$\frac{8a}{3} \times \frac{5(a+2)}{3a^2} \times \frac{15a^3}{4}$	M0M1A0
$3a + 6 = 3(a + 2)$ with no other valid working	M0M1A0
Brackets other than $(a + 2)$ may be seen $\frac{10a^2(5a+10)}{3a+6}$	M0A0
Correct answer followed by incorrect further work	M2A0
Allow one miscopy for up to M2A0	

Q13.

Answer	Mark	Comments
Alternative method 1		
common denominator $(x - 3)(x - 5)$ oe	M1	allow $(x - 3)^2(x - 5)$ oe
numerator $x(x - 5) + 6$ **or** $x^2 - 5x + 6$	M1dep	allow $x(x - 3)(x - 5) + 6(x - 3)$ oe
$\frac{(x-3)(x-2)}{(x-3)(x-5)}$	A1	$\frac{(x-3)^2(x-2)}{(x-3)^2(x-5)}$
$\frac{x-2}{x-5}$	A1	

Alternative method 2		
$\frac{1}{(x-3)}\left(x+\frac{6}{(x-5)}\right)$	M1	
$\frac{1}{(x-3)}\left(\frac{x(x-5)+6}{(x-5)}\right)$ **or** $\frac{1}{(x-3)}\left(\frac{x^2-5x+6}{(x-5)}\right)$	M1	
$\frac{(x-3)(x-2)}{(x-3)(x-5)}$	A1	
$\frac{x-2}{x-5}$	A1	

Additional Guidance
Further work eg answer of $\frac{-2}{-5}$ means the final A1 must not be awarded eg $\frac{x(x-5)}{(x-3)(x-5)} + \frac{6}{(x-3)(x-5)}$ scores M1 M1 Either ... follow the LHS of the mark scheme for the first three steps Or ... follow the RHS ... do **not** mix expressions ... the numerators and denominators must match

Q14.

Answer	Mark	Comments
Valid common denominator with at least one numerator correct	M1	eg $\frac{7x}{9x^2}$ and $\frac{a}{9x^2}$ or $\frac{7x+a}{9x^2}$ or $\frac{b}{9x \times 3x^2}$ and $\frac{2 \times 9x}{9x \times 3x^2}$ numerators and denominators may be seen as products a can be numerical or algebraic b can be numerical or algebraic
Valid common denominator with both numerators correct	M1dep	$\frac{7x}{9x^2}$ and $\frac{6}{9x^2}$ or $\frac{7 \times 3x^2}{9x \times 3x^2}$ and $\frac{2 \times 9x}{9x \times 3x^2}$ numerators and denominators may be seen as products
$\frac{7x+6}{9x^2}$ or $\frac{7x+6}{(3x)^2}$ with no further work	A1	

Additional Guidance

$\frac{21x^2+18x}{27x^3}$ or $\frac{21x+18}{27x^2}$ or $\frac{7x^2+6x}{9x^3}$	M2A0
$\frac{7x^{-1}+6x^{-2}}{9}$	M2A0
$7x + 6 / 9x^2$	M2A0

Q15.

Answer	Mark	Comments
Changes division to multiplication and inverts to $\frac{3x+12}{x^2}$	M1	may be implied
$(3x + 12 =)\ 3(x + 4)$	M1	may be implied
Correct expression written as a single fraction or a product must have factor $(x + 4)$ in a numerator and denominator $x + 4$ or correct expression written as a single fraction or a product must have denominator x^3 or x^2 or x or 1	A1	may be implied by final A1 eg $\frac{3x(x+2)(x+4)}{x+4}$ or $\frac{(3x^2+6x)(x+4)}{x+4}$ or $\frac{x}{x+4} \times \frac{x+2}{1} \times 3(x+4)$ or $\frac{x}{x+4} \times 3(x+2)(x+4)$ or $\frac{3x^4(x+2)}{x^3}$ or $x4 \times \frac{x+2}{x} \times \frac{3}{x^2}$ or $\frac{(x+2)}{x^3} \times 3x^4$ or $\frac{3x^3(x+2)}{x^2}$ or $\frac{3x^2(x+2)}{x}$ or $\frac{3x(x+2)}{1}$ or $x \times (x + 2) \times 3$

		or $3x \times (x + 2)$
$3x^2 + 6x$	A1	SC2 $\frac{x(x+2)(3x+12)}{x+4}$

Additional Guidance	
The list of examples in the first A1 is not exhaustive	
$3x^2 + 6x$ with no incorrect working	4 marks

Q16.

Answer	Mark	Comments
$x(1 - x^2)$ or $2x(1 + \mathrm{x})$ or $x(2 + 2x)$ or $\frac{1-x^2}{2+2x}$	M1	implied by 2nd M1 oe factorisation eg $-x(x2 - 1)$
$x(1 + x)(1 - x)$ or $\frac{x(1-x^2)}{2x(1+x)}$ or $\frac{1-x^2}{2(1+x)}$ or $\frac{(1+x)(1-x)}{2+2x}$	M1dep	implies M2 oe factorisation eg $-x(x + 1)(x - 1)$
$\frac{x(1+x)(1-x)}{2x(1+x)}$ or $\frac{(1+x)(1-x)}{2(1+x)}$ or $\frac{x(1-x)}{2x}$	M1dep	implies M3 oe factorisation eg $\frac{-x(x+1)(x-1)}{2x(1+x)}$
$\frac{1-x}{2}$ with M3 seen	A1	oe simplest form eg $\frac{1}{2}(1-x)$ or $\frac{1}{2}-\frac{1}{2}x$ or $\frac{-x+1}{2}$

Additional Guidance	
$\frac{x(1+x)(1-x)}{2x(1+x)}$ or $\frac{(1+x)(1-x)}{2(1+x)}$ or $\frac{x(1-x)}{2x}$	M3

is sufficient working	
$2(x + x^2)$ with no further work	M0
$\frac{x-1}{-2}$ with M3 seen or $-\frac{1}{2}(x-1)$ with M3 seen or $\frac{-(x-1)}{2}$ with M3 seen	M3 A1

Q17.

Answer	Mark	Comments
$5xy(3x - y)$	M1	
$4(3x - y)$	M1	
$\frac{5xy}{4}$	A1	

Q18.

Answer	Mark	Comments
Both fractions written with a common denominator (could be written as a single fraction) which is a multiple of 6*a* and 4 with at least one correct (term of the) numerator	M1	oe eg $\frac{20}{24a}$ or $\frac{6a^2}{24a}$ or $\frac{4(5)}{4(6a)}$ or $\frac{20+6a^2}{24a}$ allow decimals in fraction eg $\frac{5+1.5a^2}{6a}$
$\frac{10+3a^2}{12a}$	A1	

Additional Guidance	
Penalise further working	
$\frac{10+3a^2}{12}$ is likely to come from correct working	M1, A0

Q19.

Answer	Mark	Comments

(numerator =) $2x(4x^2 - 25)$ or $\dfrac{4x^2 - 25}{6x^2 - x - 35}$	B1	
(numerator =) $2x(2x + 5)(2x - 5)$ or $\dfrac{(2x+5)(2x-5)}{6x^2 - x - 35}$	B1	
$(ax + b)(cx + d)$ where $ac = 6$ and $bd = \pm 35$	M1	
$(3x + 7)(2x - 5)$	A1	
$\dfrac{2x+5}{3x+7}$	A1	

Q20.

Answer	Mark	Comments
Common denominator with at least one numerator correct	M1	eg $\dfrac{21}{6x^2} + \dfrac{8x}{6x^2}$ or $\dfrac{21x}{6x^3} + \dfrac{8x^2}{6x^3}$
$\dfrac{21 + 8x}{6x^2}$	A1	

Q21.

Answer	Mark	Comments
$\dfrac{c^3}{6c+1}$	B3	B2 $c^3(6c - 1)$ and $(6c + 1)(6c - 1)$ B1$c^3(6c - 1)$ or $(6c + 1)(6c - 1)$

Additional Guidance	
$\dfrac{c^3}{6c+1}$ followed by incorrect further work	B2

Q22.

Answer	Mark	Comments

$2(5x - y)$ or $-2(y - 5x)$ or $3(y - 5x)$ or $-3(5x - y)$	M1	
$-\frac{2}{3}$	A1	

Section 2.10

Mark schemes

Q1.

Answer	Mark	Comments
$3ef = 5e + 4$ or $ef - \frac{5e}{3} = \frac{4}{3}$	M1	
$e(3f - 5) = 4$ or $e\left(f - \frac{5}{3}\right) = \frac{4}{3}$	M1dep	oe where they are one step away from answer.
$e = \frac{4}{3f - 5}$	A1	oe eg $e = \frac{\frac{4}{3}}{\left(f - \frac{5}{3}\right)}$ or $e = \frac{-4}{5 - 3f}$

Alternative method 2		
$3f = 5 + \frac{4}{e}$	M1	
$\frac{4}{e} = 3f - 5$	M1dep	oe where they are one step away from answer
$e = \frac{4}{3f - 5}$	A1	oe eg $e = \frac{\frac{4}{3}}{\left(f - \frac{5}{3}\right)}$ or $e = \frac{-4}{5 - 3f}$

Additional Guidance	
Must have e = on the answer line for full marks	

Q2.

Answer	Mark	Comments
Alternative method 1		
$y^2 = \frac{x+2w}{3}$	M1	
$3y^2 - x = 2w$ or $\frac{3y^2 - x}{2}$ or $\frac{3y^2}{2} - \frac{x}{2}$	M1dep	
$w = \frac{3y^2 - x}{2}$ or $w = \frac{3y^2}{2} - \frac{x}{2}$	A1	

Alternative method 2		
$y^2 = \frac{x}{3} + \frac{2w}{3}$	M1	
$y^2 - \frac{x}{3} = \frac{2w}{3}$ or $\frac{3}{2}\left(y^2 - \frac{x}{3}\right)$ or $\frac{3y^2}{2} - \frac{3x}{6}$	M1dep	
$w = \frac{3}{2}\left(y^2 - \frac{x}{3}\right)$ or $w = \frac{3y^2}{2} - \frac{3x}{6}$	A1	

Additional Guidance	
Condone eg $w = \frac{3y^2 - x}{2}$ seen in working with $\frac{3y^2 - x}{2}$ on answer line	M2A1
$w = \frac{3}{2}y^2 - \frac{1}{2}x$ etc	M2A1

Q3.

(a)

Answer	Mark	Comments
$5t + 3 = 4wt + 8w$	M1	
$5t - 4wt = 8w - 3$	M1	Separation of terms in t from those not in t
$t(5 - 4w) = 8w - 3$	M1	Factorisation of terms in t
$t = \frac{8w-3}{5-4w}$	A1ft	oe eg $t = \frac{3-8w}{4w-5}$ Must have $t =$ **Only ft if third M1 and one other M1 gained**

(b)

$\frac{8\times-\frac{1}{8}-3}{5-4\times-\frac{1}{8}}$	M1	Substitution of $w = -\frac{1}{8}$ in their $\frac{8w-3}{5-4w}$ Their $\frac{8w-3}{5-4w}$ must be in terms of w
Numerator = − 4 **or** denominator = $5\frac{1}{2}$	A1ft	ft Their $\frac{8w-3}{5-4w}$ This mark can only be gained for correct evaluation of their algebraic numerator **or** their algebraic denominator
$-\frac{8}{11}$ **or** $-0.\dot{7}\dot{2}$	A1ft	ft Their $\frac{8w-3}{5-4w}$ This mark can only be gained for correct evaluation of their algebraic numerator **and** their algebraic denominator Must be an exact value in simplest form SC2 −0.72... **or** −0.73 **or** a correct evaluation of their algebraic numerator or their algebraic denominator

$5t + 3 = -\frac{4}{8}(t + 2)$	M1	oe equation
$44t = -32$	A1	oe eg $5.5t = -4$

$-\frac{8}{11}$ **or** $-0.\dot{7}\dot{2}$	A1ft	ft from their $at = b$ if M1 A0 Must be an exact value in simplest form SC2 −0.72... **or** −0.73

Q4.

(a)

Answer	Mark	Comments
$S(1 - r) = a$	B1	$\frac{a}{S} = 1 - r$
$S - Sr = a$	M1	Any valid correct step from their first step
$S - a = Sr \quad (\frac{S-a}{S} = r)$	A1	Clearly shown with no errors

(b)

$\frac{10a - a}{10a} \quad (= \frac{9a}{10a})$	M1	$10a = \frac{a}{1-r}$ oe
$\frac{9}{10}$	A1	oe

Q5.

Answer	Mark	Comments
$x(5 - 3w) = 2w + 1$	M1	
$5x - 3xw = 2w + 1$ or $5 - 3w = \frac{2w}{x} + \frac{1}{x}$	M1dep	oe eg $5x - 3xw - 2w = 1$ Expands brackets correctly or divides each term by x
$5x - 1 = 2w + 3xw$ or $5 - \frac{1}{x} = \frac{2w}{x} + 3w$	M1dep	oe eg $-3xw - 2w = 1 - 5x$ Collects terms in w (must have ≥ 2 terms containing w) Allow one sign error only dep on first M1 only
$\frac{5x-1}{2+3x} = w$	A1	oe eg $w = \frac{1-5x}{-3x-2}$

		Must have = w or w =

Q6.

Answer	Mark	Comments
$\frac{3xy}{x+y} = 16$	B1	allow 4^2
$3xy = 16(x + y)$ or $3xy = 16x + 16y$	M1	allow 'their 16' as obtained in first step
$3xy - 16y = 16x$ or $y(3x - 16) = 16x$	M1	ft with 'their 16'
$y = \frac{16x}{3x-16}$	A1	oe eg $y = \frac{-16x}{16-3x}$

Additional Guidance
They must get 4^2 or 16 to score B1 but 'their 16' is good enough to score the two M marks. For A1 it has to say 16, 4^2 is not acceptable
… any incorrect fw will lose the A mark

Q7.

Answer	Mark	Comments
Alternative method 1		
$yx = 8(w - x)$ or $y = \frac{8w-8x}{x}$	M1	
$yx = 8w - 8x$	M1dep	oe eg $yx - 8w + 8x = 0$ Implies M1 M1
$yx + 8x = 8w$ or $x(y + 8) = 8w$ or $\frac{8w}{y+8}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
$x = \frac{8w}{y+8}$	A1	oe eg $\frac{-8w}{-y-8}$ Must have x =

		SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$

Alternative method 2		
$y = \frac{8w}{x} - 8$ or $y = \frac{8w}{x} - \frac{8x}{x}$	M1	
$y + 8 = \frac{8w}{x}$	M1dep	oe eg $y + 8 - \frac{8w}{x} = 0$ Implies M1 M1
$yx + 8x = 8w$ or $x(y + 8) = 8w$ or $\frac{1}{y+8} = \frac{x}{8w}$ or $\frac{8w}{y+8}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
$x = \frac{8w}{y+8}$	A1	oe eg $\frac{-8w}{-y-8}$ Must have $x =$ SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$

Alternative method 3		
$yx = 8(w - x)$	M1	
$\frac{yx}{8} = w - x$	M1dep	oe eg $\frac{yx}{8} - w + x = 0$ Implies M1 M1
$\frac{yx}{8} + x = w$ or $x(\frac{y}{8} + 1) = w$ or $\frac{w}{\frac{y}{8}+1}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
$x = \frac{w}{\frac{y}{8}+1}$	A1	oe eg $x = \frac{-w}{-\frac{y}{8}-1}$ Must have $x =$

		SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$

Additional Guidance	
$x = \frac{8w}{y+8}$ in working with $\frac{8w}{y+8}$ on answer line	M3 A1
$x = \frac{8w}{y+1}$ in working with $\frac{8w}{y+1}$ on answer line	SC2
3rd M1 is for collecting terms in x (or x in numerator in Alt 2)	
Allow multiplications signs and 1s throughout	
Correct answer followed by incorrect further work	M3 A0

Q8.

Answer	Mark	Comments
A correct first step using algebra	M1	Here are some of the possible alternatives $\frac{1}{x} = y\left(4 - \frac{3}{y}\right)$ multiplying through by y $1 = xy\left(4 - \frac{3}{y}\right)$ multiplying through by xy $1 = 4xy - \frac{3xy}{y}$ multiplying through by xy $y = 4xy^2 - 3xy$ multiplying through by xy^2 $\frac{1}{xy} = \frac{4y - 3}{y}$ making the RHS an algebraic fraction $\frac{1 + 3x}{xy} = 4$ rearranging **and** making the LHS an algebraic fraction
Further correct algebra which	M1dep	Following two of

leads to an equation that is one step from the final answer.		the above alternatives ... $y = 4xy^2 - 3xy$ $y = x(4y^2 - 3y)$ M1dep gained $\frac{1 + 3x}{xy} = 4$ $1 + 3x = 4xy$ $1 = 4xy - 3x$ $1 = x(4y - 3)$ M1dep gained
A correct final answer in **any** form	A1	$x = \frac{1}{4y-3}$ $x = \frac{-1}{3-4y}$ $x = \frac{y}{4y^2-3y}$ $x = \frac{-y}{3y-4y^2}$ $x = \frac{1}{y\left(4-\frac{3}{y}\right)}$ $x = \frac{-1}{y\left(\frac{3}{y}-4\right)}$ $x = \frac{1}{\left(4-\frac{3}{y}\right)} \div y$

Additional Guidance

There are many ways of scoring the first M mark. They do not need to give any reasons but you need to check that what they do is valid.

For the M1dep mark you must check that their algebra is correct and will lead to a result that is one step from the final answer. 'One step from ...' means that when they divide through, they have a correct version where x is the subject.

Some of the final answers are more compact than others, but we didn't ask for any simplification so we have to accept a correct answer in any form.

... and, finally, one to look out for ... correct answer from wrong working ... 0 marks

$\frac{1}{xy} = 4 - \frac{3}{y}$ ⟶ $xy = \frac{1}{4} - \frac{y}{3}$ ⟶ $x = \frac{1}{4y} - \frac{1}{3}$ ⟶ $x = \frac{1}{4y-3}$ (creative thinking !)

Q9.

Answer	Mark	Comments
$t(w^3 - 2) = 3w^3 + a$	M1	
$tw^3 - 2t = 3w^3 + a$	M1dep	
$tw^3 - 3w^3 = a + 2t$	M1dep	
$w^3 (t - 3) = a + 2t$ or $w^3 = \frac{a+2t}{t-3}$	M1dep	
$w = \sqrt[3]{\frac{a+2t}{t-3}}$	A1	

Q10.

Answer	Mark	Comments
Alternative method 1		
$3mp = 3(2p + 1) + p + 5$ or $(m =)\ \frac{3(2p+1)}{3p} + \frac{p+5}{3p}$ or $(m =)\ \frac{6p+3+p+5}{3p}$	M1	oe fractions eliminated or common denominator eg $(m =)\ \frac{3p(2p+1)}{3p^2} + \frac{p(p+5)}{3p^2}$ or $(m =)\ \frac{6p^2+3p+p^2+5p}{3p^2}$
$3mp = 6p + 3 + p + 5$ or $3mp = 7p + 8$	M1dep	oe brackets expanded and fractions eliminated eg $3mp^2 = 7p^2 + 8p$ implies M2
$3mp – 7p = 8$ or $\frac{8}{3m-7}$ or $\frac{-8}{7-3m}$	M1dep	oe terms collected eg $p(3m – 7) = 8$ or $7p – 3mp = –8$ implies M3
$p = \frac{8}{3m-7}$ or $p = \frac{-8}{7-3m}$	A1	oe eg $\frac{8}{3m-7} = p$

Alternative method 2		
$3mp = 3(2p + 1) + p + 5$	M1	oe common denominator

or $(m =)\ \frac{3(2p+1)}{3p} + \frac{p+5}{3p}$ or $(m =)\ \frac{6p+3+p+5}{3p}$		eg $(m =)\ \frac{3p(2p+1)}{3p^2} + \frac{p(p+5)}{3p^2}$ or $(m =)\ \frac{6p^2+3p+p^2+5p}{3p^2}$
$m = \frac{7p+8}{3p}$ and $m = \frac{7}{3} + \frac{8}{3p}$ and $m - \frac{7}{3} = \frac{8}{3p}$	M1dep	simplifies numerator and isolates term in p eg $m = \frac{7p^2+8p}{3p^2}$ and $m = \frac{7}{3} + \frac{8}{3p}$ and $m - \frac{7}{3} = \frac{8}{3p}$ implies M2
$\frac{3m-7}{3} = \frac{8}{3p}$	M1dep	converts $m - \frac{7}{3}$ to a single fraction implies M3
$p = \frac{8}{3m-7}$ or $p = \frac{-8}{7-3m}$	A1	oe eg $\frac{8}{3m-7} = p$

Additional Guidance	
$p = \frac{8}{3m-7}$ in working but $\frac{8}{3m-7}$ on answer line	M3, A1
Allow recovery of missing brackets	
$p = \frac{8}{3m-7}$ followed by incorrect further work	M3, A0
Allow equivalences for A1 eg $p = \frac{\frac{8}{3}}{\frac{3m-7}{3}}$	M3, A1
Do not regard eg $3m(p) = 7p + 8$ as having unexpanded brackets	M1, M1dep

Section 2.11

Mark schemes

Q1.

	Answer	Mark	Comments
(a)	Shows substitution of $x = \frac{1}{2}$	M1	eg $2 \times \left(\frac{1}{2}\right)^3 + 11 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} - 9$ or $2 \times \frac{1}{8} + 11 \times \frac{1}{4} + 12 \times \frac{1}{2} - 9$ or $\frac{1}{4} + \frac{11}{4} + 6 - 9$
	Shows substitution of $x = \frac{1}{2}$ and evaluates to zero	A1	eg $2 \times \left(\frac{1}{2}\right)^3 + 11 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} - 9$ $= 0$ or $2 \times \frac{1}{8} + 11 \times \frac{1}{4} + 12 \times \frac{1}{2} - 9$ $= 0$ or $\frac{1}{4} + \frac{11}{4} + 6 - 9$ $= 0$

Additional Guidance	
Allow use of 0.5 and/or absence of multiplication signs eg1 $2(0.5)^3 + 11(0.5)^2 + 12(0.5) - 9 = 0$ eg2 $2\left(\frac{1}{8}\right) + 11\left(\frac{1}{4}\right) + 12\left(\frac{1}{2}\right) - 9$	M1A1 M1A0
Allow working in stages eg $2(0.5)^3 + 11(0.5)^2 + 12(0.5) = 9$ $9 - 9 = 0$	M1A1
Condone incorrect use of = eg $2(0.5)^3 + 11(0.5)^2 + 12(0.5) = 9 - 9 = 0$	M1A1
Condone $2 \times \frac{1}{2}^3$ or $2 \times \left(\frac{1^3}{2}\right)$ etc	
Ignore algebraic division or other substitution attempts	
Only stating $f\left(\frac{1}{2}\right)$ or only stating $f\left(\frac{1}{2}\right) = 0$	M0A0
Alt 1 $6x\ 2 + 9x - 8x - 12$ only 2 terms correct	M0

Calculation error(s) will be A0 eg1 $2 \times \left(\frac{1}{2}\right)^3 + 11 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} - 9 = \frac{1}{8} + \frac{11}{4} + 6 - 9 = 0$ eg2 $\frac{1}{4} + \frac{11}{4} + 6 - 9 = 4 + 6 - 9 = 0$	M1A0 M1A0
May be seen as synthetic division eg $\begin{array}{ccccc} & 2 & 11 & 12 & -9 \\ 0.5 & & 1 & 6 & 9 \\ \hline & 2 & 12 & 18 & 0 \end{array}$ (with the bottom right entry blank award M1A0) (with an error award M0A0)	M1A1

(b)

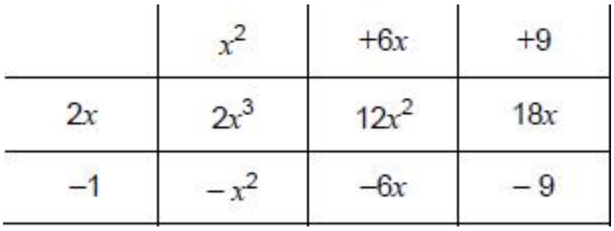
Alternative method 1		
$x^2 + 6x \ldots$ or $2 \times (-3)^3 + 11 \times (-3)^2 + 12 \times (-3) - 9$	M1	oe eg $\begin{array}{r} x^2 + 6x \ldots \\ 2x-1\overline{)2x^3+11x^2+12x-9} \end{array}$ or $(2x - 1)(x^2 + bx + c)$ and $b = 6$ or $2 \times -27 + 11 \times 9 + 12 \times -3 - 9$ or $-54 + 99 - 36 - 9$
$x^2 + 6x + 9$ **or** $(x + 3)(x + 3)$ **or** $(x + 3)^2$	M1dep	oe eg $\begin{array}{r} x^2 + 6x + 9 \\ 2x-1\overline{)2x^3+11x^2+12x-9} \end{array}$ or $(2x - 1)(x^2 + bx + c)$ and $b = 6$ and $c = 9$
$x^2 + 6x + 9$ **and** $(x + 3)(x + 3)$ or $x^2 + 6x + 9$ **and** $\frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$ or $x^2 + 6x + 9$ **and** $6^2 - 4 \times 1 \times 9$	M1dep	oe eg $x^2 + 6x + 9$ **and** $(x + 3)^2$ or $x^2 + 6x + 9$ **and** $\frac{-6}{2}$ or $x^2 + 6x + 9$ **and** $36 - 36 = 0$

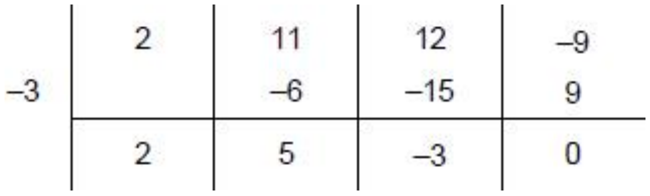
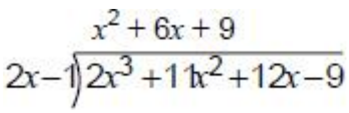
$= 0$ or $(2x – 1)(x + 3)(x + 3)$		or $(2x – 1)(x + 3)^2$
M3 **and** indication that there are exactly two solutions	A1	eg1 $x^2 + 6x + 9$ **and** $(x + 3)(x + 3)$ **and** 0.5 **and** –3 eg2 $x^2 + 6x + 9$ **and** $\frac{-6\pm\sqrt{6^2-4\times1\times9}}{2\times1}$ **and** 0.5 **and** –3 eg3 $(2x – 1)(x + 3)(x + 3)$ **and** repeated bracket so exactly two solutions/roots/answers/factors

Alternative method 2		
$6x^2 + 22x + 12 = 0$ or $(6x + 4)(x + 3) = 0$ or $\frac{-22\pm\sqrt{22^2-4\times6\times12}}{2\times6}$ or $\frac{-22\pm\sqrt{196}}{12}$	M1	condone omission of = 0 oe eg $(2x + 6)(3x + 2) = 0$ or $2(x + 3)(3x + 2) = 0$ or $-\frac{11}{6} \pm \sqrt{-2+\frac{121}{36}}$ or $-\frac{11}{6} \pm \sqrt{\frac{49}{36}}$
$x = -\frac{2}{3}$ **and** $x = −3$	M1dep	allow [–0.67, –0.66] for $-\frac{2}{3}$
$x = -\frac{2}{3}$ **and** $(−3, 0)$	M1dep	allow [–0.67, –0.66] for $-\frac{2}{3}$ ignore y-coordinate for $x = -\frac{2}{3}$ (–3, 0) may be seen on a graph
M3 **and** indication that there are exactly two solutions	A1	eg $x = -\frac{2}{3}$ **and** (–3, 0) **and** a turning point on the x-axis so two solutions/roots

Alternative method 3		
Sketch of cubic graph with	M1	condone minimum turning point

maximum turning point at (–3, 0)		at (–3, 0)
Sketch of cubic graph with maximum turning point at (–3, 0) **and** minimum turning point in the third quadrant	M1dep	
Sketch of cubic graph with maximum turning point at (–3, 0) **and** minimum turning point in the third quadrant **and** intersecting the positive x-axis at $\frac{1}{2}$	M1dep	–3 and $\frac{1}{2}$ must both be correctly labelled on the x-axis
M3 **and** indication that there are exactly two solutions	A1	eg M3 **and** 0.5 **and** –3

<table>
<tr><th colspan="2">Additional Guidance</th></tr>
<tr><td>Up to M3 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td>Alt 1 Up to the first two marks may be seen in a grid
eg

<table>
<tr><td></td><td>x^2</td><td>$+6x$</td><td>$+9$</td></tr>
<tr><td>$2x$</td><td>$2x^3$</td><td>$12x^2$</td><td>$18x$</td></tr>
<tr><td>-1</td><td>$-x^2$</td><td>$-6x$</td><td>-9</td></tr>
</table>
Condone missing + symbols in top row unless subsequently contradicted</td><td>M1M1</td></tr>
<tr><td>Alt 1 $x^2 + 6x + 9$ or $(x + 3)(x + 3)$ or $(x + 3)^2$</td><td>M1M1</td></tr>
<tr><td>Alt 1 $(2x – 1)(x + 3)(x + 3)$ or $(2x – 1)(x + 3)^2$</td><td>M1M1M1</td></tr>
<tr><td>Alt 1 $(2x – 1)(x + 3)(x + 3)$ with solutions 0.5 and –3</td><td>M1M1M1A1</td></tr>
<tr><td>Alt 1 $2x^2 + 5x – 3 = (2x – 1)(x + 3)$ 0.5 and –3</td><td>Zero</td></tr>
</table>

Alt 1 Examples of acceptable indications that there are exactly two solutions eg1 $x = 0.5, –3, –3$ (Only) two solutions eg2 $x = 0.5, –3, –3$ One root is a repeat eg3 $(2x – 1)$ gives one solution $(x + 3)(x + 3)$ gives one solution eg4 $(2x – 1)(x + 3)(x + 3)$ Two factors (only)	
Alt 1 These are not acceptable indications that there are exactly two solutions eg1 $(2x – 1)(x + 3)(x + 3)$ 3 and 0.5 eg2 $(x + 3)(x + 3)$ Exactly two solutions	
Alt 1 Ignore other substitution attempts if using factor theorem for 1st M1	
Alt 1 Allow absence of multiplication signs in factor theorem eg $2(–3)^3 + 11(–3)^2 + 12(–3) – 9$	M1
Alt 1 Condone incorrect use of = eg $2(–3)^3 + 11(–3)^2 + 12(–3) = 9 – 9$	M1
Alt 1 Allow working in stages eg $2(–3)^3 + 11(–3)^2 + 12(–3) = 9$ $9 – 9 = 0$	M1
Alt 1 Only stating f(–3) or only stating f(–3) = 0	M0
Alt 1 May be seen as synthetic division eg  $\begin{array}{c\|c\|c\|c\|c} & 2 & 11 & 12 & –9 \\ –3 & & –6 & –15 & 9 \\ \hline & 2 & 5 & –3 & 0 \end{array}$	M1
Working in (a) eg algebraic division that is not used in (b) cannot score in (b) eg (a)  $\begin{array}{r} x^2+6x+9 \\ 2x-1\overline{)2x^3+11x^2+12x-9} \end{array}$ (b) Not attempted	M0
Working in (a) eg algebraic division that is used in (b) can score in (b) eg (a) $(2x – 1)(x^2 + 6x + 9)$	M1M1

(b) Student shows an arrow from their working in (a)	

Q2.

(a)

Answer	Mark	Comments
$1^3 - 21(1) + 20 = 0$ **or** $1 - 21 + 20 = 0$	B1	Must have = 0
$4^3 - 21(4) + 20 = 0$ **or** $64 - 84 + 20 = 0$	B1	Must have = 0

(b)

$1^3 - 10(1)^2 + 29(1) - 20 = 0$ **or** $1 - 10 + 29 - 20 = 0$ Divides $x^3 - 10x^2 + 29x - 20$ by $(x - 1)$ and obtains answer $x^2 - 9x + 20$	B1	Must have = 0 B2 $(x - 1)(x - 4)(x - 5)$ **and** correct expansion of one pair of brackets eg $(x - 1)(x - 4)(x - 5)$ **and** $(x^2 - 5x + 4)(x - 5)$ B1 $(x - 1)(x - 4)(x - 5)$
$4^3 - 10(4)^2 + 29(4) - 20 = 0$ **or** $64 - 160 + 116 - 20 = 0$ Divides $x^3 - 10x^2 + 29x - 20$ by $(x - 4)$ and obtains answer $x^2 - 6x + 5$	B1	Must have = 0 B2 $(x - 1)(x - 4)(x - 5)$ **and** correct expansion of one pair of brackets eg $(x - 1)(x - 4)(x - 5)$ **and** $(x^2 - 5x + 4)(x - 5)$ B1 $(x - 1)(x - 4)(x - 5)$

(c)

$(x + 5)$ as 3rd factor of numerator	B1	Implied by final answer $\frac{x+5}{ax+b}$
$(x - 5)$ as 3rd factor of denominator	B1	Implied by final answer $\frac{cx+d}{x-5}$
$\frac{\text{their } x+5}{\text{their } x-5}$	B1ft	Do **not** award if further work

Q3.

Answer	Mark	Comments
$2^3 + a(2)^2 + b(2) + 24$	M1	oe eg $8 + 4a + 2b + 24$
$(-3)^3 + a(-3)^2 + b(-3) + 24$	M1	oe eg $-27 + 9a - 3b + 24$
$4a + 2b = -32$ **and** $9a - 3b = 3$	A1	oe Must be 2 correct equations
Multiplies equation(s) to have the same coefficient for one variable **and** attempts to eliminate by addition or subtraction eg $12a + 6b = -96$ $18a - 6b = 6$ **and** $30a = -90$	M1	Allow two errors in first stage and one error in second stage (must use the appropriate operation for elimination for their equations) oe eg substitution method used
$a = -3$ **and** $b = -10$	A1	

Alternative method		
$(x - 4)$	M1	
$x^2 - 2x + 3x - 6$ **or** $x^2 - 2x - 4x + 8$ **or** $x^2 + 3x - 4x - 12$	M1	$x^2 + x - 6$ **or** $x^2 - 6x + 8$ **or** $x^2 - x - 12$ ft their $(x - 4)$
$x^3 + x^2 - 6x - 4x^2 - 4x + 24$ **or** $x^3 - 6x^2 + 8x + 3x^2 - 18x + 24$ **or** $x^3 - x^2 - 12x - 2x^2 + 2x + 24$	M1	their $(x - 4)$ × their $(x^2 + x - 6)$ **or** $(x + 3)$ × their $(x^2 - 6x + 8)$ **or** $(x - 2)$ × their $(x^2 - x - 12)$ Allow two errors or omissions
$x^3 + x^2 - 6x - 4x^2 - 4x + 24$ **or** $x^3 - 6x^2 + 8x + 3x^2 - 18x + 24$ **or** $x^3 - x^2 - 12x - 2x^2 + 2x + 24$	A1	oe eg $x^3 - 3x^2 - 10x + 24$ Must be fully correct
$a = -3$ **and** $b = -10$	A1	

Q4.

Answer	Mark	Comments
$2a^3 - 7a^2 + 3a$	M1	Must be correct
$2a^2 - 7a + 3$	M1dep	Must be correct May also see factor a
$(2a - 1)(a - 3)$	A1	May also see factor a
3	A1ft	ft M1 M1 A0 Other solutions may be seen but 3 must be selected as their answer

Alternative method		
$(x - a)(2x^2 + 2ax - 3)$	M1	Must be correct
$- 3(x) - 2a^2(x) = - 7a(x)$	M1dep	Equating coefficients of x
$2a^2 - 7a + 3$ **and** $(2a - 1)(a - 3)$	A1	
3	A1ft	ft M1 M1 A0 Other solutions may be seen but 3 must be selected as their answer

Q5.

	Answer	Mark	Comments
(a)	$a^3 + (2a \times a^2) - (a^2 \times a) - 16 = 0$ or $2a^3 - 16 = 0$ or $a^3 - 8 = 0$	M1	
	$a^3 = 8$ or $a = \sqrt[3]{8}$ (hence $a = 2$)	A1	clearly shown

	Alternative method 1		
(b)	$(x - 2)(x^2 + ... + 8)$ (= 0)	M1	
	$(x - 2)(x^2 + 6x + 8)$ (= 0)	A1	
	$(x + m)(x + n)$ (= 0)	M1	where $mn = 8$ and $m + n = 6$

2, −2, −4	A1	
Alternative method 2		
$(x^3 + 4x^2 - 4x - 16) \div (x - 2)$ $= x^2 + ax + \ldots$	M1	Attempt at long division of polynomials a need not be correct to score M1
$x^2 + 6x + 8$	A1	
$(x + m)(x + n)$ (= 0)	M1	where $mn = 8$ and $m + n = 6$
2, −2, −4	A1	
Alternative method 3		
$(x + 4)(x^2 + \ldots)$ (= 0)	M1	
$(x + 4)(x^2 - 4)$ (= 0)	A1	
$(x + 4)(x + 2)(x - 2)$ (= 0)	M1	or $(x + 4) = 0$ or $(x^2 - 4) = 0$
2, −2, −4	A1	
Alternative method 4		
$x = 2$	B1	
testing a value of x ($x \neq 2$) to see if $f(x) = 0$	M1	
one of −2 or −4	A1	
2, −2, −4	A1	

Q6.

(a)

Answer	Mark	Comments
Alternative method 1		
$(3)^3 - 8(3)^2 + 3a + 42 = 0$ or $27 - 72 + 3a + 42 = 0$	M1	Equating to zero might not be seen until later in the working.
$3a = 3$	A1	$3a = 3$ implies $3a - 3 = 0$

Alternative method 2		
$(x^3 - 8x^2 + ax + 42) \div (x - 3)$ to give a quotient of $x^2 - 5x +$	M1	

$(a - 15)$ and a remainder of $3a - 3$		
Remainder = 0 so $3a = 3$	A1	

Alternative method 3		
$x^3 - 8x^2 + ax + 42$ $= (x - 3)(x^2 + px - 14)$ Comparing x^2 coefficients gives $p = -5$	M1	
Using $p = -5$ and comparing x coefficients gives $a = 1$	A1	

Additional Guidance	
In alt 1 ... assuming that $a = 1$ and showing that substituting $x = 3$ in the expression gives zero is only verifying the result ... and scores SC1 Similarly, assuming $a = 1$ and working as in alt 2 and alt 3 to verify the result.	SC1

(b)

Alternative method 1		
$x^3 - 8x^2 + x + 42$ $\equiv (x - 3)(x^2 + kx - 14)$	M1	Sight of quadratic with x^2 and -14 as the first and last terms
$(x + 2)$ or $(x - 7)$	A1	
$(x - 3)(x + 2)(x - 7)$	A1	any order

Alternative method 2		
Substitutes another value into the expression and tests for '= 0'	M1	their value correctly substituted eg. $2^3 - 8(2)^2 + 2 + 42 (= 20) \neq 0$
$(x + 2)$ or $(x - 7)$	A1	
$(x - 3)(x + 2)(x - 7)$	A1	any order

Alternative method 3		
Long division of polynomials getting as far as $x^2 - 5x$	M1	$(x^3 - 8x^2 + x + 42) \div (x - 3) = x^2 - 5x - 14$

$(x + 2)$ or $(x - 7)$	A1	
$(x - 3)(x + 2)(x - 7)$	A1	any order

Additional Guidance
An answer of $(x + 2)(x - 7)$ ie $(x - 3)$ missing ... implies M1 A1 An answer of $(x - 3)(x - 2)(x + 7)$ scores SC1 ... sign errors in two factors Ignore 'solutions' ie $x = 3, -2$ and 7

Q7.

(a)

Answer	Mark	Comments
$200\left(-\frac{1}{2}\right)^3 + 100\left(-\frac{1}{2}\right)^2$ $-18\left(-\frac{1}{2}\right) - 9$	M1	oe eg $200\left(-\frac{1}{8}\right) + 100\left(\frac{1}{4}\right) - 18\left(-\frac{1}{2}\right) - 9$
– 25 + 25 + 9 – 9 = 0 with M1 seen	A1	must evaluate each term and equate to zero

Additional Guidance	
Condone $\left(\frac{1}{2}\right)^2$ for $\left(-\frac{1}{2}\right)^2$	
$200\left(-\frac{1}{2}\right)^3 + 100\left(-\frac{1}{2}\right)^2 - 18\left(-\frac{1}{2}\right) - 9 = 0$	M1A0

(b)

$(100x^2 – 9)$	M1	
$(10x – 3)(10x + 3)$ or $(x =)$ $\sqrt{\frac{9}{100}}$	M1dep	oe eg $(x =)\sqrt{0.09}$
–0.5 and –0.3 and 0.3	A1	oe eg fractions

Additional Guidance	
–0.5 and –0.3 or –0.5 and 0.3 with the other solution missing implies $(100x^2 – 9)$	M1M0A0
–0.3 and 0.3 on answer line implies $(10x – 3)(10x + 3)$	M2A0

Q8.

Answer	Mark	Comments
Alternative method 1		
$(-c)^3 - 10(-c) - c$ (= 0) or $-c^3 + 10c - c$ (= 0) or $-c^3 + 9c$ (= 0)	M1	oe
$c(9 - c^2)$ (= 0) or $c(3 + c)(3 - c)$ (= 0) or $c^2 = 9$	M1dep	oe factorised expression or quadratic equation
3 with no other value(s)	A1	SC2 answer 3 with one or both of −3 and 0 and no other value

Answer	Mark	Comments
Alternative method 2		
$(x + c)(x^2 - cx - 1)$	M1	
$-1 - c^2 = -10$	M1dep	oe quadratic equation
3 with no other value(s)	A1	SC2 answer 3 with one or both of −3 and 0 and no other value

Additional Guidance	
$(-3)^3 - 10(-3) - 3 = 0$ and Answer 3 (no part marks)	M2, A1
$(-3)^3 - 10(-3) - - 3 = 0$ and Answer 3	Zero
$3^3 - 10(3) - - 3 = 0$ and Answer 3	Zero
Answer 3 with no incorrect working	M2, A1
Allow recovery of missing brackets	

Q9.

Answer	Mark	Comments

(a)

Identifies $(x =) -\frac{1}{3}$	M1	may be implied
$3\left(-\frac{1}{3}\right)^3 - 2\left(-\frac{1}{3}\right)^2$ $-7\left(-\frac{1}{3}\right) - 2 = 0$ or $-\frac{1}{9} - \frac{2}{9} + \frac{7}{3} - 2 = 0$	A1	oe must show four terms and equate to 0

(b)

Alternative method 1		
$(3x + 1)(x^2 - x \ldots)$ or $\begin{array}{r} x^2 - x - 2 \\ 3x+1\overline{)3x^3 + 4x^2 - 2x - 1} \end{array}$	M1	
$(3x + 1)(x^2 - x - 2)$ or $\begin{array}{r} x^2 - x \ldots \\ 3x+1\overline{)3x^3 + 4x^2 - 2x - 1} \end{array}$	A1	
$(3x + 1)(x + 1)(x - 2)$	A1	

Alternative method 2		
f(– 1) = 0 or f(2) = 0	M1	
f(– 1) = 0 and f(2) = 0	A1	
$(3x + 1)(x + 1)(x - 2)$	A1	

Section 2.12

Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		

$6(x^2 - 4x$ ……. or $6(x - 2)^2$ ……….	M1	oe eg $6[(x^2 - 4x)$……]
$6[(x - 2)^2 - 2^2$]…….. or $6[(x - 2)^2 - 4]$……. or $6[(x - 2)^2 - 4 + \frac{17}{6}]$ or $6[(x - 2)^2 - \frac{7}{6}]$ or $6(x - 2)^2 - 6 \times \frac{7}{6}$ or $6(x - 2)^2 - 24 + 17$	M1dep	oe the bracket is after the 2^2 and the 4 here. If they put something else inside the bracket it is incorrect unless it is equivalent to one of the fully complete versions listed
$6(x - 2)^2 - 7$	A1	

Alternative method 2		
$ax^2 + 2abx + ab^2$ $(+c)$	M1	expansion of brackets
$a = 6$ and $2ab = -24$ and $ab^2 + c = 17$	M1dep	
$b = -2$ and $c = -7$	A1	

Q2.

Answer	Mark	Comments
Alternative method 1		
sight of $2(x^2 - 8x$ ....)	M1	
sight of $2(x - 4)^2$	M1dep	
$2[(x - 4)^2 - 16] + 13$ or $2(x - 4)^2 - 32 + 13$ or $2[(x - 4)^2 - 16 + 6.5]$	M1dep	
$2(x - 4)^2 - 19$	A1	or $a = 2$, $b = -4$, $c = -19$

Alternative method 2		
$a = 2$	B1	
$-16 = 2ab$ or $-16 = 4b$ or $13 = ab^2 + c$ or $13 = 2b^2 + c$	M1	
$-16 = 2ab$ **and** $13 = ab^2 + c$ or $-16 = 4b$ **and** $13 = 2b^2 + c$	M1dep	oe
$2(x - 4)^2 - 19$	A1	or $a = 2$, $b = -4$, $c = -19$

Q3.

Answer	Mark	Comments
Alternative method 1		
$-2\left((3x+...)^2 ...\right)$	M1	from $-2\left(9x^2+6x-\frac{7}{2}\right)$ oe
$-2\left((3x+1)^2 -1^2-\frac{7}{2}\right)$	M1dep	oe
$9 – 2(3x + 1)^2$	A1	

Alternative method 2		
$-18\left(\left(x+\frac{1}{3}\right)^2 ...\right)$	M1	from $-18\left(x^2+\frac{2}{3}x-\frac{7}{18}\right)$ oe
$-18\left(\left(x+\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2-\frac{7}{18}.\right)$	M1dep	oe
$9 – 2(3x + 1)^2$	A1	

Q4.

Answer	Mark	Comments

Alternative method 1		
$(n-3)^2$	M1	Allow $(n-3)(n-3)$ for $(n-3)^2$
$(n-3)^2 - 9 + 14$ or $(n-3)^2 + 5$	A1	Allow $(n-3)(n-3)$ for $(n-3)^2$
$(n-3)^2 \geq 0$ then adding 5 so always positive or States minimum value is 5 or States (3, 5) is minimum point	A1ft	oe Allow $(n-3)(n-3)$ for $(n-3)^2$ ft M1 A0 Must see M1 and attempt $(n-3)^2 + k$ ft $(n-3)^2 + k$ where $k > 0$ SC2 States minimum value is 5 or States (3, 5) is minimum point

Alternative method 2		
Quadratic curve sketched in first quadrant with minimum point above the x-axis	M1	Labelling on axes not required
(discriminant =) −20	A1	
States no (real) roots	A1ft	oe Allow roots ⟶ solutions ft M1 A0 Must see M1 and attempt a discriminant ft discriminant < 0 SC2 States minimum value is 5 or States (3, 5) is minimum point

Alternative method 3		
$2n - 6 = 0$	M1	oe equation e.g. $2n = 6$ or $n = 3$
(second derivative =) 2	A1	
States minimum value is 5	A1ft	oe

or States (3, 5) is minimum point		ft M1 A0 Must see M1 and attempt a second derivative ft (second derivative) > 0 SC2 States minimum value is 5 or States (3, 5) is minimum point

Q5.

(a)

Answer	Mark	Comments
Alternative method 1		
$(x + 3)^2 - 9$ (+ 2)	M1	
$h = 3$ and $k = -7$	A1	

Alternative method 2		
$x^2 + 2hx + h^2$ (+ k) or $2hx = 6x$ or $2h = 6$ or $h^2 + k = 2$	M1	
$h = 3$ and $k = -7$	A1	

Additional Guidance	
$h = 3$ implies M1	

(b)

$(-3, -7)$	B1 ft	ft their h and k from part (a) only if $h \neq 0$ and $k \neq 0$

Additional Guidance	
for their h and k, the minimum point is $(-h, k)$	

(c)

$-3 \pm \sqrt{7}$	B1 ft	ft their h and k from part (a) only if $h \neq 0$ and $k \neq 0$

Additional Guidance
For their h and k, the solutions are $-h \pm \sqrt{(-k)}$

If their k is > 0 then $\sqrt{(-k)}$ will be $\sqrt{}$ of a negative number ... condone

Any use of the quadratic formula must be completely correct

Q6.

Answer	Mark	Comments
Alternative method 1		
$12(x^2 - 5x)$ … or $12(x - 2.5)^2$ …	M1	oe eg $12\{(x^2 - 5x)$ …} or $12(x^2 - 5x$ …)
$12\{(x - 2.5)^2 - 2.5^2\}$… or $12(x - 2.5)^2 - 75$ …	M1dep	oe eg $12\{(x - 2.5)^2 - 2.5^2$ …}
$12(x - 2.5)^2 - 12 \times 2.5^2 + 5$ or $12(x - 2.5)^2 - 70$	M1dep	oe eg $12(x - 2.5)^2 - 12 \times 2.5^2 + 12 \times \frac{5}{12}$
$12\left(\frac{2x-5}{2}\right)^2 - 12 \times 2.5^2 + 5$	M1dep	oe eg $12\left(\frac{2x-5}{2}\right)^2 - 12 \times 2.5^2 + 12 \times \frac{5}{12}$
$3(2x - 5)^2 - 70$ or $a = 3 \quad b = 2 \quad c = -5 \quad d = -70$ or $3(5 - 2x)^2 - 70$ or $a = 3 \quad b = -2 \quad c = 5 \quad d = -70$	A1	oe
Alternative method 2		
$3(4x^2 - 20x)$ … or $3(2x - 5)^2$ …	M1	oe eg $3\{(4x^2 - 20x)$ …} or $3(4x^2 - 20x$ …)
$3\{(2x - 5)^2 - 5^2\}$ …	M1dep	oe

or $3(2x-5)^2-75$ …		eg $3\{(2x-5)^2-5^2…\}$
$3\{(2x-5)^2-5^2\}+5$	M1dep	oe eg $3\{(2x-5)^2-5^2+\frac{5}{3}\}$
$3(2x-5)^2-3\times5^2+5$	M1dep	oe eg $3(2x-5)^2-3\times5^2+3\times\frac{5}{3}$
$3(2x-5)^2-70$ or $a=3$ $b=2$ $c=-5$ $d=-70$ or $3(5-2x)^2-70$ or $a=3$ $b=-2$ $c=5$ $d=-70$	A1	oe

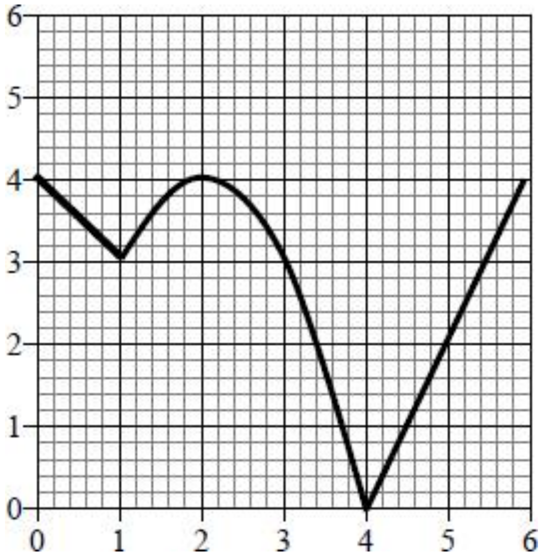
Additional Guidance	
For M marks 2.5 may be seen as $\frac{5}{2}$	
For M marks $(x-2.5)^2$ may be replaced by $(2.5-x)^2$ etc	
Expansion of given form followed by trial and improvement	
eg1 $3(2x-5)^2-70$ (or $a=3$ $b=2$ $c=-5$ $d=-70$)	5 marks
eg2 Not fully correct	Zero

Section 2.13
Mark schemes

Q1.

	Answer	Mark	Comments
(a)	Line joining (0,4) and (1,3)	B1	may be drawn free hand
	(2, 4) plotted as a maximum value	M1	needs to be some sort of graph showing a maximum value

Curve drawn through (1, 3), (2, 4), (3, 3) and (4, 0)	A1	all points should be within half a square horizontally or vertically
Line joining (4,0) and (6,4)	B1	may be drawn free hand

Additional Guidance

Maximum mark available if either or both of the straight lines include a curve (they haven’t used a ruler) is B1M1A1
Maximum mark available if any part of the quadratic curve (it must be a quadratic curve that is concave and not convex at any point) is drawn with a ruler is B2M1 (a clear vertex at (3,3) may show the use of a ruler)
Maximum mark available if any of the lines go beyond their correct domain by more than half a square is B2M1 or B1M1A1
They could lose marks for both the quadratic and straight lines
Ignore slight feathering

(b)

Alternative method 1		
Rearranging first to get $x = \frac{6 - g(x)}{3}$	M1	oe eg $x = \frac{6 - y}{3}$ or $2 - \frac{y}{3}$ $y - 6 = -3x$ is not enough to gain M1
$g^{-1}(x) = \frac{6 - x}{3}$	A1	oe eg $g^{-1}(x) = \frac{x - 6}{-3}$ or $g^{-1}(x) = -\frac{x - 6}{3}$ or $g^{-1}(x) = \frac{x}{-3} + 2$

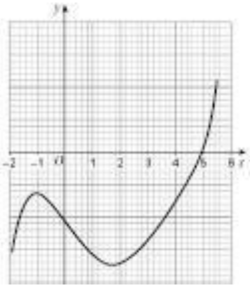
Alternative method 2		
Putting the correct terminology in to get $x = 6 - 3g^{-1}(x)$	M1	oe eg $x = 6 - 3y$ or $3y = 6 - x$
$g^{-1}(x) = \frac{6-x}{3}$	A1	oe eg $g^{-1}(x) = \frac{x-6}{-3}$ or $g^{-1}(x) = -\frac{x-6}{3}$ or $g^{-1}(x) = \frac{x}{-3}+2$

Additional Guidance	
Answer left as $y = \frac{6-x}{3}$ should gain M1 on either scheme.	M1A0
$x = \frac{6-y}{3}$ can gain M1 but not A1	M1A0
Condone $g^{-1}(x)$ missed on answer line (as long as nothing else is written in its place)	
Flow charts may be used. Mark as oe	
Penalise additional incorrect working	

Q2.

Answer	Mark	Comments
Cubic curve from $x = –2$ to $x = 6$ and maximum point at (–1, a) where a is negative and minimum point at (2, b) where b is less than a and increasing through (5, 0)	B4	B3 curve from $x = –2$ to $x = 6$ and maximum point at (–1, c) where c is any value and minimum point at (2, d) where d is less than c and d is negative and increasing through (5, 0) or a B4 response apart from cubic curve not drawn from $x = –2$ to $x = 6$

		B2 curve with maximum point at (–1, *e*) where *e* is any value and minimum point at (2, *f*) where *f* is less than *e* B1 curve with maximum point at (–1, *g*) where *g* is negative or curve with minimum point at (2, *h*) where *h* is negative or curve increasing through (5, 0) SC2 max and min correct and increasing through (5, 0) but with straight lines rather than a curve.

Additional Guidance	
	B4

Q3.

Answer	Mark	Comments
(gradient for $0 \le x \le 4$ =) $\frac{12}{4}$ **or** 3	M1	oe
(gradient for $4 < x \le 8$ =) $\frac{12}{-4}$ **or** −3	M1	oe Accept − their 3
y = their $-3x + c$ **and** substitutes (8,0) or (4,12)	M1	$y - 0$ = their $-3(x - 8)$ **or** $y - 12$ = their $-3(x - 4)$
$3x$ **and** $-3x + 24$ or $-3(x - 8)$	A2	A1 $3x$ **or** $-3x + 24$ or $-3(x - 8)$

in correct places on answer lines		in correct place on answer line **or** $y = 3x$ (for $0 \le x \le 4$) **or** $y = -3x + 24$ or $y = -3(x - 8)$ (for $4 < x \le 8$)

Q4.

Answer	Mark	Comments
($f(x)$ =) $-x^2$	B1	
($f(x)$ =) -4	B1	
($f(x)$ =) $4x - 16$	B1	
All domains correctly paired with the functions using the correct notation for the domains eg $-1 \le x < 2$	B1	Accept use of < or ≤ do not accept (eg) $-1 \le -x^2 < 2$

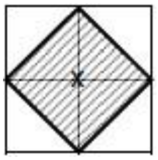
Q5.

Answer	Mark	Comments
Straight line through (−3, 0) and (0, 3)	B1	Lines must be ruled Only penalise (by 1 mark) extended lines if B1 B1 B1 SC2 Any graph that passes through (–3, 0) **and** (0, 3) **and** (1, 3) **and** (2, 1)
Straight line through (0, 3) and (1, 3)	B1	
Straight line through (1, 3) and (2, 1)	B1	

Q6.

	Answer	Mark	Comments
(a)	Straight line between (–2, 7) and (0, 3)	B1	Tolerance of ±1 small square Allow line to be extended
	Points (0, 3) (1, 4) (2, 3) (3, 0) (4, –5)	M1	Tolerance of ±1 small square May be plotted or seen in a table Points can be implied

Correct smooth parabolic curve with maximum at (1, 4)	A1	Tolerance of ±1 small square Allow (ruled) straight line between (3, 0) and (4, –5) Curve passing through all correct points within tolerance scores M1A1
Straight line between (4, –5) and (5, 0)	B1	Tolerance of ±1 small square Allow line to be extended

Additional Guidance	
Ignore extra points plotted	
Tolerance of ±1 small square means it is on the edges of or within the shaded area 	
Points only can score a maximum of M1	
Ruled straight lines for curve apart from between (3, 0) and (4, –5)	A0
If all 4 marks would be awarded but either (i) graph has a line or a curve that extends beyond the individual domains or (ii) the curve does not meet a line at a cusp	3 marks

(b)

$-5 \leq f(x) \leq 7$ or $7 \geq f(x) \geq -5$ or [–5, 7]	B2ft	Correct or ft their graph in (a) for B2 ft their graph in (a) for B1 B1ft $-5 \leq f(x)$ or $f(x) \leq 7$ on their own or embedded within an interval for f(x) or only –5 and 7 chosen eg $-5 < f(x) < 7$

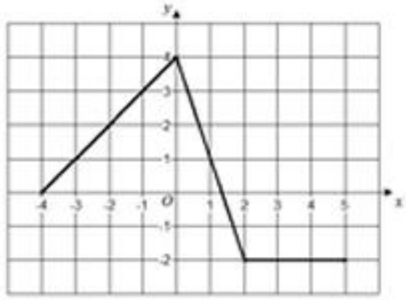
Additional Guidance

Allow f(x) to be y or f or fx eg1 –5 ≤ y ≤ 7 eg2 f ≤ 7	 B2 B1
Allow as two inequalities f(x) ≥ –5 (and/or) f(x) ≤ 7	B2
ft their graph if incomplete eg no graph drawn for –2 ≤ x < 0 but otherwise correct and answer –5 ≤ f(x) ≤ 4	B2ft
ft their graph if drawn for x values beyond [–2, 5] eg 1 straight line from (–3, 8) to (6, –1) and answer –1 ≤ y ≤ 8 eg 2 straight line from (–3, 8) to (6, –1) and answer f(x) ≤ 8	 B2ft B1ft
Straight line from (–2, 9) to (6, –7) and answer –7 ≤ y ≤ 9	B2ft
Straight line from (0, 9) to (5, –4) and answer –4 ≤ f(x) ≤ 9	B2ft
B2ft (or B1ft) can be awarded for a range beyond [–7, 9] if it is clear from working (eg a table of values) where the answer is from	
–5 to 7 inclusive is B2 whereas –5 to 7 is B1	
B1 for a correct inequality embedded eg 1 –5 < f(x) ≤ 7 eg 2 –5 ≤ f(x) ≤ 0 eg 3 –2 ≤ y ≤ 7	 B1 B1 B1
For B1 ignore incorrect notation if only –5 and 7 chosen eg 1 –5 ≤ x ≤ 7 eg 2 –5 < x ≤ 7 eg 3 –5 ≥ f(x) ≥ 7 eg 4 –5, 7	 B1 B1 B1 B1
{–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7}	B0
Working out a statistical range eg –5 to 7 = 12	B0

Q7.

Answer	Mark	Comments
Line from (−4, 0) to (0, 4)	M1	mark intention lines do not have to be straight but must pass through all integer
Line from (0, 4) to (2, −2)	M1	

		points
Line from (2, −2) to (5, −2)	M1	only condone the first instance of a line that extends beyond the given domain
Straight line from (−4, 0) to (0, 4) and straight line from (0, 4) to (2, −2) and straight line from (2, −2) to (5, −2)	A1	all straight lines must be the correct length with no other lines graph must be accurate SC3 (−4, 0) and (−3, 1) and (−2, 2) and (−1, 3) and (0, 4) and (1, 1) and (2, −2) and (3, −2) and (4, −2) and (5, −2) plotted (any other points plotted must be correct ones for the graph) SC2 (−4, 0) and (0, 4) and (2, −2) and (5, −2) plotted (any other points plotted must be correct ones for the graph)

Additional Guidance	
 (crosses do not have to be shown)	M3A1
Dashed or dotted lines can score up to M3A0	
Points may be implied by a correct line	
M mark examples eg1 2 correct lines and 1 extended line (but otherwise correct) eg2 1 correct line and 2 extended lines (but otherwise correct) eg3 3 extended lines (but otherwise correct)	 M3A0 M2A0 M1A0

Q8.

Answer	Mark	Comments
–3 2 6 14 with no other solutions	B4	B3 three correct with at most one incorrect B2 two correct with at most two

		incorrect B1 one correct with at most three incorrect SC2 –3 2 6 14 with no other values seen SC1 Two or three of –3 2 6 14 with no other values seen

Additional Guidance	
Solutions may be in any order eg1 –3 14 6 2 eg2 14 –3	 B4 B2
$x < -3$ $2 < x < 6$ $x > 14$	SC2
$2 \leqslant x \leqslant 6$	SC1
–3 2 6 14 seen in working with no other values and answer line $-3 \leqslant x \leqslant 14$	SC2

Q9.

Answer	Mark	Comments
$a = 1$	B1	
$b = 2$	B1	
$\frac{4-3}{5-2}$ or $\frac{1}{3}$	M1dep	oe eg $\frac{3-4}{2-5}$ or $\frac{-1}{-3}$
$c = \frac{1}{3}$ and $d = \frac{7}{3}$	A1	

Additional Guidance	
$(x-1)^2 + 2$	B2
$\frac{1}{3}x + \frac{7}{3}$	M1A1

Section 2.14 – 2.15

Mark schemes

Q1.

Answer	Mark	Comments
$x - 4$ or $4 - x$ seen in working	M1	from a subtraction of the quadratic and linear
$y = x - 4$ drawn	A1	
5.3 and 1.7 and $y = x - 4$ drawn	A1	Allow [5.2, 5.4] and [1.6,1.8]

Additional Guidance	
Solutions with correct graph not seen eg from formula	M0A0A0
Solutions from quadratic graph drawn	M0A0A0

Q2.

Answer	Mark	Comments
$-4 = ab^{-1}$ or $-4 = \frac{a}{b}$	M1	oe eg $a = -4b$
$-\frac{4}{3} = ab^{-2}$ or $-\frac{4}{3} = \frac{a}{b^2}$	M1	oe $a = -\frac{4}{3}b^2$
$b = 3$	A1	
$a = -12$	A1	

Q3.

	Answer	Mark	Comments
(a)	Correct line with $-1\frac{1}{2}$ labelled	B2	B1 For line through (3, 0) without $-1\frac{1}{2}$ labelled or

		for line with positive gradient through (0, $-1\frac{1}{2}$) (labelled), but not passing through (3, 0)

(b)

$x(x-3)=\frac{(x-3)}{2}$	M1	oe eg $2x^2 - 6x = x - 3$ or $2x^2 - 7x + 3 = 0$ or $x^2 - 3.5x + 1.5 = 0$ or $x^2 - 3.5x + 1.5 = 0$
$x=\frac{1}{2}$	A1	

Q4.

Answer	Mark	Comments
$2 - x$ or $x - 2$	M1	do not award M1 if you see evidence of incorrect method for finding a linear expression
$y = 2 - x$ accurately drawn	M1	
3.4	A1	accept 3.3 to 3.5
0.6	A1	accept 0.5 to 0.7

Additional Guidance
For the first M1, start by looking for evidence of a correct method. eg $x^2 - 4x + 2 + 3x - x^2 = -x + 2$ or $x^2 - 4x + 2 = 0 \longrightarrow x^2 - 3x - x + 2 = 0 \longrightarrow -x + 2 = 3x - x^2$
Attempts to solve $x^2 - 4x + 2 = 0$ by using the quadratic formula or by completing the square or by drawing a new quadratic graph (for $y = x^2 - 4x + 2$) score 0 marks You might see work which uses the quadratic formula or completing the square which leads to answers of 2 ± √2 ... and if this follows working using a correct method to find the linear graph, it can be ignored (they could be using it as a check on their answers obtained graphically), but if it looks like it is their main method, then award 0 marks, as stated above.. Ignore any y coordinates that might accompany the final x values.

Q5.

Answer	Mark	Comments
Alternative method 1		
Correct substitution $x - \frac{-3}{x} = \frac{19}{4}$ or $x\left(x - \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
$4x^2 – 19x + 12$ (= 0)	M1dep	oe eg $4x^2 + 12 = 19x$ must be integer values unless going on to complete the square
$(4x + a)(x + b)$ or $(4x − 3)(4x − 16)$	M1dep	where $ab = 12$ or $a + 4b = –$ 19
$(4x – 3)(x – 4)$	A1	
$x = \frac{3}{4}$ and 4 or $x = \frac{3}{4}$ and $y = −4$ or $x = 4$ and $y = −\frac{3}{4}$	A1	
$y = −4$ and $−\frac{3}{4}$ or $x = 4$ and $y = −\frac{3}{4}$ or $x = \frac{3}{4}$ and $y = −4$	A1	all 4 values must be correct to gain this mark

Answer	Mark	Comments
Alternative method 2		
Correct substitution $x - \frac{-3}{x} = \frac{19}{4}$ or $x\left(x - \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
$4x^2 – 19x + 12$ (= 0)	M1dep	oe eg $4x^2 + 12 = 19x$ must be integer values unless going on to complete the square

$\dfrac{19 \pm \sqrt{19^2 - 4 \times 4 \times 12}}{2 \times 4}$	M1dep	
$\dfrac{19 \pm \sqrt{169}}{8}$	A1	
$x = \dfrac{3}{4}$ and 4 or $x = \dfrac{3}{4}$ and $y = -4$ or $x = 4$ and $y = -\dfrac{3}{4}$	A1	
$y = -4$ and $-\dfrac{3}{4}$ or $x = 4$ and $y = -\dfrac{3}{4}$ or $x = \dfrac{3}{4}$ and $y = -4$	A1	all 4 values must be correct to gain this mark

Alternative method 3		
Correct substitution $x - \dfrac{-3}{x} = \dfrac{19}{4}$ or $x\left(x - \dfrac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
$4x^2 - 19x + 12$ (= 0) or $x^2 - \dfrac{19}{4}x + 3$ (= 0)	M1dep	oe eg $4x^2 + 12 = 19x$ must be integer values unless going on to complete the square
$4\left[\left(x - \dfrac{19}{8}\right)^2 \ldots\ldots\right] \ldots\ldots$ or $\left[\left(x - \dfrac{19}{8}\right)^2 \ldots\ldots\right] \ldots\ldots$	M1	oe
$4\left(x - \dfrac{19}{8}\right)^2 - \dfrac{169}{16} = 0$ or $\left[\left(x - \dfrac{19}{8}\right)^2\right] - \dfrac{169}{64} = 0$	M1dep	
$x = \dfrac{3}{4}$ and 4	A1	

or $x = \frac{3}{4}$ and $y = -4$ or $x = 4$ and $y = -\frac{3}{4}$		
$y = -4$ and $-\frac{3}{4}$ or $x = 4$ and $y = -\frac{3}{4}$ or $x = \frac{3}{4}$ and $y = -4$	A1	all 4 values must be correct to gain this mark

Alternative method 4		
Correct substitution $\frac{-3}{y} - y = \frac{19}{4}$ or $y\left(y + \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
$4y^2 + 19y + 12$ (= 0)	M1dep	oe eg $4y^2 + 12 = -19y$ must be integer values unless going on to complete the square
$(4y + a)(y + b)$ or $(4y + 3)(4y + 16)$	M1dep	where $ab = 12$ or $a + 4b = 19$
$(4y + 3)(y + 4)$	A1	
$y = -\frac{3}{4}$ and -4 or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	
$x = 4$ and $\frac{3}{4}$ or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	all 4 values must be correct to gain this mark

Alternative method 5		
Correct substitution $\frac{-3}{y} - y = \frac{19}{4}$ or $y\left(y + \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered

$4y^2 + 19y + 12$ (= 0)	M1dep	oe eg $4y^2 + 12 = -19y$ must be integer values unless going on to complete the square
$\frac{-19 \pm \sqrt{19^2 - 4 \times 4 \times 12}}{2 \times 4}$	M1dep	
$-\frac{19 \pm \sqrt{169}}{8}$	A1	
$y = -\frac{3}{4}$ and -4 or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	
$x = 4$ and $\frac{3}{4}$ or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	all 4 values must be correct to gain this mark

Alternative method 6		
Correct substitution $\frac{-3}{y} - y = \frac{19}{4}$ or $y\left(y + \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
$4y^2 - 19y + 12$ (= 0) or $y^2 - \frac{19}{4}y + 3$ (= 0)	M1dep	oe eg $4y^2 + 12 = 19y$ must be integer values unless going on to complete the square
$4\left[\left(y + \frac{19}{8}\right)^2 .....\right] .....$ or $\left[\left(y + \frac{19}{8}\right)^2 .....\right] .....$	M1	oe
$4\left(y + \frac{19}{8}\right)^2 - \frac{169}{16} = 0$ or $\left[\left(y + \frac{19}{8}\right)^2\right] - \frac{169}{64} = 0$	M1dep	

$y = -\frac{3}{4}$ and −4 or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	
$x = 4$ and $\frac{3}{4}$ or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	all 4 values must be correct to gain this mark

Additional Guidance	
Correct A marks must come from correct algebra in M marks	

Q6.

Answer	Mark	Comments
Alternative method 1		
$10x^2 + 5x(x - 2) - 7(x - 2)^2 + 23$ (= 0)	M1	oe
$10x^2 + 5x^2 - 10x - 7x^2 + 28x - 28 + 23$ (= 0)	M1dep	allow one sign error oe eg $8x^2 + 18x - 5$ (= 0)
$(4x - 1)(2x + 5)$ or $\frac{-18 \pm \sqrt{18^2 - 4 \times 8 \times -5}}{2 \times 8}$ or $-\frac{9}{8} \pm \sqrt{\frac{121}{64}}$	M1dep	oe
$x = \frac{1}{4}$ and $x = -\frac{5}{2}$ or		oe values

$x = \frac{1}{4}$ and $y = -\frac{7}{4}$ or $x = -\frac{5}{2}$ and $y = -\frac{9}{2}$	A1	
$x = \frac{1}{4}$ and $y = -\frac{7}{4}$ and $x = -\frac{5}{2}$ and $y = -\frac{9}{2}$	A1	oe values

Alternative method 2		
$10(y + 2)^2 + 5y(y + 2) – 7y^2 + 23$ (= 0)	M1	oe
$10y^2 + 40y + 40 + 5y^2 + 10y – 7y^2$ + 23 (= 0)	M1dep	allow one sign error oe eg $8y^2 + 50y + 63$ (= 0)
$(4y + 7)(2y + 9)$ or $\frac{-50 \pm \sqrt{50^2 - 4 \times 8 \times 63}}{2 \times 8}$ or $-\frac{25}{8} \pm \sqrt{\frac{121}{64}}$	M1dep	oe
$y = -\frac{7}{4}$ and $y = -\frac{9}{2}$ or $y = -\frac{7}{4}$ and $x = \frac{1}{4}$ or $y = -\frac{9}{2}$ and $x = -\frac{5}{2}$	A1	oe values
$y = -\frac{7}{4}$ and $x = \frac{1}{4}$		oe values

and $y = -\frac{9}{2}$ and $x = -\frac{5}{2}$	A1	

Q7.

Answer	Mark	Comments
Alternative method 1		
$x^2 + (2x)^2 = 20$ or $\sqrt{20 - x^2} = 2x$	M1	oe Condone absence of brackets
$5x^2 = 20$ or $5x^2 - 20$ (= 0)	M1	oe eg $x^2 = 4$ Collects terms for their quadratic to $ax^2 = b$ or $ax^2 - b$ (= 0) a and b both non-zero This mark implies the first M1
$\sqrt{\frac{20}{\text{their } 5}}$ or $x = \sqrt{4}$ or $5(x + 2)(x - 2)$ (= 0)	M1	Correct attempt to solve their quadratic oe eg $(x + 2)(x - 2)$ (= 0) If using formula must substitute correctly If using completing the square must correctly obtain $(px + q)^2 = r$ or $(px + q)^2 - r$ (= 0) p, q and r non-zero
$x = 2$ and $x = -2$ or $x = 2$ and $y = 4$ or $x = -2$ and $y = -4$	A1	Allow $x = \pm 2$
D (2, 4) and E (−2 , −4)	A1	Correct letter must be linked to correct point SC2 Both points correct by T & I SC1 One point correct by T & I

Alternative method 2		
$\left(\frac{y}{2}\right)^2 + y^2 = 20$ or $\sqrt{20 - y^2} = \frac{y}{2}$	M1	oe Condone absence of brackets
$5y^2 = 80$ or $\frac{5}{4}y^2 = 20$ or $5y^2 - 80 = 0$	M1	oe eg $y^2 = 16$ Collects terms for their quadratic to $ay^2 = b$ or $ay^2 - b$ (= 0) a and b both non-zero This mark implies the first M1
$\sqrt{\frac{80}{\text{their } 5}}$ or $y = \sqrt{16}$ or $5(y + 4)(y - 4)$ (= 0)	M1	Correct attempt to solve their quadratic oe eg $(y + 4)(y - 4)$ (= 0) If using formula must substitute correctly If using completing the square must correctly obtain $(py + q)^2 = r$ or $(py + q)^2 - r$ (= 0) p, q and r non-zero
$y = 4$ and $y = -4$ or $y = 4$ and $x = 2$ or $y = -4$ and $x = -2$	A1	Allow $y = \pm 4$
D (2, 4) and E (−2 , −4)	A1	Correct letter must be linked to correct point SC2 Both points correct by T & I SC1 One point correct by T & I

Q8.

Answer	Mark	Comments
$x - 1 = 3(y - 2)$ or	M1	oe Rearranging one of the two equations $x - 1 = 3y - 6$ or $x + 6 = 4y - 4$

$x + 6 = 4(y - 1)$		
$x - 3y = -5$ oe	M1	ft from their equations (no further errors) oe eg attempts substitution and rearranges to a suitable form (earns M2)
$x - 4y = -10$ oe	M1	
$x = 10$ or $y = 5$	A1ft	Correct elimination from their equations if at least M1 earned
$x = 10$ and $y = 5$	A1	SC1 for $x = 10$ and $y = 5$ from no (or incorrect) working

Q9.

Answer	Mark	Comments
Alternative method 1		
$3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$	M1	oe
$3x^2 + 5x - 2$ (= 0) or $3x^2 + 5x = 2$	M1dep	
$(3x + a)(x + b)$ (= 0)	M1dep	$ab = -2$ or $a + 3b = 5$
$(3x - 1)(x + 2)$ (= 0)	A1	
$x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$	A1	
$x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or $y = 6$ $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs

Alternative method 2		
$3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$	M1	oe

$3x^2 + 5x - 2$ (= 0) or $3x^2 + 5x = 2$	M1dep	
$x = \frac{-5 \pm \sqrt{[(5)^2 - 4(3)(-2)]}}{2(3)}$	M1dep	allow one sign error ... but the 2 × 3 term must be beneath the full numerator
$x = \frac{-5 \pm 7}{6}$	A1	
$x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$	A1	
$x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or $y = 6$ $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs

Alternative method 3		
$3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$	M1	oe
$3x^2 + 5x - 2$ (= 0) or $3x^2 + 5x = 2$	M1dep	
$(3 \times)\ (x + \frac{5}{6})^2$	M1dep	
$x + \frac{5}{6} = \pm \frac{7}{6}$	A1	
$x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$	A1	
$x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or $y = 6$ $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs

Alternative method 4		
$y = 3\left(\frac{2}{y}\right) + 5$ or $\frac{y(y-5)}{3}$	M1	oe
$y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$	M1dep	
$(y + a)(y + b)$ (= 0)	M1dep	$ab = -6$ or $a + b = -5$
$(y - 6)(y + 1)$ (= 0)	A1	
$y = 6$ $y = -1$ or $y = 6$ $\frac{1}{3}$ or $y = -1$ $x = -2$	A1	
$x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or $y = 6$ $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs

Alternative method 5		
$y = 3\left(\frac{2}{y}\right) + 5$ or $\frac{y(y-5)}{3}$	M1	oe
$y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$	M1dep	
$y = \frac{5 \pm \sqrt{[(-5)^2 - 4(1)(-6)]}}{2(1)}$	M1dep	allow one sign error ... but the 2 × 1 term must be beneath the full numerator
$y = \frac{5 \pm 7}{2}$	A1	
$y = 6$ $y = -1$ or $y = 6$ $\frac{1}{3}$ or $y = -1$ $x = -2$	A1	
$x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or $y = 6$ $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs

Alternative method 6		
$y = 3\left(\frac{2}{y}\right) + 5$ or $\frac{y(y-5)}{3} = 2$	M1	oe
$y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$	M1dep	
$(y - \frac{5}{2})^2$	M1dep	
$y - \frac{5}{2} = \pm \frac{7}{2}$	A1	
$y = 6$ $y = -1$ or $y = 6$ $\frac{1}{3}$ or $y = -1$ $x = -2$	A1	
$x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or $y = 6$ $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs

Additional Guidance
Trial and improvement ... 0 marks No working shown ..... 0 marks The instructions were clearly stated in the question.

Q10.

Answer	Mark	Comments
$(4 - x)^2 = 4x + 5$	M1	
$16 - 4x - 4x + x^2 = 4x + 5$	M1Dep	Allow one error but must be a quadratic in x
$x^2 - 12x + 11$ (= 0)	A1	oe Must be 3 terms
$(x - 11)(x - 1)$ (= 0)	M1	$\frac{--12 \pm \sqrt{(-12)^2 - 4(1)(11)}}{2}$ **or** $(x - 6)^2 - 36 + 11 = 0$ oe
$x = 11$ and $x = 1$	A1ft	Must have M3 to ft

		x = 11 and y = –7 **or** x = 1 and y = 3
x = 11 and y = –7 **and** x = 1 and y = 3	A1	

Alternative method		
$y^2 = 4(4 - y) + 5$	M1	
$y^2 = 16 - 4y) + 5$	M1dep	Allow one error but must be a quadratic in y
$y^2 + 4y - 21$ (= 0)	A1	oe Must be 3 terms
$(y + 7)(y - 3)$ (= 0)	M1	$\frac{-4\pm\sqrt{4^2-4(1)(-21)}}{2}$ **or** $(y + 2)^2 - 4 - 21 = 0$ oe
$y = -7$ and $y = 3$	A1ft	Must have M3 to ft x = 11 and y = −7 **or** x = 1 and y = 3
x = 11 and y = –7 **and** x = 1 and y = 3	A1	

Q11.

Answer	Mark	Comments
Alternative method 1		
$(x - 2)^2 + (2x + 1 – 1)^2 = 16$	M1	oe Eliminates y
$x^2 – 2x - 2x + 4 + 4x^2 = 16$ or $5x^2 – 4x - 12$ (= 0)	M1dep	oe Expands both brackets correctly
$(5x + 6)(x - 2)$ (= 0) or $\frac{--4\pm\sqrt{(-4)^2-4\times5\times-12}}{2\times5}$	M1	oe eg $\frac{2}{5}\pm\sqrt{\frac{64}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula

		Allow 4^2 for $(-4)^2$ Implied by correct solutions to their 3-term quadratic seen
(x =) –1.2 and (x =)2 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg (x =) $-\frac{6}{5}$ and (x =) 2 with $5x^2 - 4x - 12$ (= 0) seen
(–1.2, –1.4) and (2, 5) with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg ($-\frac{6}{5}$, $-\frac{7}{5}$) and 2, 5) with $5x^2 - 4x - 12$ (= 0) seen

Alternative method 2		
$x^2 - 2x - 2x + 4 + y^2 - y - y + 1 = 16$	M1	oe Expands both brackets correctly
$x^2 - 2x - 2x + 4 + (2x + 1)^2$ $-(2x + 1) - (2x + 1) + 1 = 16$ or $5x^2 - 4x - 12$ (= 0)	M1dep	oe Eliminates y
$(5x + 6)(x - 2)$ (= 0) or $\frac{--4 \pm \sqrt{(-4)^2 - 4 \times 5 \times -12}}{2 \times 5}$	M1	$\frac{2}{5} \pm \sqrt{\frac{64}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 4^2 for $(-4)^2$ Implied by correct solutions to their 3-term quadratic seen
(x =) –1.2 and (x =) 2 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg (x =) $-\frac{6}{5}$ and (x =) 2 with $5x^2 - 4x - 12$ (= 0) seen
(–1.2, –1.4) and (2, 5) with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg ($-\frac{6}{5}$, $-\frac{7}{5}$) and 2, 5) with $5x^2 - 4x - 12$ (= 0) seen

Alternative method 3		
$(\left(\frac{y-1}{2}\right)-2)^2+(y-1)^2=16$	M1	or Eliminates x
$\left(\frac{y-1}{2}\right)^2-2\left(\frac{y-1}{2}\right)-2\left(\frac{y-1}{2}\right)$ $y^2-y-y+1=16$ or $5y^2-18y-35$ (= 0)	M1dep	oe Expands $(\left(\frac{y-1}{2}\right)-2)^2$ and $(y-1)^2$ correctly
$(5y+7)(y-5)$ (= 0) or $\frac{--18\pm\sqrt{(-18)^2-4\times5\times-35}}{2\times5}$	M1	oe eg $\frac{9}{5}\pm\sqrt{\frac{256}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 18^2 for $(-18)^2$ Implied by correct solutions to their 3-term quadratic seen
(y =) –1.4 and (y =) 5 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5y^2-18y-35$ (= 0) seen	A1	oe eg (y =) $-\frac{7}{5}$ and (y =) 5 with $5y^2-18y-35$ (= 0) seen
(–1.2, –1.4) and (2, 5) with $5y^2-18y-35$ (= 0) seen	A1	oe eg ($-\frac{6}{5}$, $-\frac{7}{5}$) and 2, 5) with $5y^2-18y-35$ (= 0) seen

Alternative method 4		
$x^2-2x-2x+4+y^2-y-y+1=16$	M1	oe Expands both brackets correctly
$\left(\frac{y-1}{2}\right)^2-2\left(\frac{y-1}{2}\right)-2\left(\frac{y-1}{2}\right)$ $4+y^2-y-y+1=16$ or $5y^2-18y-35$ (= 0)	M1dep	oe Eliminates x

$(5y + 7)(y - 5)$ (= 0) or $\frac{--18 \pm \sqrt{(-18)^2 - 4 \times 5 \times -35}}{2 \times 5}$	M1	oe eg $\frac{9}{5} \pm \sqrt{\frac{256}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 18^2 for $(-18)^2$ Implied by correct solutions to their 3-term quadratic seen
(y =) –1.4 and (y =) 5 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5y^2 - 18y - 35$ (= 0) seen	A1	oe eg (y =) $-\frac{7}{5}$ and (y =) 5 with $5y^2 - 18y - 35$ (= 0) seen
(–1.2, –1.4) and (2, 5) with $5y^2 - 18y - 35$ (= 0) seen	A1	oe eg ($-\frac{6}{5}$, $-\frac{7}{5}$) and 2, 5) with $5y^2 - 18y - 35$ (= 0) seen

Additional Guidance	
Answers only (no valid working)	Zero
Both solutions from scale drawing	5 marks
(2, 5) is often seen without seeing any correct method	Zero
Allow one miscopy for up to M3A0A0	

Section 2.16

Mark schemes

Q1.

Answer	Mark	Comments
Elimination of one variable making an equation with at least two terms correct	M1	eg1 (elimination of b by adding 1st and 2nd equations) $5a + 3c = -1$ with at least two terms correct eg2 (elimination of a by doubling

		1st equation and subtracting 3rd equation) $5b - 7c = -1$ with at least two terms correct
Elimination of one variable making an equation with at least two terms correct and elimination of the same variable making a different equation with at least two terms correct	M1dep	eg1 (elimination of b by adding 1st and 2nd equations and elimination of b by trebling 3rd equation and subtracting 1st equation) $5a + 3c = -1$ with at least two terms correct and $5a + 11c = 23$ with at least two terms correct eg2 (elimination of a by doubling 1st equation and subtracting 3rd equation and elimination of a by doubling 3rd equation and subtracting 2nd equation) $5b - 7c = -1$ with at least two terms correct and $5b + c = 23$ with at least two terms correct
Correct equation in one variable with two correct equations in the same two variables	M1dep	eg $3c - 11c = -1 - 23$ or $-8c = -24$ or $c = 3$ with $5a + 3c = -1$ and $5a + 11c = 23$
Two correct values with two correct equations in the same two variables	A1	eg $c = 3$ and $a = -2$ with $5a + 3c = -1$ and $5a + 11c = 23$
$a = -2$ $b = 4$ $c = 3$ with two correct equations in the same two variables	A1	eg $a = -2$ $b = 4$ $c = 3$ with $5a + 3c = -1$ and $5a + 11c = 23$

Additional Guidance	
The two correct equations in the same two variables referred to in the scheme are a pair from one of these columns	

<table>
<tr><td>
<table>
<tr><td>$15b - 13c = 21$</td><td>$5a + 3c = -1$</td><td>$13a + 9b = 10$</td></tr>
<tr><td>$5b - 7c = -1$</td><td>$5a + 11c = 23$</td><td>$7a + 11b = 30$</td></tr>
<tr><td>$5b + c = 23$</td><td>$10a + 14c = 22$</td><td>$2a - 14b = -60$</td></tr>
</table>
All equations have equivalents
eg equivalents for $5a + 3c = -1$ include $-5a - 3c = 1$ and $5a = -1 - 3c$
</td><td></td></tr>
<tr><td>All equations in two variables must have terms collected
eg $a + 4a - 2c + 5c = 4 - 5$ requires simplification to $5a + 3c = -1$</td><td></td></tr>
<tr><td>$0a + 15b - 13c = 21$ is equivalent to $15b - 13c = 21$ etc</td><td></td></tr>
<tr><td>Equations with two terms correct include
eg1 (For $5b + c = 23$) $5b + c = 10$ and $-5b - c = 2$ and $5b - 3c = 23$
eg2 (For $5a + 3c = -1$) $5a + 6c = -1$ and $-5a - 3c = 4$ and $5a = 2 - 3c$</td><td></td></tr>
<tr><td>For equations with two terms correct the signs can be ignored if the modulus of the numbers in the correct equation are unchanged
eg For the correct equation $5b - 7c = -1$ (so modulus 5, 7 and 1) equations with two terms correct include
$5b + 7c = 1$ and $5b - 7c = 1$ and $-5b - 7c = 1$ and $-5b - 7c + 1 = 0$</td><td></td></tr>
<tr><td>Up to M3 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td>Elimination of variables may be seen from other approaches
eg rearranges 1st equation to $a = 4 - 3b + 2c$ and substitutes into the 2nd and 3rd equations</td><td></td></tr>
<tr><td>Correct values with no working</td><td>Zero</td></tr>
<tr><td>Matrix method involving row reduction is equivalent to the methods in the mark scheme</td><td></td></tr>
<tr><td>Correct inverse matrix seen with three correct solutions</td><td>M3A2</td></tr>
</table>

Q2.

Answer	Mark	Comments
Alternative method 1 Eliminates b from first two equations before eliminating a second variable		
Correct attempt to eliminate b from LHS of first two equations	M1	eg $2(4a - b + 3c) + 3a + 2b - c$ or $11a + 5c$ adding or subtracting the two equations can be implied from two terms correct
Correct attempt to eliminate a or c from LHS of third equation and their equation in a and c	M1dep	eg $11a + 5c + 2a - 5c$ or $2(11a + 5c) - 11(2a - 5c)$
Correct equation in a or c	M1dep	eg $13a = 52$ or $65c = 195$ implied by $a = 4$ or $c = 3$ with M2
Two correct values with M3	A1	eg $a = 4$ and $c = 3$ with M3
$a = 4$ and $b = -2$ and $c = 3$ with M3	A1	

Alternative method 2 Eliminates a or c before eliminating a second variable		
Two correct attempts to eliminate the same variable (a or c) from LHS	M1	eg (eliminating a) $4a - b + 3c - 2(2a - 5c)$ and $2(3a + 2b - c) - 3(2a - 5c)$ or $-b + 13c$ and $4b + 13c$
Correct attempt to eliminate a second variable from LHS of their two equations	M1dep	eg $-b + 13c - (4b + 13c)$
Correct equation in one variable	M1dep	eg $-5b = 10$ implied by $b = -2$ with M2
Two correct values with M3	A1	eg $b = -2$ and $a = 4$ with M3 or $b = -2$ and $c = 3$ with M3
$a = 4$ and $b = -2$ and $c = 3$ with M3	A1	

Additional Guidance	
For the first two marks ignore the RHS of the equations	
First two method marks may be seen in one attempt eg Alt1 2(4a – b + 3c) + 3a + 2b – c + 2a – 5c	M1M1
Elimination may be seen from other approaches eg1 Alt 1 (equates expressions for 2b from first two equations) 2(4a + 3c – 27) = 5 – 3a + c eg2 Alt 2 (rearranges third equation to a = 2.5c – 3.5 and substitutes into first two equations) 4(2.5c – 3.5) – b + 3c and 3(2.5c – 3.5) + 2b – c	M1 M1
Correct values with no working	M0A0

Q3.

Answer	Mark	Comments
Alternative method 1		
Correct attempt to eliminate two variables from left hand side	M1	eg 2(2a + b − c) − (4a − 3b − 2c)
Correct attempt to eliminate two variables	M1dep	eg 2(2a + b − c) − (4a − 3b − 2c) = 2 × 8 − (− 9) or 5b = 25
Solves their equation	M1dep	eg b = 25 ÷ 5 or b = 5
Substitutes their value into two equations and correct method to eliminate a variable	M1	eg 2a + 5 − c = 8 and 6a + 15 + c = 0 and 8a + 20 = 8
$a = -\frac{3}{2}$ and b = 5 and c = −6	A1	oe

Alternative method 2		
Two correct attempts to eliminate same variable from left hand side	M1	eg 3(2a + b − c) + (4a − 3b − 2c) and 4a − 3b − 2c + 6a + 3b + c
Two correct attempts to	M1dep	eg 3(2a + b − c) + (4a − 3b − 2c)

eliminate same variable		$= 24 - 9$ and $4a - 3b - 2c + 6a + 3b + c = 0 - 9$ or $10a - 5c = 15$ and $10a - c = -9$
Correct attempt to eliminate a variable from their two equations	M1dep	eg $10a - 5c - (10a - c) = 15 - 9$ or $-4c = 24$ or $c = -6$
Substitutes their value into two equations and correct method to eliminate a variable	M1	eg $2a + b + 6 = 8$ and $4a - 3b + 12 = -9$ and $2(2a + b + 6) - (4a - 3b + 12)$ $= 2 \times 8 - (-9)$
$a = -\frac{3}{2}$ and $b = 5$ and $c = -6$	A1	oe

Section 2.17

Mark schemes

Q1.

Answer	Mark	Comments
–3 –2 –1 with no other values	B3	any order B2 –3 –2 –1 with one other value or any two of –3 –2 –1 with no other values or inequality for which the only integer values are –3 –2 –1 eg $-4 < x < 0$ or $-3 \leqslant x \leqslant -1$ or $-4 < x \leqslant -1$ B1 $-4 < x < 4$ or –3 –2 –1 (0) 1 2 3 with no other values

		or one of –3 –2 –1 with no other values or $x^2 < \frac{48}{3}$ or $x^2 < 16$ or $3(x + 4)(x - 4) < 0$ or $(x + 4)(x - 4) < 0$

Additional Guidance	
B1 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
Answer –3 –2 –1 with no other values (no need to check working)	B3
$-4 < x < 0$ is equivalent to the two inequalities $x > -4$ $x < 0$ etc	B2
For B1 allow equivalent factorised inequalities or equivalent inequalities with coefficient 1 for x^2 eg1 $(3x + 12)(x - 4) < 0$ eg2 $3(4 + x)(4 - x) > 0$ eg3 $x^2 - \frac{48}{3} < 0$	B1 B1 B1
(–4, 0) or [–3, –1] etc	B2
(–4, 4)	B1
Only $x > -4$ or only $x < \pm4$ or only $x < 4$	B0
Condone B3 response in working with any inequality on answer line	B3
Condone B3 response in working with 3 on answer line (3 is likely to be the number of integers)	B3
Only invalid inequalities with no or incorrect answer	B0
Only equations with no or incorrect answer	B0

Q2.

$(x - 4)(x - 7)$ or $\frac{--11 \pm \sqrt{(-11)^2 - 4 \times 1 \times 28}}{2 \times 1}$	M1	oe

or $\frac{11}{2} \pm \sqrt{\frac{9}{4}}$		
Identifies 4 and 7	A1	may be on a graph or implied by an inequality using 4 and 7
$x < 4$ $x > 7$	A1	do not allow incorrect notation eg $4 > x > 7$

Additional Guidance	
$x < 4$ with M1 not scored	Zero
$x > 7$ with M1 not scored	Zero
Both $x < 4$ and $x > 7$ in working but only one on answer line	M1A 1A0
$x < 4$ and $x > 7$	M1A2
$x < 4$ and $x > 7$	M1A2

Q3.

Answer	Mark	Comments
Alternative method 1		
$(x \pm a)(x \pm b)$	M1	$ab = 96$ or $a + b = -20$
$(x - 8)(x - 12)$ or $(x =)$ 8 and $(x =)$ 12	A1	
9, 10 and 11	A1	A0 if extra values seen

Alternative method 2		
$(x - 10)^2 - 100$ $(+ 96)$ (< 0)	M1	oe eg $(x - 10)^2 - 4$ (< 0)
$8 < x < 12$ or $(x =)$ 8 and $(x =)$ 12	A1	
9, 10 and 11	A1	A0 if extra values seen

Alternative method 3		
$\frac{20 \pm \sqrt{\{(\ 20)^2 - 4 \times 1 \times 96\}}}{2}$ or	M1	accept $(20)^2$ or $(-20)^2$ for b^2 in the discriminant

$\frac{20 \pm \sqrt{(-20)^2 - 4 \times 1 \times 96}}{2}$		
8 and 12	A1	
9, 10 and 11	A1	A0 if extra values seen

Additional Guidance
9, 10 and 11 using Trial and Improvement - all correct is 3 marks, otherwise 0 marks. No working ... treat as Trial and Improvement For alt 3 ... substitution in the formula must be correct

Q4.

Answer	Mark	Comments
$2x^2 - x - 3$ or $2x^2 - 3x + 2x - 3$	M1	
$4 > - x - 3$	M1dep	oe eg $7 > - x$
$x > -7$ or $-7 < x$	A1	

Additional Guidance	
= used instead of > throughout and not recovered on answer line	M2A0

Q5.

Answer	Mark	Comments
$-18 < 5x$ or $8 - 26 < 5x$ or $-5x < 26 - 8$ or $-5x < 18$ or $x > -3.6$ or $-x < 3.6$	M1	$5x$ or x term isolated on one side of a correct inequality
-3	A1	

Additional Guidance	
Trial and improvement (with no incorrect working) with correct answer. Could be as little as one trial	M1, A1
Trial and improvement with incorrect answer or choice	M0, A0

$-5x < 18$ but $x < -3.6$ (error) answer −3 (common double error, answer should be −4 following the first error)	M1, A0
$8 - 5x = 26$ leading to $x = -3$	M1, A1
$8 - 5x = 26$ not leading to $x = -3$	M0, A0

Q6.

Answer	Mark	Comments
Alternative method 1		
$-\frac{11}{5} < x \le \frac{5}{5}$ or $-2.2 < x \le \frac{5}{5}$	M1	oe eg $x \le \frac{5}{5}$ and $x > -\frac{11}{5}$
$-\frac{11}{5} < x \le 1$ or $-2.2 < x \le 1$ or $-2 \le x \le 1$ or −2, −1, 0, 1	A1	oe eg $x \le 1$ and $x > -\frac{11}{5}$
$6x - 4x \le 4 - 7$ or $2x \le -3$	M1	oe Collects terms
$x \le -\frac{3}{2}$ or $x \le -1.5$ or $x < -\frac{3}{2}$ or $x < -1.5$ or $x \le -2$ or −2, −3 (, −4, ...)	A1	$-2.2 < x \le -1.5$ or $-2 \le x \le -1.5$ implies M1A1M1A1
−2 with no other values given	A1	Must have gained M1A1M1A1

Alternative method 2		
Shows that −2 satisfies either $-11 < 5x \le 5$ or $6x + 7 \le 4x + 4$	M1	eg $-11 < -10 \le 5$ or $5x = -10$ and yes
Shows that −2 satisfies both $-11 < 5x \le 5$ and $6x + 7 \le 4x + 4$	A1	
Shows that −1 does not satisfy $6x + 7 \le 4x + 4$ **or** shows that −3 does not	M1	eg $-6 + 7 > -4 + 4$

satisfy $-11 < 5x \leq 5$		
Shows that −1 does not satisfy $6x + 7 \leq 4x + 4$ **and** shows that −3 does not satisfy $-11 < 5x \leq 5$	A1	
−2 with no other values given	A1	Must have gained M1A1M1A1

Alternative method 3		
$-\frac{11}{5} < x \leq \frac{5}{5}$ or $-2.2 < x \leq \frac{5}{5}$	M1	oe eg $x \leq \frac{5}{5}$ and $x > -\frac{11}{5}$
$-\frac{11}{5} < x \leq 1$ or $-2.2 < x \leq 1$ or $-2 \leq x \leq 1$ or −2, −1, 0, 1	A1	oe eg $x \leq 1$ and $x > -\frac{11}{5}$
Shows that −2 satisfies $6x + 7 \leq 4x + 4$ **or** shows that −1 does not satisfy $6x + 7 \leq 4x + 4$	M1	eg $6 \times -2 + 7 = -5$ and $4 \times -2 + 4 = -4$ ✓
Shows that −2 satisfies $6x + 7 \leq 4x + 4$ **and** shows that −1 does not satisfy $6x + 7 \leq 4x + 4$	A1	
−2 with no other values given	A1	Must have gained M1A1M1A1

Alternative method 4		
$6x - 4x \leq 4 - 7$ or $2x \leq -3$	M1	oe Collects terms

$x \le -\frac{3}{2}$ or $x \le -1.5$ or $x < -\frac{3}{2}$ or $x < -1.5$ or $x \le -2$ or −2, −3 (, −4, ...)	A1	
Shows that −2 satisfies $-11 < 5x \le 5$ **or** shows that −3 does not satisfy $-11 < 5x \le 5$	M1	eg $-11 < -10 \le 5$ or $5x = -10$ and yes
Shows that −2 satisfies $-11 < 5x \le 5$ **and** shows that −3 does not satisfy $-11 < 5x \le 5$	A1	
−2 with no other values given	A1	Must have gained M1A1M1A1

Additional Guidance	
Allow eg max 1 and min −2.2 for $-2.2 < x \le 1$, unless contradicted by a list of values	
Condone omission of non-critical values from lists eg −2, −1, 1	
Using = signs when solving inequalities can score M marks only unless recovered	
Incorrect notation eg ≤ for < can score M marks only	
If answers to trials evaluated they must be correct	
Choose the scheme that favours the student	
−2 identified as the only integer with no valid working	Zero

Q7.

	Answer	Mark	Comments
(a)	$\frac{6}{3} \le w < \frac{18}{3}$ **or** $2 \le w$	M1	

or ..... $w < 6$		
$2 \le w < 6$ **or** $2 \le w \le 5$	A1	
2 3 4 5	A1ft	ft M1 A0 and inequality of form $a \le w < b$ or $a \le w \le b$ SC2 Answer 2 3 4 5 6 **or** 3 4 5 with M0 SC1 Answer 6 9 12 15 with M0 SC1 $\frac{6}{3} < w \le \frac{18}{3}$

(b)

16	B1	

(c)

their min from (a) – 3	M1	
– 1	A1ft	ft their min from (a)

Q8.

Answer	Mark	Comments
Alternative method 1		
$(a + 2)(a - 2)$ or 2 and −2 identified	M1	2 and −2 may be seen on a graph or within inequalities
$8 - 2b < 2$ or $b > 3$	M1	oe Allow any inequality symbol Allow inequality symbol to be = M3 $-2 < 8 - 2b < 2$
$-2 < 8 - 2b$ or $b < 5$	M1	
$3 < b < 5$	A1	SC3 $2 < b < 6$ or $-4 < b < 12$

Additional Guidance	
Both inequalities $b < 5$ and $3 < b$ given as their answer	M3 A1
$a < 2$	M0
$8 - 2b = 2$	M1

$b = 3$	M0 A0
Must use 2 in 2nd M1	
Must use −2 in 3rd M1	
3 or 5 identified implies M1	
3 and 5 identified	M1 M1 M1
Working with = throughout can gain a maximum of M1 M1 M1 A0 unless recovered	
Condone use of any letter other than a	

Alternative method 2		
$(8 - 2b)^2 < 4$	M1	Allow any inequality symbol Allow inequality symbol to be = Must see 4
$64 - 16b - 16b + 4b^2$ or $64 - 32b + 4b^2$ or $60 - 16b - 16b + 4b^2$ or $60 - 32b + 4b^2$	M1	oe Correct expansion or correct expansion − 4
$(2b - 10)(2b - 6)$ or $(b - 5)(b - 3)$ or 3 and 5 identified	M1	Correct factorisation of $60 - 32b + 4b^2$ or correctly substitutes into quadratic formula or correctly completes the square to an equation
$3 < b < 5$	A1	SC3 $2 < b < 6$ or $-4 < b < 12$

Additional Guidance	
Both inequalities $b < 5$ and $3 < b$ given as their answer	M3 A1
Must expand correctly for 2nd M1	
Must factorise correctly for 3rd M1	
3 and 5 identified	M1 M1 M1

Working with = throughout can gain a maximum of M1 M1 M1 A0 unless recovered	
Condone use of any letter other than a	

Section 2.18

Mark schemes

Q1.

Answer	Mark	Comments
$w^{13}x^7 \div w^3x^2$ or $w^{10}x^5$ or $x^2y^5 = w^{10}x^7$ or $y^5 = \frac{w^{10}x^7}{x^2}$ or $w^3y^5 = w^{13}x^5$ or $y^5 = \frac{w^{13}x^5}{w^3}$	M1	oe eg $\frac{w^{13}x^7}{w^3x^2}$ may be embedded eg $\sqrt[5]{w^{10}x^5}$
$w^2x^{(1)}$	A1	oe eg xw^2

Additional Guidance	
$y = w^{10}x^5$	M1A0

Q2.

Answer	Mark	Comments
$\frac{8}{27}x^9y^3$ or $\frac{8x^9y^3}{27}$	B2	oe B1 Two of the three components correct and simplified

Additional Guidance	
Allow multiplication signs for B2 and B1	
Allow $0.\dot{2}9\dot{6}$ or $0.\dot{2}\dot{9}\dot{6}$ as a correct component	
$0.296x^9y^3$	B1

$\frac{8}{27}x^9y^3$ followed by incorrect further work (only penalise B2 responses)	B1
$8x^9y^3 \div 27$	B1
$(\frac{2}{3})^3x^9y^3$	B1
$\frac{8}{27}x^9$	B1
$8x^9 \times 27y^3$	B1
$\frac{8}{27}x^9 + y^3$	B0

Q3.

Answer	Mark	Comments
$q^{-3}(\times)\,r^{-2}$ **or** $\frac{1}{q^3(\times)r^2}$	B2	B1 q^{-3} **or** r^{-2} **or** $(q^6(\times)r^4)^{-\frac{1}{2}}$ **or** $(q^{-6}(\times)r^{-4})^{\frac{1}{2}}$ **or** $\frac{1}{\sqrt{q^6(\times)r^4}}$ **or** $\sqrt{\frac{1}{q^6(\times)r^4}}$ **or** $(q^3(\times)\,r^2)^{-1}$ **or** $p^{-1} = q^3\,(\times)\,r^2$ **or** $\frac{1}{p} = q^3\,(\times)\,r^2$ **or** $p^2 = q^{-6}\,(\times)\,r^{-4}$ **or** $p^2 = \frac{1}{q^6(\times)r^4}$

Q4.

Answer	Mark	Comments
c^{5p} or c^{12} or $5p = 12$	M1	

2.4 or $\frac{12}{5}$ or $2\frac{2}{5}$	A1	oe

Q5.

Answer	Mark	Comments
Alternative method 1		
$a^{\frac{16}{12}}$ or $a^{\frac{4}{3}}$	M1	oe eg $a^{\frac{8}{6}}$
$a^{\frac{10}{12}}$ or $a^{\frac{5}{6}}$	A1	
a^5	A1	

Alternative method 2		
$a^{\frac{18}{4}} \times a^{\frac{42}{12}}$ or $a^{\frac{96}{12}}$ or a^8	M1	oe eg $a^{\frac{9}{2}} \times a^{\frac{7}{2}}$
$\frac{a^8}{a^3}$	A1	oe eg $\frac{a^{\frac{96}{12}}}{a^3}$
a^5	A1	

Q6.

Answer	Mark	Comments
$32c^2d^2$ or $32(cd)^2$	B3	B2 (numerator =) $64c^3d^6$ or single term answer with two of 32, c^2 and d^2 (not in a denominator) B1 single term answer with one of 32, c^2 and d^2 (not in a denominator) SC2 factorised correct expression eg $16cd(2cd)$

Additional Guidance	
$2c^2d^2$ or $32c^2d$ or $32c^2$ or $\frac{32d^2}{c^3}$ or $\frac{c^2d^2}{32}$ or $64(cd)^2$ etc	B2
$32c^3d$ or c^2 or $\frac{d^2}{c}$ or $\frac{c^2d}{32}$ or $\frac{32}{c^2}$ etc	B1
$\frac{32c^2d^2}{1}$ or $\frac{32(cd)^2}{1}$	B2
Allow denominator of 1 in a B2 or B1 answer eg $\frac{32c^2d}{1}$	B2
Multiplication signs in a correct expression eg $32 \times c^2 \times d^2$	B2
Allow multiplication signs in a B2, SC2 or B1 answer eg $32 \times c^3 \times d$	B1
Do not accept 25 for 32 eg $25c^2d$	B1
If answer line scores B1 or B0 check working lines for possible response for up to 2 marks	
$32c^2d^2$ in working with different answer on answer line	B2

Q7.

Answer	Mark	Comments
2 and 3	B1	coefficients
x and x^3	B1	
y and y^4	B1	

Additional Guidance	
$2xy + 3x^3y^4$ or $xy(2 + 3x^2y^3)$ scores B3 If no B marks awarded then $3x^3(2y^{-2} + 3x^2y)$ or $3x^3y(2y^{-3} + 3x^2)$ or $3x^3y^{-2}(2 + 3x^2y^3)$ or $3x^2(2xy^{-2} + 3x^3y)$ or $3x^2y(2xy^{-3} + 3x^3)$ or $3x^2y^{-2}(2x + 3x^3y^3)$ seen in the working for the numerator Penalise incorrect further working for the B marks	B3 SC1

Q8.

Answer	Mark	Comments
0.2 or $\frac{1}{5}$ or 5^{-1}	B2	B1 $125^{-\frac{1}{3}}$ or $\sqrt[-3]{125}$ or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1}{125}}$ or $\frac{1}{125^{\frac{1}{3}}}$ or $\frac{1}{\sqrt[3]{125}}$ or $\left(\frac{1}{5^3}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1}{5^3}}$ or $\frac{1^{\frac{1}{3}}}{5}$ or $\frac{\sqrt[3]{1}}{5}$ or $\frac{1}{y^3} = 125$ or $y^3 = \frac{1}{125}$ or $\frac{1}{y} = 5$ or $\frac{1}{y} = \sqrt[3]{125}$ or $\frac{1}{y} = 125^{\frac{1}{3}}$

Q9.

Answer	Mark	Comments
$7\frac{1}{9} = \frac{64}{9}$	B1	Can be done at any stage
$x^{\frac{2}{3}} = \frac{9}{64}$ or $(\sqrt[3]{x})^2 = \frac{9}{64}$ or $\sqrt[3]{(x^2)} = \frac{9}{64}$	M1	oe or the reciprocals $1 \div x^{\frac{2}{3}} = \frac{64}{9}$ or $\frac{1}{(\sqrt[3]{x})^2} = \frac{64}{9}$ or $\frac{1}{\sqrt[3]{(x^2)}} = \frac{64}{9}$
$x = \left(\frac{9}{64}\right)^{\frac{3}{2}}$	M1	oe or the reciprocals $\frac{1}{x} = \left[\frac{64}{9}\right]^{3/2}$

or $\sqrt[3]{x} = \sqrt{\left[\frac{9}{64}\right]}$ or $x^2 = \left[\frac{9}{64}\right]^3$		or $\frac{1}{\sqrt[3]{x}} = \sqrt{\left[\frac{64}{9}\right]}$ or $\frac{1}{x^2} = \left[\frac{64}{9}\right]^3$
$x = \left(\frac{3}{8}\right)^3$ or $\frac{1}{x} = \left[\frac{8}{3}\right]^3$	A1	
$(x =) \pm \frac{27}{512}$ or $\frac{27}{512}$ or $-\frac{27}{512}$	A1	SC3 for $\frac{512}{27}$

Q10.

Answer	Mark	Comments
6^2 (= 36)	M1	
$\sqrt{x}$ = their 36 − 33	M1	oe
9	A1	

Q11.

Answer	Mark	Comments
8 seen as 2^3 or 16 seen as 2^4	M1	oe eg 2^{3a}
2^{3a} and 2^4 seen	M1	oe eg $2^{3a + 4}$
$a^2 - 3a - 4$ (= 0)	M1	oe equation eg $a^2 = 3a + 4$ ft if all three terms expressed as powers of 2 and a^2 term correct
−1 and 4 with correct method seen	A1	

Additional Guidance	
Trial and improvement or answer(s) only	Zero
First 2 M marks can be awarded even if subsequent method is not clear	
2nd M1 may be implied	

eg $2^{a^2} = 2^{2a}$ $2^3 = 8$ $2^4 = 16$ $2a = 3a + 4$ ($3a + 4$ implies 2nd M1) (a^2 term not correct so 3rd mark is M0 $a = -4$	M1 M1 M0 A0
$16 = 2^4$ $(2^3)^a = 2^{a^3}$ $a^2 = a^3 + 4$	M1 M0 M1 M0

Q12.

Answer	Mark	Comments
x^7	B2	B1 $\sqrt{x^{14}}$ or $(x^{14})^{\frac{1}{2}}$ or $\sqrt{x^{5+9}}$ or $(x^{5+9})^{\frac{1}{2}}$ or $x^{\frac{14}{2}}$ or $x^{\frac{5+9}{2}}$ or $x^{\frac{5}{2}} \times x^{\frac{9}{2}}$ or $x^{2.5} \times x^{4.5}$

Q13.

Answer	Mark	Comments
$(-2)^3$ or -8 seen	B1	
$-\sqrt{x}$ = (their −8) − 3 or $-\sqrt{x}$ − 11 or $\sqrt{x} = 11$	M1	
121	A1	

Additional Guidance
-2^3 (no brackets) is B0 unless −8 seen For M1 it must say $\sqrt{x}$ = ... or $-\sqrt{x}$ = ... Note: ... (their −8) cannot be −2 ... and it must be **correct** manipulation from their −8 eg $3 - \sqrt{x} = (-2)^3$ or $3 - \sqrt{x} = -8$ B1 $\sqrt{x} = -11$ M0 (error in manipulating terms) $x = 121$ A0 (correct answer from wrong

working)

Q14.

Answer	Mark	Comments
$5x^6$ or $(-)6x^5$ or $ax^6 - bx^5$ with $a > 0$ and $b > 0$	M1	
$5x^6 - 6x^5$	A1	

Additional Guidance	
$\frac{5x^6 - 6x^5}{1}$	M1 A0
$\frac{5x^6}{1}$ or $(-)\frac{6x^5}{1}$	M1 A0

Q15.

Answer	Mark	Comments
$\left(\frac{56}{4}\right)^3$ or 14^3 or $4^3x = 56^3$ or $64x = 175\ 616$ or $\frac{56^3}{x} = 4^3$	M1	oe oe equation in $x^{(1)}$ or $\frac{1}{x^{(1)}}$
2744	A1	

Additional Guidance	
$\sqrt[3]{x} = \frac{56}{4}$ or $\sqrt[3]{x} = 14$ with no correct further work	M0
$56x^{-\frac{1}{3}} = 4$	M0
Solving $\frac{56}{3x} = 4$	M0
Answer 14^3 with 2744 not seen in working	M1A0
Embedded solution	M1A0

Q16.

Answer	Mark	Comments
Alternative method 1 Powers of 3		
$(3^2)^{0.5p}$ or $(3^3)^{2p-1}$ or $3^{2\times0.5p+4}$	M1	oe powers of 3 eg 3^p or 3^{6p-3} or 3^{p+4} brackets not needed if intention clear eg $3^{20.5p}$
$(3^2)^{0.5p}$ and 3^4 and $(3^3)^{2p-1}$ or $3^{2\times0.5p+4}$ and $(3^3)^{2p-1}$	M1dep	oe powers of 3 eg 3^p and 3^4 and 3^{6p-3} or 3^{p+4} and 3^{6p-3}
$2 \times 0.5p + 4 = 3(2p - 1)$ or $p + 4 = 6p - 3$	M1dep	oe equation dep on M2
1.4 or $\frac{7}{5}$	A1	oe

Alternative method 2 Powers of 9		
$9^{0.5p+2}$ or $(9^{1.5})^{2p-1}$	M1	oe power of 9 eg $9^{3p-1.5}$ brackets not needed if intention clear eg $9^{1.52p-1}$
9^2 and $(9^{1.5})^{2p-1}$ or $9^{0.5p+2}$ and $(9^{1.5})^{2p-1}$	M1dep	oe powers of 9 eg 9^2 and $9^{3p-1.5}$ or $9^{0.5p+2}$ and $9^{3p-1.5}$
$0.5p + 2 = 1.5(2p - 1)$ or $0.5p + 2 = 3p - 1.5$	M1dep	oe equation dep on M2
1.4 or $\frac{7}{5}$	A1	oe

Alternative method 3 Powers of 27		
$\left(27^{\frac{2}{3}}\right)^{0.5p}$	M1	oe power of 27 eg $27^{\frac{2}{3}\times 0.5p}$ or $27^{\frac{1}{3}p}$ brackets not needed if intention clear eg $27^{\frac{2^{0.5p}}{3}}$
$\left(27^{\frac{2}{3}}\right)^{0.5p}$ and $27^{\frac{4}{3}}$	M1dep	oe powers of 27 eg $27^{\frac{2}{3}\times 0.5p}$ and $27^{\frac{4}{3}}$ or $27^{\frac{1}{3}p}$ and $27^{\frac{4}{3}}$ M2 $27^{\frac{2}{3}\times 0.5p+\frac{4}{3}}$ or $27^{\frac{1}{3}p+\frac{4}{3}}$
$\frac{2}{3}\times 0.5p+\frac{4}{3}=2p-1$ or $\frac{1}{3}p+\frac{4}{3}=2p-1$	M1dep	oe equation dep on M2
1.4 or $\frac{7}{5}$	A1	oe

Alternative method 4 Powers of 81		
$(81^{0.5})^{0.5p}$ or $(81^{0.75})^{2p-1}$ or $81^{0.5\times 0.5p+1}$	M1	oe power of 81 eg $81^{0.25p}$ or $81^{1.5p-0.75}$ or $81^{0.25p+1}$ brackets not needed if intention clear eg $81^{0.50.5p}$
$(81^{0.5})^{0.5p}$ and $(81^{0.75})^{2p-1}$ or $81^{0.5\times 0.5p+1}$ and $(81^{0.75})^{2p-1}$	M1dep	oe powers of 81 eg $81^{0.25p}$ and $81^{1.5p-0.75}$ or $81^{0.25p+1}$ and $81^{1.5p-0.75}$
$0.5\times 0.5p+1=0.75(2p-1)$	M1dep	oe equation

or $0.25p + 1 = 1.5p - 0.75$		dep on M2
1.4 or $\frac{7}{5}$	A1	oe

Additional Guidance	
Mark positively if potentially more than one scheme used	
Answer 1.4	M3 A1
Correct equation implies M3	
Just seeing expressions not in an equation and not as powers scores zero eg Alt 1 $6p - 3$ and $p + 4$ not in an equation and not as powers of 3	M0 M0 M0
Allow recovery of missing brackets	
Use of logs with answer not 1.4 - escalate	

Q17.

Answer	Mark	Comments
Alternative method 1		
$3^{\frac{1}{2}} \times 3^{\frac{1}{2}} + 3^{\frac{1}{2}} \times 3^{\frac{3}{2}} + 3^{\frac{1}{2}} \times 3^{\frac{3}{2}} + 3^{\frac{3}{2}} \times 3^{\frac{3}{2}}$ or $\sqrt{3}\sqrt{3} + \sqrt{3}\sqrt{27} + \sqrt{3}\sqrt{27} + \sqrt{27}\sqrt{27}$	M1	oe allow an error in one term
3 or 9 or 27	M1dep	
48	A1	

Alternative method 2		
$\sqrt{3}$ and $3\sqrt{3}$	M1	$3\sqrt{3}$ must come from correct working
$(4\sqrt{3})^2$	M1dep	
48	A1	

Alternative method 3		
$\left(3^{\frac{1}{2}}\right)^2 (1+3)^2$	M1	oe
3 × 4^2	M1dep	oe
48	A1	

Additional Guidance
Alt 1 mark scheme ... likely to see a 3 (or 9 or 27) somewhere, so need to be careful that the M1 mark has been earned before awarding A1
In alt 1, for the first M1, we want to see an attempt at the full expansion of the correct terms Probably 4 terms, but there could be 3 if they combine the middle two terms. eg (√3 + 27)(√3 + 27) scores M0 because it ought to be √27 not 27

Q18.

Answer	Mark	Comments
$2\sqrt{x}-10=2^3$ or $2\sqrt{x}-10=8$ or $2\sqrt{x}=18$	M1	
$\sqrt{x}=\frac{2^3+10}{2}$ or $\sqrt{x}=\frac{8+10}{2}$ or $\sqrt{x}=9$ or $4x=18^2$ or $x=9^2$	M1dep	
$x=81$	A1	± 81 scores A0

Q19.

Answer	Mark	Comments
$2x^2-3x=7$	M1	at least two terms correct
$2x^2-3x-7$ (= 0)	A1	oe 3-term quadratic equation

$\frac{--3 \pm \sqrt{(-3)^2 - 4 \times 2 \times -7}}{2 \times 2}$ or $\frac{3}{4} \pm \sqrt{\frac{65}{16}}$	M1	oe correct attempt to solve their 3-term quadratic equation
2.77	A1	2.77 and − 1.27 is A0

Q20.

Answer	Mark	Comments
Alternative method 1 Processes the brackets then divides		
$\frac{5x}{10} + \frac{6x}{10}$	M1	oe valid common denominator with both numerators correct eg $\frac{10x}{20} + \frac{12x}{20}$
$\frac{11x}{10}$	A1	oe single term eg $\frac{22x}{20}$ or $1.1x$ may be implied eg by single term with roots evaluated that is equivalent to $\frac{11}{5x^2}$
$\frac{x^{6\div2}}{2}$ or $\frac{x^3}{2}$	M1	may be implied eg by multiplication by $\frac{2}{x^3}$
their $\frac{11x}{10} \times \frac{2}{x^3}$ or $\frac{22x}{10x^3}$ or $\frac{22}{10x^2}$ or $\frac{11x}{5x^3}$ or $\frac{22}{10}x^{-2}$	M1dep	oe multiplication eg $\frac{11x}{10} \times 2x^{-3}$ their $\frac{11x}{10}$ can be unprocessed dep on 2nd M1
$\frac{11}{5x^2}$ or $\frac{11}{5}x^{-2}$ or $2.2x^{-2}$	A1	allow $2\frac{1}{5}x^{-2}$ or $\frac{2.2}{x^2}$

Alternative method 2 Divides then expands the brackets		
$\frac{x^{6\div2}}{2}$ or $\frac{x^3}{2}$	M1	may be implied

		eg by multiplication by $\frac{2}{x^3}$
$\left(\frac{x}{2}+\frac{3x}{5}\right)\times\frac{2}{x^3}$	M1dep	oe multiplication eg $\left(\frac{x}{2}+\frac{3x}{5}\right)\times 2x^{-3}$
$\frac{2x}{2x^3}+\frac{6x}{5x^3}$ or $\frac{1}{x^2}+\frac{6}{5x^2}$	M1dep	oe expansion of brackets
$\frac{10x}{10x^3}+\frac{12x}{10x^3}$ or $\frac{5}{5x^2}+\frac{6}{5x^2}$ or $\frac{22x}{10x^3}$ or $\frac{22}{10x^2}$ or $\frac{11x}{5x^3}$ or $\frac{22}{10}x^{-2}$	M1dep	oe valid common denominator with both numerators correct eg $\frac{10x^4}{10x^6}+\frac{12x^4}{10x^6}$ or $\frac{22x^4}{10x^6}$ roots must be processed
$\frac{11}{5x^2}$ or $\frac{11}{5}x^{-2}$ or $2.2x^{-2}$	A1	allow $2\frac{1}{5}x^{-2}$ or $\frac{2.2}{x^2}$

Additional Guidance	
Any single fraction with roots evaluated that is equivalent to $\frac{11}{5x^2}$	4 marks
Allow inclusion of ± from the square root for up to 4 marks	
$\frac{11}{5x^2}$ in working with answer $\frac{11}{5}x^2$	4 marks
Alt 1 $\frac{11x}{10}$ subsequently squared and not recovered	M1A1 M0M0A0

Q21.

Answer	Mark	Comments
$x + 1 = 6x^2$ or $6x^2 - x - 1$ (= 0)	M1	oe
$(3x + 1)(2x - 1)$ or $\frac{--1\pm\sqrt{(-1)^2-4\times6\times-1}}{2\times6}$	M1dep	

or $\frac{1}{12} \pm \sqrt{\frac{25}{144}}$		
$-\frac{1}{3}$ and $\frac{1}{2}$	A1	oe values

Additional Guidance	
Incorrect quadratic	M0M0A0

Q22.

Answer	Mark	Comments
Alternative method 1 Uses powers of 2		
$(16^x =)\ 2^{4x}$ or $((16^x)^x =)\ (2^4)^{x^2}$	M1	implied by $((16^x)^x =)\ 2^{4x^2}$ may be implied by 3rd M1
$((16^x)^x =)\ 2^{4x^2}$	M1dep	implied by $2^{4x^2 + 3x}$ may be implied by 3rd M1
Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $4x^2 = -3x$ or $4x^2 + 3x = 0$ or $4x = -3$ or $2^{4x^2} = 2^{-3x}$ or $2^{4x^2 + 3x} = 2^0$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Alternative method 2 Uses powers of 16		
$((16^x)^x =)\ 16^{x^2}$ or $\left(\frac{1}{2^{3x}} =\right) \frac{1}{\left(16^{\frac{1}{4}}\right)^{3x}}$	M1	implied by $\left(\frac{1}{2^{3x}} =\right) \frac{1}{16^{\frac{3x}{4}}}$ or $\left(\frac{1}{2^{3x}} =\right) 16^{-\frac{3x}{4}}$ may be implied by 3rd M1

$((16^x)^x =)\ 16^{x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) \frac{1}{16^{\frac{3x}{4}}}$	M1	oe eg $((16^x)^x =)\ 16^{x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) 16^{-\frac{3x}{4}}$ may be implied by 3rd M1
Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $x^2 = -\frac{3}{4}x$ or $4x^2 + 3x = 0$ or $16^{x^2} = 16^{-\frac{3x}{4}}$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Alternative method 3 Uses powers of 4		
$(16^x =)\ 4^{2x}$ or $((16^x)^x =)\ (4^2)^{x^2}$ or $\left(\frac{1}{2^{3x}} =\right) \frac{1}{\left(4^{\frac{1}{2}}\right)^{3x}}$	M1	implied by $((16^x)^x =)\ 4^{2x^2}$ or $\left(\frac{1}{2^{3x}} =\right) \frac{1}{4^{\frac{3x}{2}}}$ or $\left(\frac{1}{2^{3x}} =\right) 4^{-\frac{3}{2}x}$ may be implied by 3rd M1
$((16^x)^x =)\ 4^{2x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) \frac{1}{4^{\frac{3x}{2}}}$	M1dep	oe $((16^x)^x =)\ 4^{2x^2}$ **and** $\left(\frac{1}{2^{3x}} =\right) 4^{-\frac{3}{2}x}$ may be implied by 3rd M1
Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $2x^2 = -\frac{3}{2}x$ or $4x^2 + 3x = 0$ or $4^{2x^2} = 4^{-\frac{3}{2}x}$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Alternative method 4 Takes the xth root of each side and uses powers of 2		
$(16^x =)\ 2^{4x}$ or $16^x = \left(\frac{1}{2^{3x}}\right)^{\frac{1}{x}}$	M1	oe eg $16^x = \sqrt[x]{\frac{1}{2^{3x}}}$ or $16^x = \frac{1}{2^3}$ or $16^x = 2^{-3}$ may be implied by 3rd M1
$2^{4x} = \left(\frac{1}{2^{3x}}\right)^{\frac{1}{x}}$	M1dep	oe eg $2^{4x} = \frac{1}{2^3}$ may be implied by 3rd M1
Correct quadratic equation or correct linear equation or correct equation involving indices with the same base	M1dep	eg $4x = -3$ or $2^{4x} = 2^{-3}$ do not allow if the equation is from incorrect working do not allow if the only equation is $x = -\frac{3}{4}$
M3 and $-\frac{3}{4}$	A1	oe ignore inclusion of answer 0

Additional Guidance	
Up to M2 may be awarded for correct work with no, or incorrect answer, even if this is seen amongst multiple attempts	
Allow $2^{4\times x\times x}$ for 2^{4x^2} etc	
Responses using other powers eg powers of 8 can be escalated	Escalate
Ignore simplification or conversion if correct answer seen	

Q23.

Answer	Mark	Comments
a^{4m} or a^{10m} or $4m = 10m$	M1	oe eg $a^{4\times m}$
0	A1	

Additional Guidance

Allow *a* to be replaced by any value greater than 1	

Section 2.19

Mark schemes

Q1.

Answer	Mark	Comments
$5n^2 – 5n + 3n − 3$	M1	oe 4 terms with 3 correct including a term in n^2
$5n^2 – 5n + 3n − 3$	A1	Fully correct oe eg $5n^2 – 2n − 3$
$6n^2 – 3$	A1	
$3(2n^2 – 1)$ **or** states that both terms are multiples of 3	A1	oe

Q2.

Answer	Mark	Comments
Alternative method 1 Expands the given brackets		
$((2n + 1)^2 =)\ 4n^2 + 2n + 2n + 1$ or $((2n – 1)^2 =)\ 4n^2 – 2n – 2n + 1$	M1	oe expansion eg $((2n + 1)^2 =)\ 4n^2 + 4n + 1$ may be seen in a grid may be seen embedded in second mark ignore any denominator
$4n^2 + 2n + 2n + 1 – 4n^2 + 2n + 2n – 1$ or $4n^2 + 4n + 1 – 4n^2 + 4n – 1$ or $4n^2 + 2n + 2n + 1$ $– (4n^2 – 2n – 2n + 1)$ **and** $8n$ with no errors seen or	M1dep	terms in any order ignore any denominator

$4n^2 + 4n + 1 - (4n^2 - 4n + 1)$ **and** $8n$ with no errors seen		
M2 seen and valid explanation	A1	eg1 M2 seen and $\frac{8n}{4} = 2n$ eg2 M2 seen and $8n$ is even and when divided by 4 it is even

Alternative method 2 Difference of two squares		
$(2n + 1 + 2n - 1)(2n + 1 - (2n - 1))$ or $(2n + 1 + 2n - 1)(2n + 1 - 2n + 1)$	M1	ignore any denominator
M1 seen and $4n \times 2$ with no errors seen	M1dep	ignore any denominator
M2 seen and valid explanation	A1	eg1 M2 seen and $\frac{4n \times 2}{4} = 2n$ eg2 M2 seen and $\frac{8n}{4} = 2n$ eg3 M2 seen and $8n$ is even and when divided by 4 it is even

Additional Guidance	
Do not allow missing brackets even if recovered	
Alt 1 $4n^2 + 4n + 1 - 4n^2 - 4n + 1$	M1M0
Alt 1 $4n^2 + 2n + 2n + 1 - (4n^2 - 2n - 2n + 1)$ $= 4n^2 + 4n + 1 - 4n^2 - 4n - 1 = 8n$ ($8n$ but error seen)	M1 M0
Alt 1 Only $8n$	M0M0
Alt 1 2nd M1 Allow unnecessary brackets eg $4n^2 + 4n + 1 - (4n^2 - 4n + 1) = (4n^2 - 4n^2) + (4n + 4n) + (1 - 1)$	M1M1
Alt 2 $(2n + 1 + 2n - 1)(2n + 1 - 2n - 1)$	M0M0
Alt 2 $(2n + 1 + 2n - 1)(2n + 1 - (2n - 1))$ $= (2n + 1 + 2n - 1)(2n + 1 - 2n - 1) = 4n \times 2$ ($4n \times 2$ but error seen)	M1 M0

Alt 2 $(2n + 1 + 2n - 1)(2n + 1 - (2n - 1)) = 8n$	M1M0
Alt 2 Only $4n \times 2$	M0M0
Response only referring to odds and evens or only involving substitution	M0M0A0
Assuming the expression simplifies to $2k$ and working back could score up to M1M1	
Setting up an equation eg $(2n + 1)^2 - (2n - 1)^2 = 4$ could score up to M1M1	
For A1 do not allow incorrect use of = eg $4n^2 + 2n + 2n + 1 - 4n^2 + 2n + 2n - 1$ $= \frac{8n}{4} = 2n$	M1M1 A0

Q3.

Answer	Mark	Comments
$4n^2 + 6n + 6n + 9$ or $4n^2 + 12n + 9$	M1	allow one error implied by $4n^2 + 12n + k$ or $an^2 + 12n + 9$
$8n^3 + 12n^2 + 24n^2 + 36n + 18n + 27$	M1dep	oe ft their $4n^2 + 6n + 6n + 9$ allow one error
$8n^3 + 36n^2 + 54n + 27$ or $9n^3 + 36n^2 + 54n + 27$	A1	
$9n^3 + 36n^2 + 54n + 27$ and $9(n^3 + 4n^2 + 6n + 3)$	A1	oe eg $(9n^3 + 36n^2 + 54n + 27) \div 9$ $= n^3 + 4n^2 + 6n + 3$ or $9n^3 + 36n^2 + 54n + 27$ and all coefficients are divisible by 9

Q4.

Answer	Mark	Comments
$(5n - 3)^2 + 1$	M1	
$25n^2 – 15n – 15n + 9 + 1$	M1	Allow 1 error Must have an n^2 term
$25n^2 – 30n + 10$	A1	
$5 (5n^2 – 6n + 2)$	B1ft	oe e.g., shows that all terms divide by 5 or explains why the expression is a multiple of 5

Alternative method 1		
Use of $an^2 + bn + c$ for terms of quadratic sequence i.e., any one of $a + b + c = 5$ $4a + 2b + c = 50$ $9a + 3b + c = 145$	M1	
$3a + b = 45$ $5a + b = 95$	M1	For eliminating c
$25n^2 – 30n + 10$	A1	
$5(5n^2 – 6n + 2)$	B1ft	oe e.g., shows that all terms divide by 5 or explains why the expression is a multiple of 5

Alternative method 2		
5 50 145 290 45 95 145 2nd difference of 50 ÷ 2 (= 25)	M1	$25n^2$
Subtracts their $25n^2$ from terms of sequence –20 –50 –80	M1	$–30n$
$25n^2 – 30n + 10$	A1	
$5(5n^2 – 6n + 2)$	B1ft	oe

		e.g., shows that all terms divide by 5 or explains why the expression is a multiple of 5

Q5.

Answer	Mark	Comments
Alternative method 1		
$8(c^2 + 2)$ or $3(c^2 + 2)$	M1	
$\frac{8(c^2+2)}{3(c^2+2)}$	A1	
$\frac{8}{3} + \frac{1}{3} = 3$	A1	
Alternative method 2		
Converts to a valid common denominator with at least one numerator correct eg1 $\frac{3(8c^2+16)}{3(3c^2+6)} + \frac{3c^2+6}{3(3c^2+6)}$ eg2 $\frac{8c^2+16+c^2+2}{3c^2+6}$	M1	oe Other valid common denominators include $9c^2 + 18$ and $3(c^2 + 2)$
Makes into a single fraction with terms collected eg1 $\frac{27c^2+54}{3(3c^2+6)}$ eg2 $\frac{9c^2+18}{3c^2+6}$	A1	oe
Shows that fraction simplifies to 3	A1	oe Must see a correct common quadratic factor and = 3

eg1 $\frac{9(3c^2+6)}{3(3c^2+6)}=3$ eg2 $\frac{3(3c^2+6)}{3c^2+6}=3$ eg3 $\frac{9(c^2+2)}{3(c^2+2)}=3$		

Additional Guidance
Answer of 3 does not gain marks without correct working for M1 A1 (1st) seen
Do not allow $\frac{3}{1}$ unless subsequently becomes 3

Q6.

Answer	Mark	Comments
Alternative method 1		
$9x^2 + 15x + 15x + 25 - 50x$ or $9x^2 + 30x + 25 - 5x^2 - 50x$ or $9x^2 + 15x + 15x + 25$ and $-5x^2 - 50x$ or $5x^2 + 50x$	M1	allow only one error in sign, omission or coefficient but not in more than one of these could be written as 2 separate expansions or in a grid
$4x^2 - 20x + 25$	A1	
$4x^2 - 20x + 25$ **and** $(2x - 5)^2$ or $(2x - 5)(2x - 5)$ or $4(x - 2.5)^2$ or $x = 2.5$ or $b^2 - 4ac = 0$ from quadratic formula	M1dep	factorises or completes the square or uses the quadratic formula correctly. Answer required for M1 dep
$(2x - 5)^2$ or $4(x - 2.5)^2$ (are squared terms) and so are always ≥ 0	A1	oe there must be a stated conclusion eg equal roots and positive quadratic so must be

		greater than or equal to zero

Alternative method 2		
$9x^2 + 15x + 15x + 25 - 50x$ or $9x^2 + 30x + 25 - 5x^2 - 50x$ or $9x^2 + 15x + 15x + 25$ and $-5x^2 - 50x$ or $5x^2 + 50x$	M1	allow only one error in sign, omission or coefficient but not in more than one of these could be written as 2 separate expansions or in a grid
$4x^2 - 20x + 25$	A1	
$4x^2 - 20x + 25$ and $\frac{d}{dx} = 8x - 20$ and is zero when $x = 25$	M1dep	uses calculus to find stationary point
Tests for minimum by using eg $x = 2$ and $x = 3$ **or** by using 2nd derivative or concludes argument by saying this is a positive quadratic curve with minimum point (2.5, 0), hence always ≥ 0	A1	oe there must be a stated conclusion

Q7.

Answer	Mark	Comments
Alternative method 1		
$2(2 - 5x) + 3(3x - 1)$ or $4 - 10x$ or $9x - 3$	M1	
$4 - 10x + 9x - 3 = 1 - x$	M1dep	
$(1 - x)^2 = 1 - 2x + x^2$	A1	must see working for M2
$2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$	B1	

Alternative method 2

$4(2 - 5x)^2 + 6(2 - 5x)(3x - 1)$ $+ 6(2 - 5x)(3x - 1) + 9(3x - 1)^2$	M1	oe allow $+ 12(2 - 5x)(3x - 1)$ for $+ 6(2 - 5x)(3x - 1) + 6(2 - 5x)(3x - 1)$
$4(4 - 10x - 10x + 25x^2)$ $+ 6(6x - 2 - 15x^2 + 5x)$ $+ 6(6x - 2 - 15x^2 + 5x)$ $+ 9(9x^2 - 3x - 3x + 1)$ $= 16 - 40x - 40x + 100x^2 + 36x - 12$ $- 90x^2 + 30x + 36x - 12 - 90x^2$ $+ 30x + 81x^2 - 27x - 27x + 9$	M1dep	oe must see expansions must see working for 1st M1 allow $+ 12(6x - 2 - 15x^2 + 5x)$ for $+ 6(6x - 2 - 15x^2 + 5x)$ $+ 6(6x - 2 - 15x^2 + 5x)$
$1 - 2x + x^2$	A1	must see working for M2
$2 - 5x + 3x - 1 + x2 = 1 - 2x + x^2$	B1	

Alternative method 3		
$2(2 - 5x) + 3(3x - 1)$ or $4 - 10x$ or $9x - 3$	M1	oe
$(4 - 10x + 9x - 3)^2$ $= 16 - 40x + 36x - 12 - 40x + 100x^2$ $- 90x^2 + 30x + 36x - 90x^2 + 81x^2$ $- 27x - 12 + 30x - 27x + 9$	M1dep	oe must see expansions
$1 - 2x + x^2$	A1	must see working for M2
$2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$	B1	

Additional Guidance	
Allow working down both sides of an equation/identity	
M2A1 is for working on $(2A + 3B)^2$	

B1 is for working on $A + B + C$	
$1 - 2x + x^2$ with working for M2 seen and $2 - 5x + 3x - 1 + x^2 = x^2 - 2x + 1$	4 marks
$1 - x^2 = 1 - 2x + x^2$ (do not allow missing brackets even if recovered)	

Section 2.20 – 2.21

Mark schemes

Q1.

	Answer	Mark	Comments
(a)	$33n^2 = 32(n^2 + 2)$ or $\frac{64-n^2}{11n^2+22}=0$	M1	oe (both denominators should be cleared for the first method)
	8	A1	ignore –8 in working as long as only 8 stated in answer

Additional Guidance	
May use T&I and will be 2 marks if they get the correct answer (0 marks without the answer)	

(b)	3	B1	

Additional Guidance	
Condone $\frac{3}{1}$	

Q2.

	Answer	Mark	Comments
(a)	$1420 - 5n = 0$ or $5n = 1420$ or $\frac{1420}{5}$	M1	oe eg $5(284 - n) = 0$
	284	A1	

Additional Guidance	
$\frac{1420 - 5n}{1420 + 5n} = 0$	Zero
$1420 - 5n = 0(1420 + 5n)$	Zero
$n = 284$	M1A1
$1420 - 5n = 0$ and $1420 + 5n = 0$ with correct equation not selected	Zero
±284 is A0	
Embedded answer	M1A0

(b)

−1	B1	

Additional Guidance	
$-\frac{5}{5}$	B0
$-1 \quad n \longrightarrow \infty$	B1
$-1 \longrightarrow \infty$	B0
$x \longrightarrow -1$ (any letter other than n)	B1

Q3.

	Answer	**Mark**	**Comments**
(a)	105 (numerator) or 145 (denominator)	M1	
	$\frac{21}{29}$	A1	

(b)

Alternative method 1		
$\dfrac{2 + \frac{7}{n^2}}{3 - \frac{2}{n^2}}$	M1	
$\frac{7}{n^2}$ and $\frac{1}{n^2}$ both $\longrightarrow 0$ as $n \longrightarrow \infty$	A1	

Alternative method 2		
as $n \longrightarrow \infty \quad 2n^2 + 7 \longrightarrow 2n^2$ and $3n^2 - 2 \longrightarrow 3n^2$	B1	
limiting value is $\frac{2n^2}{3n^2} = \frac{2}{3}$	B1	

Q4.

	Answer	Mark	Comments
(a)	$32n > 11(3n - 7)$	M1	allow $32n = 11(3n - 7)$
	$32n > 33n - 77$ or $77 > n$	M1dep	oe must be correct inequality unless recovered
	76	A1	

Additional Guidance	
$n = 77$ with final answer 76	M2A1
$n = 77$ with final answer not 76	M1M0A0

(b)	$\frac{32}{3}$	B1	oe value

Additional Guidance	
Ignore conversion to decimal if $\frac{32}{3}$ seen	

Q5.

Answer	Mark	Comments
Alternative method 1		
$(n - 3)^2$	M1	Allow $(n - 3)(n - 3)$ for $(n - 3)^2$
$(n - 3)^2 - 9 + 14$ or	A1	Allow $(n - 3)(n - 3)$ for $(n - 3)^2$

$(n - 3)^2 + 5$		
$(n - 3)^2 \geq 0$ then adding 5 so always positive or States minimum value is 5 or States (3, 5) is minimum point	A1ft	oe Allow $(n - 3)(n - 3)$ for $(n - 3)^2$ ft M1 A0 Must see M1 and attempt $(n - 3)^2 + k$ ft $(n - 3)^2 + k$ where $k > 0$ SC2 States minimum value is 5 or States (3, 5) is minimum point

Alternative method 2		
Quadratic curve sketched in first quadrant with minimum point above the x-axis	M1	Labelling on axes not required
(discriminant =) −20	A1	
States no (real) roots	A1ft	oe Allow roots ⟶ solutions ft M1 A0 Must see M1 and attempt a discriminant ft discriminant < 0 SC2 States minimum value is 5 or States (3, 5) is minimum point

Alternative method 3		
$2n - 6 = 0$	M1	oe equation e.g. $2n = 6$ or $n = 3$
(second derivative =) 2	A1	
States minimum value is 5 or States (3, 5) is minimum point	A1ft	oe ft M1 A0 Must see M1 and attempt a second derivative ft (second derivative) > 0 SC2 States minimum value is 5

		or States (3, 5) is minimum point

Q6.

(a)

Answer	Mark	Comments
Alternative method 1		
1 − a + 2a = 1 + a and 3(1 + a) = 3 + 3a	B1	oe Allow 3(1 − a + 2a) = 3 + 3a if no incorrect working seen
Alternative method 2		
$\frac{3 + 3a}{3}$ = 1 + a and 1 + a − 2a = 1 − a	B1	oe

Additional Guidance	
Allow 1a for a throughout	
Alt 1 a + 2a = 3a 3 × 1 = 3 3 + 3a (incorrect working seen)	B0
Alt 1 −a + 2a = a 3 × a = 3a 3 × 1 = 3 3 + 3a	B1
3(1 + a) = 3 + 3a	B0
Alt 1 1 − a + 2a = 1 + a 3 × 1 + a = 3 + 3a (incorrect working seen)	B0
Alt 1	B1

$1 - a + 2a = 1 + a$ $1 + a$ $\frac{\times 3}{3 + 3a}$	
Must use algebra	

(b)

Alternative method 1		
$9 + 15a$ or $3(3 + 5a)$ or $3(3 + 3a + 2a)$	M1	oe
their $(9 + 15a) = 16$ and their $15a = 16 -$ their 9	M1	Must expand any brackets correctly and collect terms correctly their $(9 + 15a)$ must be at least two terms
$\frac{7}{15}$ or $0.4\dot{6}$ or 0.47	A1ft	ft from M1 M0 or M0 M1 with 1 error Allow 0.466... or 0.467 SC1 $\frac{13}{3}$ or 4.33… oe

Additional Guidance	
$\frac{7}{15}$ (may be seen in working) with subsequent attempt at evaluation	M1 M1 A1
$3(3 + 5a) = 16$ $9 + 5a = 16$ (error in expansion) $5a = 7$ $a = 1.4$ (1 error)	M1 M0 A1ft
$3(3 + 5a) = 16$ $6 + 15a = 16$ (error in expansion) $15a = 22$ (error in collection) $a = \frac{22}{15}$ (2 errors)	M1 M0 A0ft
May just state a 3rd term but cannot use $3 + 3a$ for the 3rd term $9 + 8a = 16$	M0

$8a = 7$ (no brackets to expand and collects term correctly) $a = \frac{7}{8}$ (2 errors)	M1 A1ft
For A1ft accept answers rounded to at least 2sf if not an integer	
$3(3 + 5a) = 6 + 5a$ is two errors so not possible to award A1ft	
$1 - a = 16$	M0 M0 A0

Alternative method 2		
$3(3 + 5a)$ or $3(3 + 3a + 2a)$	M1	oe
their $(3 + 5a) = \frac{16}{\text{their } 3}$ and their $5a = \frac{16}{\text{their } 3} -$ their 3	M1	Must divide by their 3 correctly and collect terms correctly their $(3 + 5a)$ must be at least two terms
$\frac{7}{15}$ or $0.4\dot{6}$ or 0.47	A1	ft from M1 M0 or M0 M1 with 1 error Allow 0.466... or 0.467 SC1 $\frac{13}{3}$ or 4.33… oe

Additional Guidance	
$\frac{7}{15}$ (may be seen in working) with subsequent attempt at evaluation	M1 M1 A1
$3(3 + 5a) = 16$ $9 + 5a = \frac{16}{3}$ (error in division by 3) $5a = \frac{16}{3} - 9$ $a = -\frac{11}{15}$ (1 error)	M1 M0 A1ft
$3(3 + 5a) = 16$ $9 + 5a = \frac{16}{3}$ (error in division by 3) $5a = \frac{16}{3} + 9$ (error in collection)	M1 M0 A0ft

$a = \frac{43}{15}$ (2 errors)	
For A1ft accept answers rounded to at least 2sf if not an integer	
3(3 + 5a) = 6 + 5a is two errors so not possible to award A1ft	

Q7.

Answer	Mark	Comments
$7 + 12\sqrt{5} + 6(9 - 2\sqrt{5})$ or $12\sqrt{5} + 6(-2\sqrt{5}) = 0$ or $12\sqrt{5} \div 2\sqrt{5} = 6$ or states that need to add 6 lots of $(9 - 2\sqrt{5})$ or 7th term	M1	oe eg 7 + 6 × 9 or 7 + 54 or 6 × −2 = −12 allow $7 + 12\sqrt{5} + (n - 1)(9 - 2\sqrt{5})$ with $n = 7$ allow $7 + 12\sqrt{5} + n(9 - 2\sqrt{5})$ with $n = 6$

Additional Guidance	
61 in working lines with 7(th) on answer line	M1 A0
If repeatedly adding $(9 - 2\sqrt{5})$ they must stop after adding 6 lots or clearly select the relevant one	
Answer 6 or 6th term with M1 not seen	M0 A0
Ignore any conversions to decimals	
Beware $(9 - 2\sqrt{5})(9 + 2\sqrt{5}) = 61$	M0 A0

Q8.

Answer	Mark	Comments
$(n = 1)\quad 4a = \frac{10 \times 1 - 2}{3}$	M1	$(n = 2)\quad 9a = \frac{10 \times 2 - 2}{3}$ **or**

		$(n = 3)\quad 14a = \dfrac{10 \times 3 - 2}{3}$ **or** $(n = 4)\quad 19a = \dfrac{10 \times 4 - 2}{3}$
$\dfrac{2}{3}$	A1	oe

Alternative method		
$5an - a = \dfrac{10n - 2}{3}$	M1	oe
$\dfrac{2}{3}$	A1	oe

Q9.

Answer	Mark	Comments
$k^2 = 2(14k + 30)$	M1	oe correct equation with fractions eliminated
$k^2 - 28k - 60$ (= 0)	M1dep	oe equation
$(k + 2)(k - 30)$ (= 0) or $\dfrac{--28 \pm \sqrt{(-28)^2 - 4 \times 1 \times -60}}{2 \times 1}$ or $14 \pm \sqrt{256}$	M1	oe correct attempt to solve their 3-term quadratic equation
30	A1	30 and −2 is A0

Q10.

(a)

Answer	Mark	Comments
$30 + 12k$ or $12k + 30$	B1	allow factorised eg $6(5 + 2k)$

Additional Guidance	
$30 + 12k$ seen in working but incorrect answer eg $5 + 2k$ or –2.5	B0
Answer line $30 + 12k$ and expression for the nth term eg $30 + 4nk – 4k$	B0
$30 + 8k + 4k$	B0

30 + 12k unambiguously indicated as 4th term (eg in given sequence) with answer line blank	B1

Alternative method 1 Works out a correct expression for the 100th term		
30 + 99 × 4k or 30 + 396k or 100 × 4k + 30 – 4k	M1	oe eg 30 + (100 – 1) × 4k or 30 + 4k + 98 × 4k or 30 + 8k + 97 × 4k or 30 + 12k + 96 × 4k
99 × 4k = 525 – 30 or 396k = 495 or 495 ÷ 396	M1dep	oe terms must be collected in an equation eg 396k – 495 = 0
1.25 or $\frac{5}{4}$ or $1\frac{1}{4}$	A1	oe eg $\frac{495}{396}$

Alternative method 2 Uses a common difference (eg d)		
30 + 99 × d or 30 + 99d	M1	oe eg 30 + (100 – 1) × d
$4k = \frac{525-30}{99}$ or $4k = \frac{495}{99}$ or 4k = 5 or 5 ÷ 4	M1dep	oe terms must be collected in an equation eg 4k – 5 = 0
1.25 or $\frac{5}{4}$ or $1\frac{1}{4}$	A1	oe eg $\frac{495}{396}$

Alternative method 3 Uses their (a) to work out an expression for the 100th term		
their (a) + 96 × 4k or their (a) + 384k	M1	their (a) must be in terms of k their (a) cannot be 30 + 4k or 30 + 8k
Collection of terms for their (a) + 384k = 525	M1dep	their (a) must be of the form c + dk $c \neq 0$ $d \neq 0$
Solution to their equation rounded to 1 dp or better	A1ft	ft their (a) and M2

Additional Guidance

Ignore simplification or conversion if correct answer seen	
Alt 1 Do not allow M1 if seen embedded eg in formula for S_n	
Alt 3 (a) $12k$ (b) $12k + 384k$ $396k = 525$ 1.326	M1M0A0ft
Alt 3 (a) $30 + 16k$ (b) $30 + 16k + 384k$ $400k = 525 – 30$ 1.238	M1M1A1ft
Alt 3 (a) $12k + 60$ (b) $12k + 60 + 96 × 4k$ $396k = 525 – 60$ 1.2	M1M1A1ft

Section 2.22
Mark schemes

Q1.

Answer	Mark	Comments
Alternative method 1		
2nd difference = 8 **or** $a = 4$	M1	sight of $4n^2$ implies this mark
subtract their $4n^2$ **or** sight of three of 6 17 28 39	M1	subtracting 4 16 36 64 the coefficient of their $4n^2$ will come from half the value of their 2nd difference
subtract their $11n$ **or** $b = 11$ **or** tests $4n^2 + 11n$ and compares to original sequence **or** sight of three of 15 38 69 108	M1dep	dep on 2nd M mark
$4n^2 + 11n − 5$	A1	

Alternative method 2		
Any three of these $a + b + c = 10$ $4a + 2b + c = 33$ $9a + 3b + c = 64$ $16a + 4b + c = 103$	M1	

Any two of these $3a + b = 23$ $5a + b = 31$ $7a + b = 39$	M1dep	
$a = 4$ and $b = 11$	A1	
$4n^2 + 11n - 5$	A1	

Alternative method 3		
$a = 4$	M1	
$3a + b = 33 - 10$ **and** substitutes their a in this equation	M1	oe
$b = 11$	A1	
$4n^2 + 11n - 5$	A1	

Additional Guidance
SC3 for $4n^2 - 11n + 5$ Condone $4x^2 + 11x - 5$ **or** eg $4x^2 + 11n - 5$ (mixed letters)

Q2.

Answer	Mark	Comments
Alternative method 1		
(Second differences =) – 2 or $-n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
0 – – 1 1 – – 4 0 – – 9 (– 3 – – 16) or 1 5 9 (13) or – 1 – 0 – 4 – 1 – 9 – 0 (– 16 – – 3) or –1 –5 – 9 (–13)	M1dep	subtracts $-n^2$ from the given terms or subtracts the given terms from $-n^2$
$-n^2 + 4n - 3$	A1	oe eg $4n - 3 - n^2$

Alternative method 2		
Any three of	M1	using nth term = $an^2 + bn + c$

$a + b + c = 0$ $4a + 2b + c = 1$ $9a + 3b + c = 0$ $16a + 4b + c = -3$		
$3a + b = 1$ and $5a + b = -1$ or $a = -1$ and $b = 4$	M1dep	oe obtains two equations in the same two variables
$-n^2 + 4n - 3$	A1	oe eg $4n - 3 - n^2$

Alternative method 3		
(Second differences =) – 2 or $-n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
$3a + b = 1$ and substitutes $a = -1$	M1dep	oe eg $-3 + b = 1$ or $b = 4$
$-n^2 + 4n - 3$	A1	oe eg $4n - 3 - n^2$

Additional Guidance	
Condone use of U_n	M2A1
Condone working in different variable(s) eg $-n^2 + 4x - 3$	M2A1
Answer $-n^2 \ldots$ scores at least M1	
Condone $-n^2 + 4n - 3 = 0$ or $n = -n^2 + 4n - 3$	M2A1

Q3.

	Answer	Mark	Comments
(a)	**Alternative method 1 (grid)**		
	1 5 9 4 4 and $2n^2$	M1	
	–4 –9 (–14 –19) and $-5n$ (+ c)	M1dep	subtract $2n^2$

$2n^2 - 5n + 1$	A1	

Alternative method 2 (simultaneous equations)		
Any 3 of: $a + b + c = -2$ $4a + 2b + c = -1$ $9a + 3b + c = 4$ $16a + 9b + c = 13$	M1	using n^{th} term = $an^2 + bn + c$
$3a + b = 1$ or $5a + b = 5$	M1dep	or any other equation with an unknown eliminated
$b = -5$, $c = 1$ so $2n^2 - 5n + 1$	A1	

Alternative method 3 (using terms)		
1 5 9 4 4 so $a = 2$	M1	using n^{th} term = $an^2 + bn + c$
$3a + b = 1$ and $a = 2$ substituted in this equation	M1dep	oe
$b = -5$, $c = 1$ so $2n^2 - 5n + 1$	A1	

Additional Guidance	
Condone other letters used eg $2x^2 - 5x + 1$ or even $2n^2 - 5x + 1$	
After finding $a = 2$ they may find the 0th term to get $c = 1$	M2
$2n^2 + 5n - 1$ from Alt 1 but subtracting the wrong way round	SC2

(b)

$n^2 + 10n - 2000 < 0$	M1	the correct inequality needed for this mark and must be written in this form
$(n - 40)(n + 50)$ or $(n + 5)^2 - 25 - 2000$ or $\frac{-10 \pm \sqrt{10^2 - 4 \times 1 \times -2000}}{2}$	M1	oe inequality not needed for this mark condone + instead of ± as the negative solution has no meaning here

39	A1	

Additional Guidance	
Do not accept T&I	
Incorrect use of inequalities can be recovered by a correct use of inequalities later in the method such as n < 40 near the end	M2
Incorrect use of inequalities can be recovered for full marks. An answer of 39 after a method that uses an incorrect inequality or = shows inequality has been recovered	M2A1
An incorrect solution with incorrect use of inequalities can only be awarded the second M mark	M0M1A0
Correct answer not coming from correct working will not gain any marks	M0A0
For students who try to complete the square accept $(n + 5)^2 < 2025$ as an oe giving M2 but $(n + 5)^2 = 2025$ would only gain M0M1 unless recovered in the answer	

Q4.

Answer	Mark	Comments
(a) **Alternative method 1**		
Second differences −4	M1	Implied by $-2n^2$
Subtracts $\frac{\text{their} - 4}{2}n^2$ from given sequence or 304 608 912	M1	At least 3 correct values implies correct method (next term is 1216)
$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter
Alternative method 2		
Any 3 of $a + b + c = 302$ $4a + 2b + c = 600$ $9a + 3b + c = 894$ $16a + 4b + c = 1184$	M1	Using $an^2 + bn + c$
Correctly eliminates the same letter using two different pairs of equations	M1	

eg $3a + b = 600 - 302$ and $5a + b = 894 - 600$		
$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter Allow $a = -2$ $b = 304$ $c = 0$ if $an^2 + bn + c$ seen earlier

Additional Guidance	
Condone mixed letters and/or inclusion of = 0 eg1 $-2n^2 + 304x$ eg2 $-2n^2 + 304n = 0$	M1M1A1 M1M1A1
Alt 1 2nd differences = 4 300 592 876 1152	 M0 M1 A0

Alternative method 3		
$a = -2$	M1	Using $an^2 + bn + c$
$3a + b = 600 - 302$ and substitutes their a	M1	oe eg $b = 304$ May also see $a + b + c = 302$ used to obtain c
$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter

Alternative method 4		
Second differences −4	M1	
$302 + (600 - 302)(n - 1) +$ $0.5 \times$ their $-4(n - 1)(n - 2)$	M1	Using $a + d(n - 1) + 0.5c(n - 1)(n - 2)$ a is 1st term d is 2nd term − 1st term c is second differences
$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$

		Allow any letter

Additional Guidance	
Condone mixed letters and/or inclusion of = 0 eg1 $-2n^2 + 304x$ eg2 $-2n^2 + 304n = 0$	 M1 M1 A1 M1 M1 A1

(b)	$n(-2n + 304)$ or $2n(-n + 152)$ or $2n = 304$	M1	oe Factorises correctly to two linear factors or substitutes correctly in quadratic formula or correctly completes the square to a correct equation or simplifies to $an = b$ ft their quadratic
	152	A1	

Additional Guidance	
152 and 0	M1 A0
M1 Factorising may be seen after division eg if (a) correct $n(-n + 152)$	 M1
Their quadratic must have at least two terms for M1	
Only ft for M1 A0	
If their quadratic in (a) is incorrect, check for M1 A0 using their answer (correct to at least 1dp) if method not shown	
Do not award M1 if their quadratic from (a) has solution $n = 0$	